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# A NOTE ON AN ABSOLUTE DIFFERENCE OF CUBIC AND SQUARE SUM LABELING

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**Abstract**

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In [4], we introduced the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few graphs for absolute difference of cubic and square sum labeling.

## 1. Introduction

All graphs in this paper are finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph $G$. The graph whose cardinality of the vertex set is called the order of $G$, denoted by $p$ and the cardinality of the edge set is called the size of the graph $G$, denoted by $q$.

A graph with $p$ vertices and $q$ edges is called a $(p, q)$ graph.

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Key Words : *Graph labeling, Sum square graph, Square sum graphs, Cubic graphs.*

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2] and [3]. Some basic concepts are taken from Frank Harary [1]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph [4]. In [4], [5], [6], [7], [8], [9], [10], [11], [12] and [13], it is shown that planar grid, web graph, kayak paddle graph, snake graphs, armed crown, fan graph, friendship graph, windmill graph, cycle graphs, wheel graph, gear graph, helm graph, 2-tuple graphs, middle graphs, total graphs and shadow graphs have an adcss-labeling. In this paper we investigated an Absolute Difference of cubic and Square Sum labeling of some classes of graphs.

**Definition: 1.1 [4]** : Let $G = (V(G), E(G))$ be a graph. A graph $G$ is said to be an absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection
$f : V(G) \to \{1, 2, \ldots, p\}$ such that the induced function
$f^*_{adcss} : E(G) \to$ multiples of 2 is given by
$f^*_{adcss}(uv) = \left| f(u)^3 + f(v)^3 - \left(f(u)^2 + f(v)^2\right) \right|$ is injective.

**Definition: 1.2** : A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.

## 2. Main Results

**Definition 2.1** : A barbell graph $B(p, n)$ is the graph obtained by connecting $n$-copies of a complete graph $K_p$ by a bridge.

**Theorem 2.1** : The barbell graph $B(p, n)$ is the absolute difference of the css-labeling.

**Proof** : Let $G = B(p, n)$ and let $v_1, v_2, \ldots, v_{np}$ are the vertices of $G$.
Define a function
$f : V \to \{1, 2, 3, \ldots, np\}$ by

$$f(v_i) = i, \; i = 1, 2, \ldots, np$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$f^*_{adcss}\,(v_{pi-p+k}\; v_{pi-p+j}) = (pi-p+k)^2(pi-p+k-1) + (pi-p+j)^2(pi-p+j-1)$$
$$k = 1, 2, 3, \ldots, p-1, \; j = k+1, k+2, \ldots, p, \; i = 1, 2, 3, \ldots, n$$

$f^*_{adcss}(v_{pi-p+1}\ v_{pi+1}) = (ip-p+1)^2(ip-p)+(ip+1)^2(ip),\ i = 1,2,3,\ldots,n-1$
All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence $B(p,n)$ admits absolute difference of css-labeling.

**Example 2.1** : $G = B(p = 5,\ n = 3)$

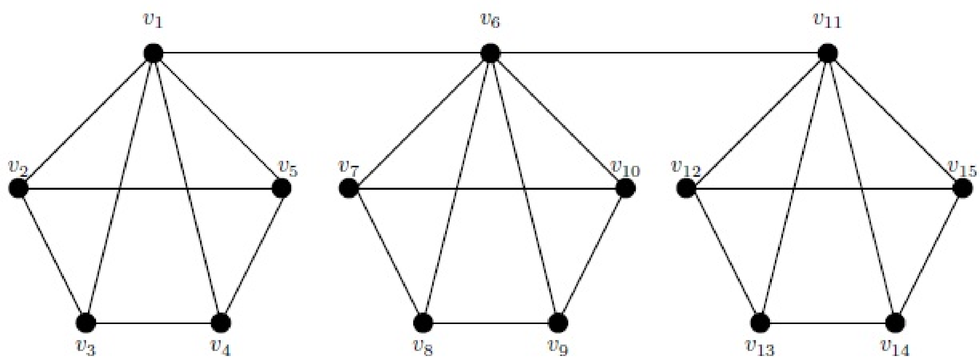


Fig-1

**Definition 2.2** : A gear graph is an extension of a wheel graph, by adding a vertex between each pair of adjacent vertices in the $n$- cycle of a wheel graph. We will denote a gear graph by $G_n$, where $n$ is the number of vertices in the cycle of the wheel graph. Hence the number of vertices in any gear graph $G_n$ is $2n+1$.

**Theorem 2.2**: The gear graph $G_n$ is the absolute difference of the css-labeling.

**Proof:** Let $G = G_n$ and let $v_1,\ v_2,\ldots,v_{2n+1}$ are the vertices of $G$.
Define a function
$f : V \to \{1,2,3,\ldots,2n+1\}$ by

$$f(v_i) = i,\ i = 1,2,\ldots,2n+1$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$\begin{aligned} f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1)+(i+1)^2 i, && i = 2,\ldots,2n \\ f^*_{adcss}(v_1\ v_{2i}) &= (2i)^2(2i-1), && i = 1,2,\ldots,n \\ f^*_{adcss}(v_2\ v_{2n+1}) &= 4+(2n+1)^2(2n) \end{aligned}$$

All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence $G_n$ admits absolute difference of css-labeling.

**Example 2.2** : $G = G_5$

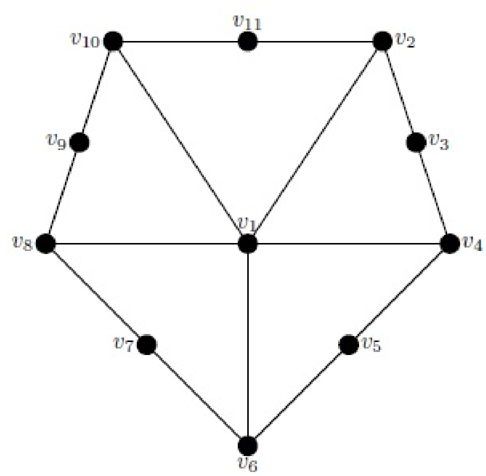


Fig 2

**Definition 2.3** : To form a Helm graph, denoted here by $H_n$, we take the wheel graph $W_n$ and append a pendant edge to each vertex of the $n$-cycle.

**Theorem 2.3**: The Helm graph $H_n$ is the absolute difference of the css-labeling.

**Proof** : Let $G = H_n$ and let $v_1,\ v_2, \ldots, v_{2n+1}$ are the vertices of $G$.

Define a function

$f : V \to \{1, 2, 3, \ldots, 2n+1\}$ by

$$f(v_i) = i,\ \ i = 1, 2, \ldots, 2n+1$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$\begin{aligned}
f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1) + (i+1)^2 i, && i = 2, \ldots, n \\
f^*_{adcss}(v_1\ v_{i+1}) &= (i+1)^2 i, && i = 1, 2, \ldots, n \\
f^*_{adcss}(v_i\ v_{n+i}) &= i^2(i-1) + (n+i)^2(n+i-1), && i = 2, \ldots, n+1 \\
f^*_{adcss}(v_2\ v_{n+1}) &= 4 + (n+1)^2 n
\end{aligned}$$

All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence $H_n$ admits absolute difference of css-labeling.

**Example 2.3** : $G = H_4$

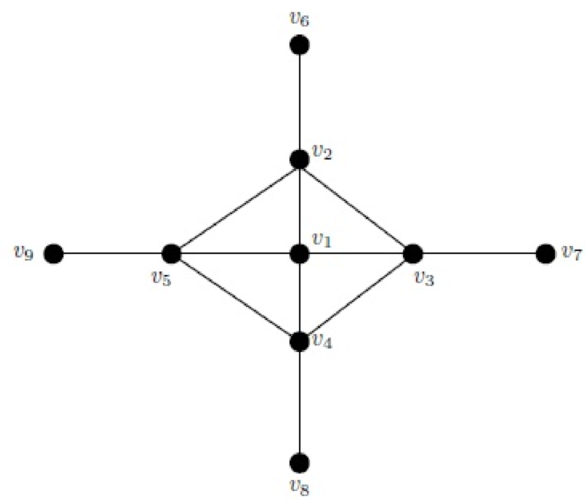


Fig-3

**Theorem 2.4** : The graph corona of a cycle $C_n$ is the absolute difference of the css-labeling.

**Proof:** Let $G = C_n \odot K_1$ and let $v_1,\ v_2, \ldots, v_{2n}$ are the vertices of $G$.

Define a function $f : V \to \{1, 2, 3, \ldots, 2n\}$ by

$$f(v_i) = i,\ i = 1, 2, \ldots, 2n$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$\begin{aligned} f^*_{adcss}(v_i\ v_{n+i}) &= i^2(i-1) + (n+i)^2(n+i-1), \qquad i = 1, 2, \ldots, n. \\ f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1) + (i+1)^2 i, \qquad i = 1, 2, \ldots, n-1. \\ f^*_{adcss}(v_1\ v_n) &= n^2(n-1) \end{aligned}$$

All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence corona of cycle $C_n$ admits absolute difference of css-labeling.

**Example 2.4 :** $G = C_5 \odot K_1$

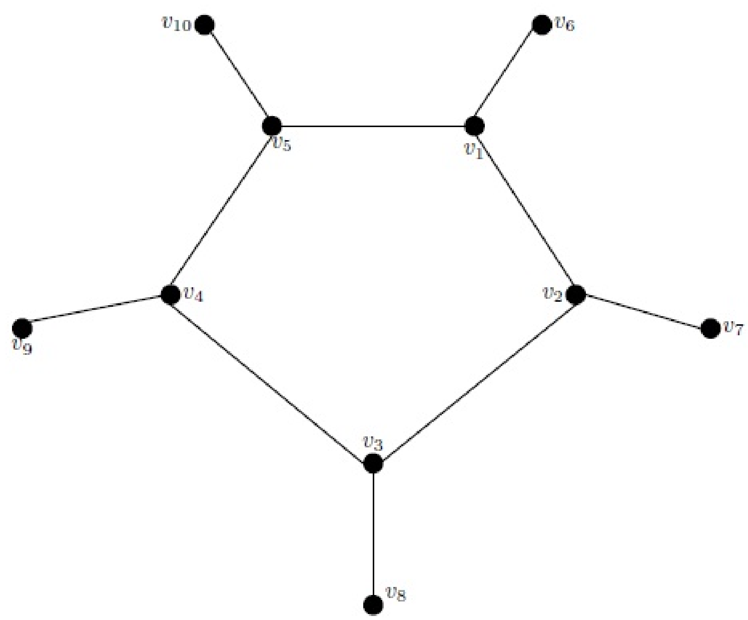


Fig-4

**Definition 2.4 :** The graph $K_{2,n}^{(m)}$ is the one point union of $m$-copies of the complete bipartite graph $K_{2,n}$ . Let $v_1,\ v_2, \dots v_m$ be the vertices of the graph $K_{2,n}^{(m)}$ with degree $n$. Join the vertices $(v_1, v_2),\ (v_2, v_3), \dots, (v_{m-1}, v_m)$ and $(v_m, v_1)$, we get another graph denoted by $C(K_{2,n}^{(m)})$.

**Theorem 2.5 :** The graph $C(K_{2,n}^{(m)})$ is the absolute difference of the css-labeling.

**Proof :** Let $G = C(K_{2,n}^{(m)})$ and let $v_1,\ v_2, \dots, v_{mn+m+1}$ are the vertices of $G$.
Here $|V(G)| = mn + m + 1$ and $|E(G)| = (2n + 1)m$
Define a function
$f : V \longrightarrow \{1, 2, 3, \dots, nm + m + 1\}$ by

$$f(v_i) = i,\ i = 1, 2, \dots, nm + m + 1$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$\begin{aligned} f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1) + (i+1)^2 i, & i &= nm+2, nm+3, \dots, nm+m+1 \\ f^*_{adcss}(v_i\ v_1) &= i^2(i-1), & i &= 2, 3, \dots, nm+1 \end{aligned}$$

$$f^*_{adcss}(v_{nm+2}\ v_{nm+m+1}) = (nm+2)^2(nm+1) + (nm+m+1)^2(nm+m)$$

$$f^*_{adcss}(v_{(j-1)n+i+1}v_{nm+j+1}) = \{(j-1)n+i+1\}^2\{(j-1)n+i\} + (nm+j+1)^2(nm+j)$$

$$j = 1, 2, \ldots m,\ i = 1, 2, \ldots n$$

All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence $C(K^{(m)}_{2,n})$ admits absolute difference of css-labeling.

**Example 2.5** : $C(K^m_{2,5}),\ n = 5,\ m = 4$

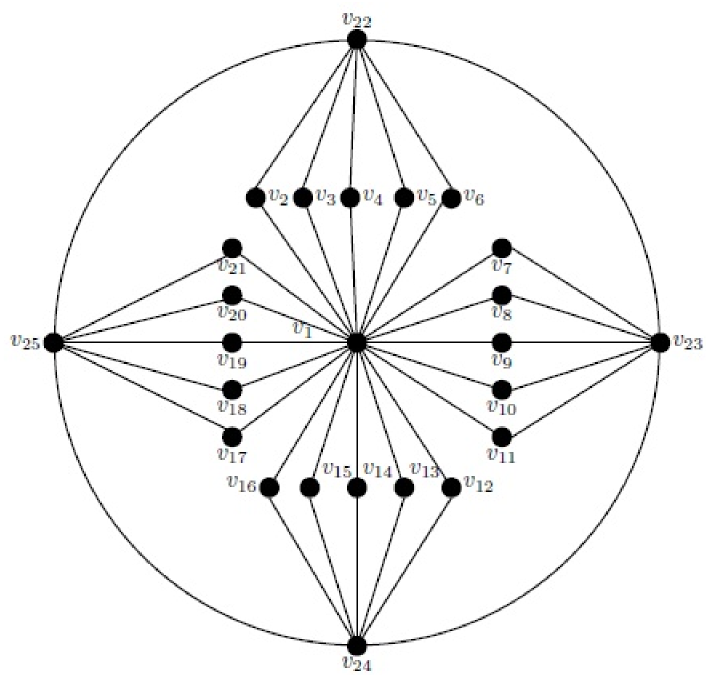


Fig-5

**Theorem 2.6** : The graph $K_{1,n} \odot 2P_m$ is the absolute difference of the css-labeling.

**Proof:** Let $G = K_{1,n} \odot 2P_m$ and let $v_1,\ v_2, \ldots, v_{2mn+n+1}$ are the vertices of $G$.

Here $|V(G)| = 2mn + n + 1$ and $|E(G)| = (2m + 1)n$.

Define a function

$f : V \longrightarrow \{1, 2, 3, \ldots, 2mn + n + 1\}$ by

$$f(v_i) = i,\ i = 1, 2, \ldots, 2mn + n + 1$$

For the vertex labeling $f$, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$f^*_{adcss}(v_i\ v_{(i-1)m+n+1}) = i^2(i-1)+\{(i-1)m+n+1)\}^2\{(i-1)m+n\}, i=1,2,\ldots,n$$

$$f^*_{adcss}(v_i\ v_{(n+i-1)m+n+1}) = i^2(i-1)+\{(n+i-1)m+n+1)\}^2\{(n+i-1)m+n\}, \quad i=1,2,\ldots,n$$

$$f^*_{adcss}(v_{(j-1)m+n+i}\ v_{(j-1)m+n+i+1}) = \{(j-1)m+n+i\}^2\{(j-1)m+n+i-1\} + \{(j-1)m+n+i+1\}^2\{(j-1)m+n+i\} \quad j=1,2,\ldots n,\ i=1,2,\ldots m-1$$

$$f^*_{adcss}(v_{(j-1)m+nm+n+i}\ v_{(j-1)m+nm+n+i+1}) = \{(j-1)m+nm+n+i\}^2\{(j-1)m+nm+n+i-1\}+\{(j-1)m+nm+n+i+1\}^2\{(j-1)m+nm+n+i\} \quad j=1,2,\ldots n,\ i=1,2,\ldots m-1$$

$$f^*\{v_{2mn+n+i}\ v_i\} = \{2mn+n+1\}^2\{2mn+n\}+i^2(i-1), \quad i=1,2,3,\ldots,n$$

All edge values of $G$ are distinct, which are multiples of 2. That is the edge values of $G$ are in the form of an increasing order. Hence $K_{1,n}\odot 2P_m$ admits absolute difference of css-labeling.

**Example 2.6** : $G=K_{1,4}\odot 2P_5$

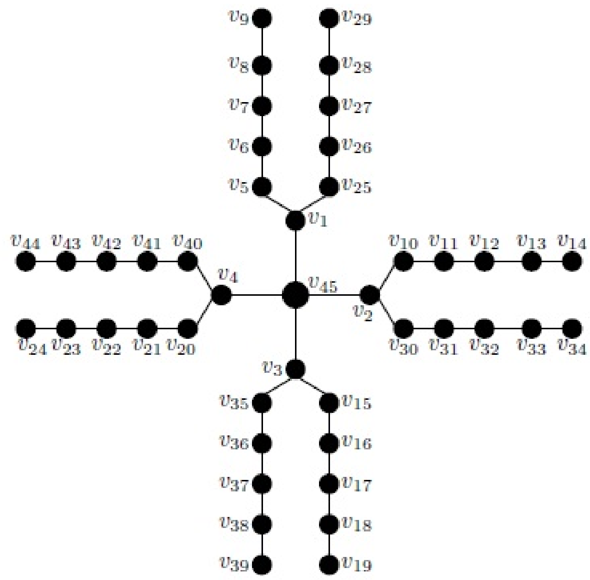


Fig-6

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