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EFFECT OF THERMAL MODULATION ON CONVECTIVE INSTABILITY IN ELASTICO-VISCOUS FLUID SATURATED LAYER

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Abstract

The effect of thermal modulation on the onset of convection in a horizontal porous layer saturated with Walter's liquid-B' is investigated by a linear stability analysis. The modified Darcy law with viscoelastic correction is used to describe the fluid motion in the porous layer. The following three cases have been considered for discussion: (i) When the oscillating temperature field is symmetric, i.e., the wall temperatures are modulated in phase. (ii) When the oscillating temperature field is asymmetric, corresponding to an out-of-phase modulation (iii) When only the temperature of the bottom wall is modulated the upper wall being held at a fixed constant temperature. The perturbation method is used to find the critical Rayleigh number and the corresponding wave number for small amplitude thermal modulation. The stability of the system characterized by a correction Rayleigh number is calculated as a function of elasticity parameter, Darcy number, Prandtl number and the frequency of modulation. It is found that the onset of convection can be delayed or advanced by the factors represented by these parameters.

1. Introduction

The problem of natural convection in a Walter's liquid-B' horizontal fluid layer and a porous layer has been investigated by many authors because of its applications in applied geophysics. The onset of thermal convection in a fluid saturated porous medium has attracted the interest of engineers and scientists for a long time due to its numerous applications in fields such as geothermal energy utilization, oil reservoir modeling, building thermal insulation, nuclear waste disposals and mantle convection, to mention a few. The problem has been investigated extensively by several researchers and the growing volume of work in this area is well documented by Ingham and Pop (1998), Nield and Bejan (2006) and Vafai (2005).

The study of convection in a porous medium is of great practical importance in the fields such as chemical engineering, geothermal activities, oil recovery techniques, biomechanics and biological processes and so on. In these practical applications, naturally occurring phenomena are usually unsteady because of the periodicity of the principal driving forces. More specifically, if heat is introduced slowly, the basic temperature gradient is uniform, the instability usually manifests in the form of rolls and numbers of studies on this (Cheng, 1978) are available. If heat is introduced rapidly, the basic temperature gradient is non-uniform, being a function of position and time, the instability may manifest in the form of columnar instability. The effect of non-uniform basic temperature gradient on the onset of convection in horizontal fluid layers (Venezian, 1969; Rosenblat and Herbert, 1970; Finucane and Kelly, 1976) and in porous layers (Nield, 1975; Rudraiah et al, 1980, 1982, 1990) has been investigated and they have shown that a non-uniform temperature gradient controls (i.e., either augments or suppresses) the convection.

There are many investigations available on the effect of time dependent boundary temperature on the onset of Releigh-Benard convection. Most of the findings related to this problem have been reviewed by Davis [1976]. A linear stability analysis in case of small amplitude to temperature modulation is performed by Venezian [1969]. He has established that the onset of convection can be delayed or advanced by the out of or in phase modulation of the boundary temperatures, respectively. It has been found that at low frequencies the equilibrium state becomes unstable, because at low frequencies the disturbances grow to a sufficient size that the inertia effects become more important.

Rosenblat and Herbert [1970] found the asymptotic solution of the low frequency and arbitrary amplitude thermal modulation problem. The solution is discussed from the viewpoint of the stability or otherwise of the basic state and possible stability criteria are analyzed. Comparison is made with known experimental results. Rosenblat and Tanaka [1971] have studied the effect of thermal modulation on the onset of Rayleigh-Benard convection when the temperature gradient has both a steady and time periodic component. They have solved the problem using Galerkin technique and discussed the stability using Floquet theory. It has been found that, in general, there is enhancement of the critical value of a suitably defined Rayleigh number.

Finucane and Kelly [1976] performed both theoretical and experimental investigation of the thermal modulation in a horizontal fluid layer. A numerical analysis of the linear stability equations indicated that the linear assumption is valid at the low frequencies of modulation. A nonlinear analysis employing the shape assumption and free boundary conditions was developed and examined numerically. They found both experimentally and numerically that a low frequencies the modulation is destabilizing, whereas high frequency modulation is stabilizing. Roppo et al. [1984] have performed weakly nonlinear stability analysis and found that the modulation produces a range of stable hexagons near the critical Rayleigh-number. These authors have reported that for low frequencies the modulation is destabilizing, whereas at high frequencies some stabilization is apparent. Recently, Schmitt and Lucke (1991), Or and Kelly (1999), Li (2001) and Or (2001) have also investigated the effect of modulation on the thermal convection in a horizontal fluid layer.

On the other hand, the studies related to the effect of thermal modulation on the onset of convection in a fluid saturated porous medium have not received much attention. On the other hand the studies related to the effect of thermal modulation on the onset of convection in a fluid-saturated porous medium have received marginal attention. The effect of time dependent wall temperature on the onset of convection in a porous medium has been studied by Caltagirone (1976), and Rudraiah and Malashetty (1990). Quite recently non-Newtonian fluids housed in fluid-based systems, with and without porous matrix, have been extensively used in application situations and hence warrant the attention they have been duly getting. In the asthenosphere and the deeper mantle it is well known now that viscoelastic behavior is an important rheological process (see

Lowrie, 1997). The other application areas of viscoelastic fluid saturated porous media are flow through composites, timber wood, snow systems and rheology of food transport. The present problem housed in a porous medium suggests an elastohydrodynamical model for geophysical applications and the likes of it (see Lowrie, 1997; Siddheshwar and Srikrishna, 2001; Yoon et al., 2004). Regulation of convection in these application situations is important and the study of this is the motive for the paper.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. During recent years the theory of polar fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the fluid flow with suspended particles. The study of such fluids have applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants and colloidal and suspension solutions. In the category of non-Newtonian fluids Walter's liquid- B' has distinct features, such as polar effects. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant.

We investigate the effect of thermal modulation on the onset of convection in a horizontal porous layer saturated with a Walters's liquid-B'. The amplitude and frequency of the modulation are externally controlled parameters and hence the onset of convection can be delayed or advanced by a proper tuning of these parameters. The problem has potential application in achieving major enhancement of mass, momentum and heat transfer in the geothermal context and related areas.

2. Mathematical Formulation

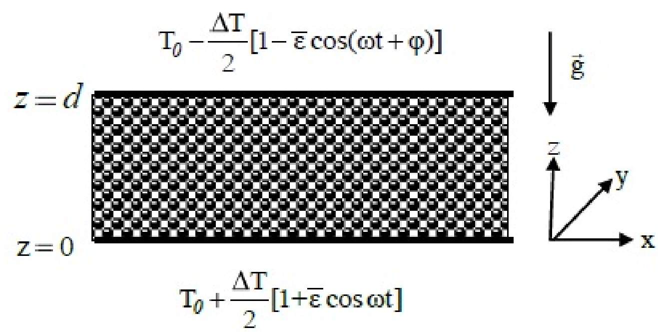
We consider Walter's liquid-B' saturated horizontal porous layer of thickness d in the presence of gravity. The wall temperatures are externally imposed and are given by

$$T = T_0 + \frac{1}{2}\Delta T(1 + \bar{\epsilon}\cos\omega t) \quad \text{at} \quad z = 0 \tag{1}$$

$$T = T_0 - \frac{1}{2}\Delta T[1 - \bar{\epsilon}\cos(\omega t + \varphi)] \quad \text{at} \quad z = d \tag{2}$$

where $\bar{\epsilon}$ represents a small amplitude of the thermal modulation, ω the frequency, φ the phase angle and T_0 is the reference temperature. The time dependent parts denote the modulation imposed on the adverse thermal gradient caused by the temperatures

$T_0+\Delta T/2$ and $T_0-\Delta T/2$ at the lower and upper walls respectively. The fluid saturated porous medium is assumed to have coinciding principal axes of permeability and thermal conductivity. One of the axes is directed upwards in the z direction. The x and y axes are defined by the directions of the other two principal axes.



The governing basic equations

$$\rho_0\left[\frac{1}{\epsilon}\frac{\partial\vec{a}}{\partial t}+\frac{1}{\epsilon^2}(\vec{q}\cdot\nabla\vec{q}\right]=-\nabla_\rho+\rho\vec{g}-\frac{1}{k}\left(\mu-\mu_v\frac{\partial}{\partial t}\right)\vec{q}. \tag{3}$$

$$A\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa_t\nabla^2T \tag{4}$$

$$\nabla\cdot\vec{q}=0 \tag{5}$$

$$\rho=\rho_0\{1-\alpha(T-T_0)\} \tag{6}$$

where $A=(\rho_0c_p)_m/(\rho_0c_p)_f=[(1-\epsilon)(\rho_0c_p)_s+\epsilon(\rho_0c_p)_f]/(\rho_0c_p)_f$ is the ratio of heat capacities of the fluid saturated porous medium to that of the fluid, ϵ is the porosity of the medium, c_p is the specific heat, T is the temperature, κ is the effective thermal diffusivity, α is the volumetric expansion coefficient, μ_v is the viscoelasticity of Walters' liquid-B' and ρ_0 is the reference density. The subscripts m, s and f refer respectively to the porous medium, solid and fluid.
The basic state is quiescent and the temperature T_b and the pressure p_b satisfy

$$\rho_b\vec{g}+\nabla p_b=0 \tag{7}$$

$$A\frac{\partial T_b}{\partial t}=\kappa_t\frac{\partial^2T_0}{\partial z^2}. \tag{8}$$

The solution of Eq. (7) satisfying the thermal conditions Eq. (1) and Eq. (2) is

$$T_b = T_1(z) = \bar{\epsilon} T_2(z,t) \tag{9}$$

where

$$T_1(z) = \frac{\Delta T}{2}(1 - 2z/d) \tag{10}$$

$$T_2(z) = Re\left\{\left[b(\lambda)e^{\lambda z/d} + b(-\lambda)e^{-\lambda z_t d}\right]e^{-\omega t}\right\}. \tag{11}$$

Here

$$\lambda = (1-i)\left(\frac{A\omega d^2}{2\kappa}\right)^{1/2} \tag{12}$$

$$b(\lambda) = \left(\frac{\Delta T}{2}\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right) \tag{13}$$

and Re stands for the real part.

We give an infinitesimal disturbance of the form

$$\vec{q} = \vec{q}', \quad T = T_b + T', \; p = p_b + p' \tag{14}$$

where q', T' and p' represent the perturbed quantities. Substituting Eq. (14) in Eq. (3), eliminating the pressure and retaining the vertical component, we get (after ignoring the primes).

$$\left(\frac{1}{\epsilon}\frac{\partial}{\partial t} + \frac{\mu}{K\rho_0}\left(1 - \frac{\mu_v}{\mu}\frac{\partial}{\partial t}\right)\right)\nabla^2 w = \alpha g \nabla_h^2 T. \tag{15}$$

Substituting Eq. (14) in Eq. (4), linearizing we obtain (after ignoring the primes)

$$A\frac{\partial T}{\partial t} = \kappa_t \nabla^2 T - \frac{\partial T_b}{\partial z} w. \tag{16}$$

Non- dimensionalizing the equations by setting

$$x^* = \frac{x}{d}, \; T^* = \frac{T}{\Delta T}, \; w^* = \frac{w}{\kappa/d}, \quad t^* = \frac{t}{d^2 A/\kappa}, \quad \omega^* = \frac{Ad^2}{\kappa}\omega \tag{17}$$

and substituting in Eqs.(15) and (16), we obtain

$$\left[\frac{1}{Pr}\frac{\partial}{\partial t} + Da^{-1}\left(1 - \frac{\Gamma}{Pr}\frac{\partial}{\partial t}\right)\right]\nabla^2 w - R\nabla_h^2 T = 0 \tag{18}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T = -\frac{\partial T_b}{\partial z} w \tag{19}$$

where $\Gamma = \mu_v \epsilon / \rho_0 d^2$ is the Elastic parameter, $Pr = vA\epsilon/\kappa_t$ is the modified Prandtl number, $R = \alpha g \Delta T d^3 / v\kappa_t$ is the Rayleigh number and $Da = K/d^2$ is the Darcy number. The boundary conditions are:

$$w = T = 0 \ \text{ at‘ } z = 0, 1. \tag{20}$$

Eliminating T from Eq. (18) using Eq. (19), we obtain the following equation

$$\left[\frac{\partial}{\partial t} - \nabla^2\right]\left[\frac{1}{Pr}\frac{\partial}{\partial t} + Da^{-1}\left(1 - \frac{\Gamma}{Pr}\frac{\partial}{\partial t}\right)\nabla^2 w + R\nabla_h^2 w \frac{\partial T_b}{\partial z}\right] = 0. \tag{21}$$

The dimensionless temperature gradient is given by

$$\frac{\partial T_b}{\partial z} = -1 + \bar{\epsilon} f \tag{22}$$

where

$$f = Re\left\{\left[A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}\right]e^{-\omega t}\right\}$$

the modulation temperature gradient.

$$A(\lambda) = \left(\frac{\lambda}{2}\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right), \quad \lambda = (1 - i)\left(\frac{\omega}{2}\right)^{1/2}. \tag{23}$$

3. Perturbation Procedure With Small Amplitude Approximation

The aim of this section is to determine the eigen functions w and the eigenvalues R of Eq. (21) from the basic temperature distribution given by Eq. (22) that departs from the linear profile $\partial T_b/\partial z = -1$ in modulated system by the quantities of the order $\bar{\epsilon}$. It follows that the eigenfunction and the eigenvalues of the present problem differ from those associated with usual Darcy-Benard convection by quantities of order ϵ. From Eq. (21) we can also see that when the temperature profile is linear, as far as stationary instability is concerned, the viscoelastic properties of the fluid have no effect on the onset of linear instability. We therefore assume the solution of Eq. (21) in the form

$$(R, w) = (R_0, w_0) + \bar{\epsilon}(R_1, w_1) + \bar{\epsilon}^2(R_2, w_2) + \cdots \tag{24}$$

where R_0 is the Rayleigh number corresponding to Darcy-Benard convection. Substituting Eq.(24) into Eq. (21) and equating different powers of ϵ, we obtain the following system of equations:

$$Lw_0 = 0 \tag{25}$$

$$Lw_1 = R_1 \nabla_h^1 w_0 - R_0 f \nabla_h^2 w_0 \tag{26}$$

$$Lw_2 = R_1 \nabla_h^2 w_1 + R_2 \nabla_h^2 w_0 - R_0 f \nabla_h^2 w_1 - R_1 f \nabla_h^2 w_0 \tag{27}$$

where

$$L = \left[\frac{\partial}{\partial t} - \nabla^2\right] \left[\frac{1}{Pr}\frac{\partial}{\partial t} + Da^{-1}\left(1 - \frac{\Gamma}{Pr}\frac{\partial}{\partial t}\right)\nabla^2 - R_0 \nabla_h^2\right].$$

The function w_0, which corresponds to no modulation i.e, $\bar{\epsilon} = 0$ is solved. The marginal stable solution should be (chandrashekar 1961).

A general solution of Eq.(25) is

$$w_0^{(n)} = \sin(n\pi z)\exp[i(lx + my)] \tag{28}$$

where l and m are the horizantal wave numbers with the corresponding eigenvalues

$$R_0^{(n)} = \frac{Da^{-1}(n\pi^2 + a^2)^2}{a^2}.$$

For a fixed value of wave number a, the least eigenvalue occurs at $n = 1$ and is given by

$$R_0 = \frac{Da^{-1}(\pi^2 + a^2)^2}{a^2}. \tag{29}$$

We note that R_0 attains its minimum value, R_{0c} at $a = a_c$, where

$$R_w = 4\pi^2 Da^{-1} \tag{30}$$

$$a_c = \pi. \tag{31}$$

These are the values reported by Lapwood (1948) for convection in porous layer.

Equation (26) is inhomogeneous and its solution poses a problem due to the presence of resonance terms. The solvability condition requires that time independent part of the right-hand side of Eq. (26) should be orthogonal to w_0. The term independent of time on the right hand side is $R_1 \nabla_h^2 w_0$ so that $R_1 = 0$. It follows that all the odd coefficients, i.e., $R_1, R_3, \cdots$ in Eq. (24) must vanish. If we expand the right-hand side of Eq. (26) in a Fourier series of the form

$$e^{\lambda z}\sin(m\pi z) = \sum_1^\infty g_{nm}(\lambda)\sin(n\pi z) \tag{32}$$

then

$$g_{nm}(\lambda) = 2\int_0^1 e^{\lambda z}\sin(m\pi z)\sin(n\pi z)dz = \frac{-4nm\pi^2\lambda[1 + (-1)^{n+m+1}e^\lambda]}{[\lambda^2 + (n+m)^2\pi^2][\lambda^2 + (n-m)^2\pi^2]}. \tag{33}$$

We thus obtain

$$L[\sin(n\pi z)e^{-i\omega t}] = L(\omega, n)\sin(n\pi z)e^{-i\omega t} \tag{34}$$

where

$$L(\omega, n) = \frac{\omega^2(n^2\pi^2 + a^2)}{Pr} - \frac{\omega^2 Da^{-1}\Gamma(n^2\pi^2 + a^2)}{Pr} - Da^{-1}(n^2\pi^2 + a^2)^2 + Da^{-1}(\pi^2 + a^2)^2$$
$$+ i\omega\left[Da^{-1}(n^2\pi^2 + a^2) + \frac{(n^2\pi^2 + a^2)^2}{Pr} - \frac{Da^{-1}\Gamma(n^2\pi^2 + a^2)^2}{Pr}\right]. \tag{35}$$

From Eq. (26) we have

$$Lw_1 = R_0 a^2\left\{Re\left[\sum_n A(\lambda)g_{n1}(\lambda) + A(-l)g_{n1}(-\lambda)\right]\sin(n\pi z)e^{-i\omega t}\right\}. \tag{36}$$

We obtain w_1 by inverting the operator L term by term, in the form

$$w_1 = R_0 a^2\left[Re\sum_n \frac{B_n(\lambda)}{L(\omega, n)}\sin(n\pi z)z^{-i\omega t}\right] \tag{37}$$

where

$$B_n(\lambda) = A(\lambda)g_{n1}(\lambda) + A(-\lambda)g_{n1}(-\lambda).$$

The solution of the homogenous equation corresponding to Eq. (36) involves a term proportional to $\sin(\pi z)$. However, addition of such a term to the complete solution of Eq. (36) merely amounts to a renormalization of ω because all the terms proportional to $\sin(\pi z)$ can then be grouped to define a new w_0 with corresponding definition for w_1, w_2, etc. Hence, we can assume that w_0 is orthogonal to all other w_n's. From Eq. (27) we get

$$Lw_2 = a^2 R_0 f w_1 - a^2 R_2 w_0. \tag{38}$$

We shall not require the solution of this equation, but merely use it to determine R_{2c}, the first nonzero correction to R_2. The solubility condition requires that the steady part of the right-hand side is orthogonal to $\sin(\pi z)$. Thus,

$$R_2 = 2R_0\int_0^1 \overline{fw_1}\sin(\pi z)dz \tag{39}$$

where the upper bar denotes the time average. From Eq. (26) we have

$$\overline{fw_1}\sin\pi z = \frac{1}{R_0 a^2}\overline{w_2 Lw_1}. \tag{40}$$

Using Eq. (36) and Eq. (37) and finding the time average we obtain $\overline{w_1 L w_1}$, which yields from Eqs.(39) and (40),

$$R_2 = \frac{R_0^2 a^2}{4} \sum \frac{|B_n(\lambda)|^2}{|L(\omega,n)|^2}[L(\omega,n) + L^*(\omega,n)] \tag{41}$$

where $L^*(\omega,n)$ is the complex conjugate of $L(\omega,n)$. The critical value of R_2 is obtained at the wave number given by Eq.(31).

We calculate R_{2c} for the following three different cases.

Case (i) : Oscillating temperature field is symmetric(in phase, $\phi = 0$)

When the oscillating temperature field is symmetric so that the wall temperatures are modulated in phase with $\varphi = 0$. In this case,

$$\begin{aligned} |B(\lambda)|^2 &= \frac{16n^2\pi^4\omega^2}{[\omega^2+(n+1)^4\pi^4][\omega^2+(n-1)^4\pi^4]} \\ &= |b_n|^2 \;\; \text{(say) if } n \text{ is even} \\ &= 0 \;\; \text{if } n \text{ is odd} \end{aligned} \tag{42}$$

Then

$$R_{2c} = \frac{R_0 a_c^2}{2} \sum_n |b_n|^2 \frac{A}{(A^2+B^2)} \tag{43}$$

where

$$A = Re[L(\omega,n)] = \frac{\omega^2(n^2\pi^2+a^2)}{Pr} - \frac{\omega^2 Da^{-1}\Gamma(n^2\pi^2+a^2)}{Pr} - Da^{-1}(n^2\pi^2+a^2)^2 + Da^{-1}(\pi^2+a^2)^2 \tag{44a}$$

and

$$B = Im[L(\omega,n)] = \omega\left[Da^{-1}(n^2\pi^2+a^2) + \frac{(n^2\pi^2+a^2)^2}{Pr} - \frac{Da^{-1}\Gamma(n^2\pi^2+a^2)^2}{Pr}\right]. \tag{44b}$$

The summation extends over even values of n.

Case(ii) : Wall temperature field is asymmetric (out of phase, $\phi = \pi$)

When the wall temperature field is asymmetric corresponding to out-of-phase modulation with $\varphi = \pi$, we obtain

$$\begin{aligned} |B_n(\lambda)|^2 &= |b_n|^2 \quad \text{if } n \text{ is odd} \\ &= 0 \quad \text{if } n \text{ is even} \end{aligned} \tag{45}$$

Then R_{2c} has the same expression as Eq. (43), with the summation extending over odd values of n only.

Case (iii) : Only lower wall temperature is modulated while the upper one is held at constant temperature $(\phi = -i\infty)$

When only the lower wall temperature is oscillating, while the upper wall is held at constant temperature, with $\varphi = -i\infty$, we have $|B_n(\lambda)|^2 = \frac{|b_n|^2}{4}$. Then, R_{2c} is given by Eq. (43) in which the summation extends over all values of n. The variations of R_{2c} with ω for different physical parameters are shown in the figures and results are discussed in the final section.

4. Subcritical Instability

The critical value of the Rayleigh number, R_c is determined to order of $\bar{\epsilon}^2$ by evaluating R_{0c} and R_{2c} and is of the form

$$R_c = R_{0c} + \bar{\epsilon}^2 R_{2c} \tag{46}$$

where

$$R_{0c} = \frac{Da^{-1}(\pi^2 + a^2)^2}{a^2}, \; R_{2c} = \frac{Da^{-1}(\pi^2 + a^2)^2}{2a^2} \sum_n |B_n(\lambda)|^2 \left(\frac{A}{(A^2 + B^2)} \right)_{a=a_c} \qquad (47), (48)$$

If R_{2c} is positive, supercritical instabilities exist and R_c has a minimum at $\bar{\epsilon} = 0$. When R_{2c} is negative, subcritical instabilities are possible. In this case, R_{0c} should be greater than $\bar{\epsilon}^2 R_{2c}$ for R_c to become positive. That is

$$R_{0c} > \bar{\epsilon}^2 R_{2c}. \tag{49}$$

which leads to

$$\bar{\epsilon}^2 < \frac{2}{Da^{-1}(\pi^2 + a^2)^2} \times \frac{1}{\sum_n |b_n(\lambda)|^2 \frac{A}{(A^2+B^2)}}. \tag{50}$$

Now we can calculate the maximum range for ϵ by assigning values to the physical parameters involved in the above condition. Thus, the range of the amplitude of modulation, which causes subcritical instabilities in different physical situations, can be explained. To this effect, graphs have been plotted and the results are discussed in the next section.

5. Results and Discussion

The effect of modulated basic temperature gradient on the onset of convection in a Walter's liquid-B' fluid saturated porous layer is investigated to understand the control of convection. A perturbation technique is used to find the critical thermal Rayleigh number as a function of frequency of the modulation, elasticity parameter, Darcy number and Prandtl number. The analysis presented in this paper is based on the assumption that the amplitude of the modulating temperature is small. The stability of the system is characterized by the sign of the correction Rayleigh number R_{2c}. A positive R_{2c} represent a stable system while a negative R_{2c} indicates an unstable one as compared to the system in the absence of modulation. It is evident that the instability occurs at the minimum temperature gradient at which a balance can be maintained between the kinetic energy dissipated by viscosity and the internal energy released by the buoyancy force. This minimum temperature is called critical temperature, the corresponding Rayleigh number is called critical Rayleigh number, where the Rayleigh number is defined as the ratio of buoyancy force to viscous force. Physically, it represents the balance of energy released by buoyancy force to the energy dissipated by viscous friction and thermal dissipation. Once if the temperature gradient crosses this critical temperature, i.e., the internal energy released by the buoyancy force is greater than the kinetic energy dissipated by viscosity, then the instability occurs. Before that the system remains stable. Thus the onset of convective instability is characterized by Rayleigh number. Hence we say that as the critical Rayleigh number increases, convection occurs late and hence the system becomes stable and as the critical Rayleigh number decreases, convection occurs early and hence the system becomes unstable.

In the present chapter, the eigenvalue R is written in terms of power series in (Venezian, 1969) and is determined to the order of $\bar{\epsilon}^2$, accordingly we have $R = R_0 + \bar{\epsilon}^2 R_2$. Here we note that R_1 is shown to be zero in section 3. R_{oc} and R_{2c} are, respectively, given by equations (30) and (43).

If R_{2c} is positive, then the value of total R is greater than the value of R which is present in the problem of convection without modulation. In this case we say that the system is stable. In a similar way, if R_{2c} is negative, we say that the system is unstable. In this chapter, the values of R_{2c} will be obtained for the following three cases:

(a) When the oscillating temperature field is symmetric, i.e., the wall temperatures are modulated in phase, $\overline{\varphi} = 0$.

(b) When the field is asymmetric, corresponding to an out-of-phase modulation, $\overline{\varphi} = \pi$.

(c) When only the temperature of the bottom wall is modulated, the upper wall being held at a fixed constant temperature. In this case it is convenient to take the wall temperature to be $\frac{\Delta T}{2}[1 + \overline{\epsilon}\cos\omega t]$ at the bottom and $\frac{\Delta T}{2}$ at the top. This can be recovered from the equations by setting $\overline{\varphi} = -i\infty$.

The correction Rayleigh number R_{2c} is obtained as a function of Elastic parameter Γ, frequency of temperature modulation ω, Darcy number Da and Prandtl number Pr and is depicted in figures 1-9.

The results obtained for the above cases are depicted in Figures 1-9 as a function of frequency of temperature modulation for different values of physical parameters.

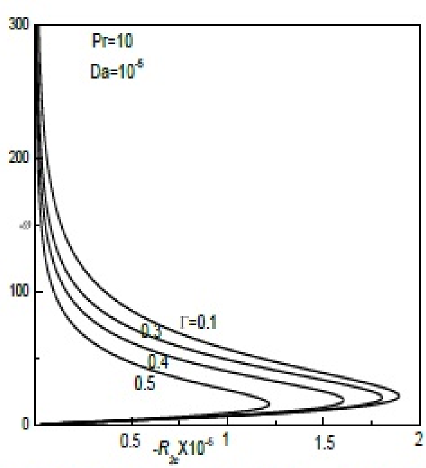
Figure 1 is a plot of $(-R_{2c})$ versus ω for different values of elasticity parameter Γ_P when $Pr = 10$ and $Da = 10^{-5}$ for the case of symmetric modulation of the wall temperature. We observe that, in general, R_{2c} is negative over the whole range of frequencies, indicating that the symmetric temperature modulation has a destabilizing effect on the system as compared to the un-modulated system. That is, convection sets in at lower values of Rayleigh number than the classical Darcy-Benard problem in the presence of thermal modulation. Further, it is noted that as the elasticity parameter Γ_P increases, the magnitude of correction Rayleigh number R_{2c} decreases indicating that the effect of elasticity parameter is to suppress the destabilizing effect of thermal modulation. Besides, the curves for different values of Γ_P are very close to zero when the modulation frequency is very small. Hence, the modulation has very little effect on the stability of the system when ω approaches to zero value. As ω increases, $|R_{2c}|$ increases to its maximum value initially and then decreases with further increase in ω. When ω is very large, all the curves for different Γ_P coalesce and $|R_{2c}|$ decrease to zero. This means that the modulation with large frequency will have no substantial effect on the stability characteristics of the system. This figure also indicates that the peak negative value of R_{2c} decreases with an increase in the value of Γ_P.

The results obtained for the case of asymmetric modulation with $Pr = 10$ and $Da = 10^{-5}$ are presented in Figure 2. We note that the curves of $(-R_{2c})$ versus ω for different values of elasticity parameter Γ_P do not coalesce as the modulation frequency approaches to zero value. Moreover, $|R_{2c}|$ decreases monotonically with an increase in the value of ω without attaining any peak value for a fixed value of elasticity parameter Γ_P and all the curves for different Γ_P coalesce at higher values of ω. Also, in contrast to the symmetric temperature modulation case, the elasticity parameter plays a dual role in deciding the stability of the system depending on the value of ω.

Figure 3 displays the variation of (R_{2c}) versus for different values of Γ_P with $Pr = 10$ and $Da = 10^{-5}$ for the case of only lower wall temperature modulation. Here also we observe that R_{2c} is negative over the whole range of frequencies as noticed in the case of symmetric and asymmetric modulation of the wall temperature. From this figure it is observed that at low frequencies, $|R_{2c}|$ increases with increasing Γ_P, while this trend is reversed at higher values of frequencies.

The effect of Prandtl number on the correction Rayleigh number R_{2c} with $Da = 10^{-5}$ and $\Gamma_P = 0.1$ for the cases of $\varphi = 0, \pi$ and $-i\infty$ is shown in Figures 4, 5 and 6 respectively as a function of ω. When the oscillating temperature field is symmetric (i.e., $\varphi = 0$), it is observed that all curves are very close to zero when the modulation frequency is very small (see Figure 4). Hence, the symmetric temperature modulation has little effect on the stability of the system when ω approaches to zero. This figure also indicates that the peak value of $|R_{2c}|$ decreases marginally with increasing Pr and the curves for different Pr coalesce. Also, increase in the value of Prandtl number is to reduce marginally the destabilizing effect of symmetric temperature modulation. In the case of asymmetric temperature modulation (see Figure 5), $|R_{2c}|$ does not attain any peak value with increasing ω as noticed in the symmetric temperature modulation. Besides, increase in Pr is to reduce the value of $|R_{2c}|$ at lower as well as moderate values of ω but it has no impact on the onset of convection at higher values of ω. The results observed in the case of lower wall temperature modulation are qualitatively similar to those of symmetric wall temperature modulation (see Figure 6).

The variation of $(-R_{2c})$ as a function of for different values of Darcy number Da is shown in Figures 7, 8 and 9 for symmetric temperature modulation, asymmetric wall temperature modulation and only lower wall temperature modulation, respectively when $Pr = 10$ and $\Gamma_P = 0.1$. From the figures it is evident that effect of increase in Da has qualitatively similar effect as that of Prandtl number. The peak value of $|R_{2c}|$ decreases rapidly with an increase in the value of Da from 1×10^{-5} to 3×10^{-4} (see Figures 7 and 9).



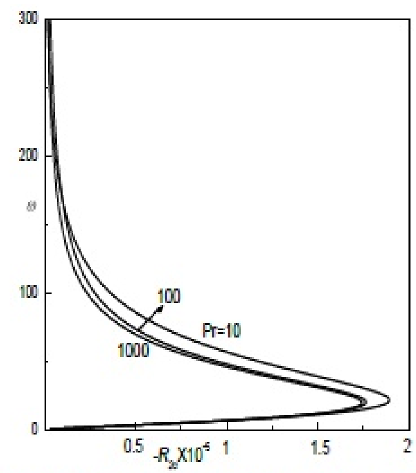
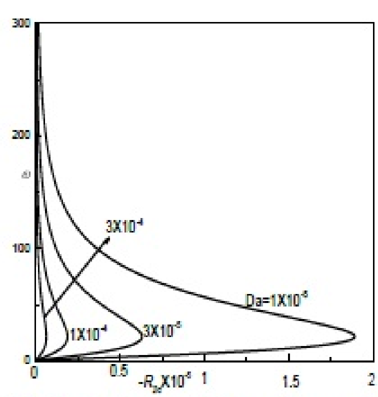


Fig. 1. Variation of R_{2c} with ω for different values of the elasticity parameter Γ for Symmetric temperature modulation.

Fig. 2. Variation of R_{2c} with ω for different values of the Prandtl number Pr for Symmetric temperature modulation .



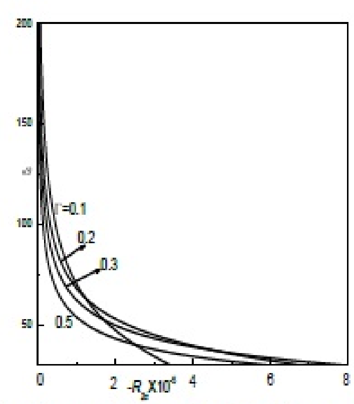
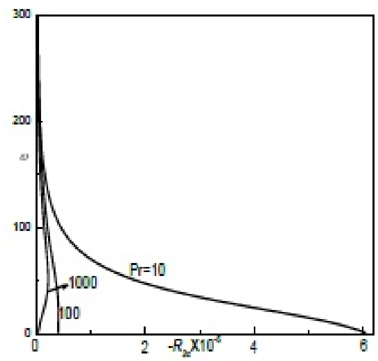


Fig. 3. Variation of R_{2c} with ω for different values of the Da for Symmetric temperature modulation..

Fig. 4. Variation of R_{2c} with ω for different values of the Γ for Asymmetric temperature modulation.



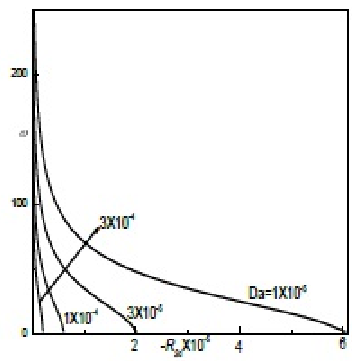
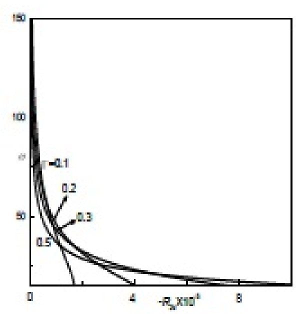
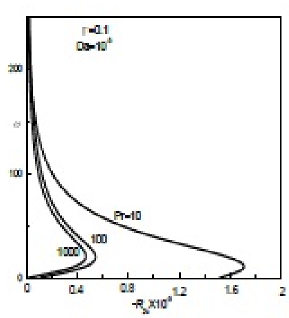


Fig.5. Variation of R_{2c} with ω for different values of the Prandtl number Pr for Asymmetric temperature modulation.

Fig. 6. Variation of R_{2c} with ω for different values of the Da for Asymmetric temperature modulation.





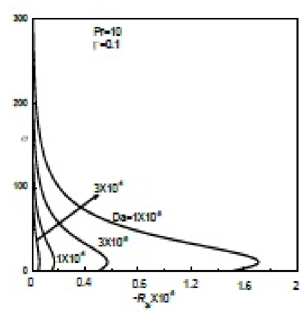


Fig. 7. Variation of R_{2c} with ω for different values of the Γ for only-lower wall temperature modulation.

Fig. 8. Variation of R_{2c} with ω for different values of the Prandtl number Pr only-lower wall temperature modulation.

Fig. 9. Variation of R_{2c} with ω for different values of the Da only-lower wall temperature modulation.

5. Conclusions

The effect of thermal modulation on the onset of convection in a horizontal layer of porous medium saturated with Walters liquid-B is studied using a linear stability analysis. The analytic expression obtained for R_{2c} is computed for various values of physical parameters for the cases of (i) oscillating wall temperature field is symmetric (i.e., the wall temperatures are modulated in phase, $\varphi = 0$), (ii) oscillating wall temperature field is asymmetric (i.e., the wall temperatures are modulated out-of-phase, $\varphi = \pi$) and (iii) only lower wall temperature is modulated and the upper wall being held at a fixed constant temperature (i.e., $\varphi = -i\infty$) and the following conclusions may be drawn: (i) The effects of all three types of modulations namely, symmetric, asymmetric, and only lower wall temperature modulations are found to be destabilizing. (ii) The effect of thermal modulation disappears at large frequencies in all the cases of thermal modulation. (iii) Increase in the value of Pr and Da is to decrease $|R_{2c}|$ in all the cases, while increase in Γ_P reduces $|R_{2c}|$ only in the case of symmetric temperature modulation. (iv) The critical correction Rayleigh number $R_{2c} \to 0$ with increase in faster for large Da.

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