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RELIABILITY OF GENERAL SERIES-PARALLEL AND SEQUENCIAL SERIES-PARALLEL SYSTEMS AND THEIR OPTIMIZATION

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Abstract

In this paper we derive the explicit formula for sequential series-parallel system by using induction method and we also obtain the optimal number of redundant components of the system of general series-parallel system and sequential series-parallel system under the consideration of cost minimization. The comparative study of both types of reliability-configuration is carried out from the cost point of view. To show the applicability of the reliability models under study the numerical illustrations have also been obtained.

1. Introduction

Reliability is the probability that a system will perform its intended work satisfactorily for a specified operating condition. Reliability of complex system is studied under various structural framework such as series structures, parallel structures, series-parallel structures, parallel-series structures, sequential structures, combined structures. Series

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system is characterized by the property that the failure of one of these components means the failure of whole system. Due to this fact system may have less durability so to avoid this it is usual to use redundant system is characterized by the property that the failure of one of these components means the failure of whole system. Due to this fact system may have less durability so to avoid this it is usual to use redundant components on parallel to guarantee a certain level of reliability. From time to time study of optimization of reliability of complex systems, their analysis and evaluation have been getting the attention of several authors so it is worth noting to mention some of the contributions. Djerdjour and Rekab [5] presented a nonlinear integer programming model n the evaluation of reliability of series- parallel system. Ruan and Sun [15] derived exact solution for cost minimization by using greedy algorithm. Castro et al. [1] introduced an algebraic method based on Grobnerbase to obtain exact solution of cost minimization of series - parallel system. Xiao et al. [21] proposed sensitivity analysis method based on P-boxes interval algorithm, linear aggression analysis. Chun et al. [3] developed Chun-song-paulino(CSP) method to compute parameter sensitivities of system failure probability by using sequential compounding method. Lee et al. [10] compared of max-min approach and Nakagawa and Nakishima method for reliability of series-parallel system.Vargas et al. [18] obtained the exact solution of cost minimization of series-parallel system with multiple component choices using an algebraic method and they also compared to other approaches in the literature and standard nonlinear solvers. El. Said and Raghab [6] studied the availability and reliability of the of the preventive maintainable model with two types failures (type 1 and type 2) by using supplementary techniques and laplace transforms.

Redundancy is a common approach to improve the reliability and availability of a system. Adding redundancy increases the cost and complexity of a system design and with the high reliability. If cost does not play vital role in this case use of redundant components may be the attractive options such as machinery of aero plane, equipments of nuclear power plants. As we know that use of redundant obviously increase the reliability but motto of the scientist is to design the structure of redundant components so as to minimize the number of components and cost ultimately. Many researchers are engaged to model the structure of redundant components of complex system. Tian et al. [16] applied a practical approach for joint reliability redundancy optimization of multistate series-parallel systems. Liang and Smith [11] solved redundancy allocation problem by using an ant colony meta-heuristic optimization method. Yalaoui et al. [22] investigated the problem as an linear integer programming in redundancy allocation in a parallel-series system reliability by with the help of decomposition approach. Linmin [12] introduced the availability equivalence factor method to compare different system designs wherein two types of availability equivalence factors of the repairable multi-state series-parallel system are derived. Dhillon [4] made the study of income optimization analysis of a repairable system composed of two units in parallel and one warm standby. Tian et al.[?] developed recursive algorithms for efficient performance evaluations of the multi-state k-out-of-n system models. Wang et al. [19] investigated a multi-state Markov repairable system with redundancies to evaluate the reliability. Li and Zuo [20] presented a recursive algorithms for reliability evaluation of multi-state weighted k-out-of-n systems. Hu et al. [8] extended the concept of availability equivalence from general binary repairable system to discrete multistate parallel - series system with different performance rate by universal generating function technique.

In this paper we set up the system of equations and derive the explicit formula for sequential series - parallel system by using induction method and we also obtain the optimal number of redundant components of the system of general series - parallel system and sequential series - parallel system under the consideration of cost minimization. The comparative study of both types of reliability- configuration is carried out from the cost point of view.

2. Mathematical model

m number of subsystem.

n_i number of different types of available components for the i-th subsystem,$i = 1, 2, ..., m$.

R_{i_j} reliability of the j-component for the i-th subsystem,$i = 1, ..., m, j = 1, ..., n_i$.

R_0 admissible level of reliability of the whole system.

x_{i_j}number of redundant of j-th components used in i-th subsystem, $i = 1, ..., m, j = 1, ..., n_i$

c_{i_j}cost of j-th component for i-th sub system,$i = 1, ..., m, j = 1, ..., n_i$

u_{i_j}upper bounds of number of j-th components for the i-th subsystem,$i = 1, ..., m, j = 1, ..., n_i$

λ_{i_j} rate of failure of j-th component for the i-th subsystem, $i = 1, ..., m, j = 1, ..., n_i$
In our model, we make the following assumptions:

- Components have two states: working or failed.
- The reliability of each component is known and is deterministic.
- Failure of individual components are independent.
- Failed components do not damage other components or the system, and they are not repaired.

2.1 Reliability of series - parallel system

A series parallel system consists of m disjoint subsystem that are connected in series and subsystem i for $1 \leq i \leq m$ consists of n_i components that are connected in parallel. The reliability block diagram of a series parallel system is given in Figure. In such a series parallel system, there are m minimal cuts and they do not have any components in common.The reliability of this system can be obtained by:

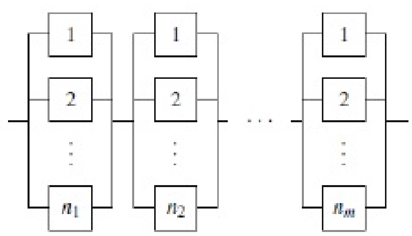

Figure 1: Series-parallel system

$$R_{sp} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n_i}(1 - R_{i_j})]; R_{sp} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n_i}(1 - e^{-\lambda_{i_j} t})] \tag{2.1}$$

2.2. Reliability of Perfect Sensing and Switching Mechanism

Perfect Sensing and Switching Mechanism When the sensing and switching mechanism is perfect, that is, instantaneous and failure free, a standby component is switched into operation as soon as the active component becomes failed. The system fails when the last component fails in active operation.

For a cold standby system with n different components.
when $n = 1$,$R_1(t) = e^{-\lambda_t}$
when $n = 2$,

$$R_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1(t)} + \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_2(t)} = [\sum_{j=1}^{2} e^{-\lambda_j(t)} \prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j}]$$

when $n = 3$,

$$\begin{aligned} R_3(t) &= \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1(t)} + \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_2(t)} \\ &\quad - \frac{\lambda_1 \lambda_2 [(\lambda_2 - \lambda_3) e^{-\lambda_1(t)} + (\lambda_3 - \lambda_1) e^{-\lambda_2(t)} + (\lambda_1 - \lambda_2) e^{-\lambda_3(t)}]}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_1)} \\ &= [\sum_{j=1}^{3} e^{-\lambda_j(t)} \prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j}] \end{aligned}$$

Therefore in general for n number of components in sequential standby system, the system reliability is

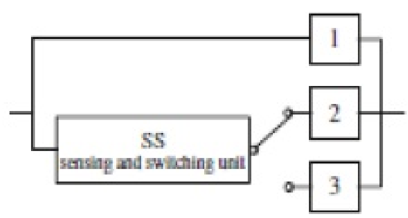

Figure 2: sequensing standby system

$$R_s(t) = \sum_{j=1}^{n} e^{-\lambda_j t} \prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j} \tag{2.2}$$

2.3 Sequential Series-parallel System

A sequential series - parallel system consists of m disjoint subsystem that are connected in series and subsystem i for $1 \leq i \leq m$ consists of n_i components that are connected in parallel. The reliability block diagram of a series.parallel sequencing system is given in figure. In such a sequential series - parallel system, there are m minimal cuts and they do not have any components in common.

For i=1 and $n_1 = 1$,

$$R_s(t) = e^{-\lambda_{1_1} t} = [\sum_{j=1}^{n_1=1} e^{-\lambda_{1_j} t} \prod_{\alpha \neq j} \frac{\lambda_{1_\alpha}}{\lambda_{1_\alpha} - \lambda_{1_j}}]$$

For i=2 and $n_1 = 1, n_2 = 2$

$$\begin{aligned} R_s(t) &= [e^{-\lambda_{2_1} t}[\frac{\lambda_{2_2}}{\lambda_{2_2} - \lambda_{2_1}}] + e^{-\lambda_{2_2} t}[\frac{\lambda_{2_1}}{\lambda_{2_1} - \lambda_{2_2}}]] * e^{-\lambda_{1_1} t} \\ &= [\sum_{j=1}^{n_2=2} e^{-\lambda_{2_j} t} \prod_{\alpha \neq j} \frac{\lambda_{2_\alpha}}{\lambda_{2_\alpha} - \lambda_{2_j}}] * e^{-\lambda_{1_1} t} \\ &= \prod_{i=1}^{2} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}} \end{aligned}$$

For i=3 and $n_1 = 1, n_2 = 2, n_3 = 3$

$$\begin{aligned} R_s(t) &= [e^{-\lambda_{3_1} t}[\frac{\lambda_{3_2}}{\lambda_{3_2} - \lambda_{3_1}} \frac{\lambda_{3_3}}{\lambda_{3_3} - \lambda_{3_1}}] + e^{-\lambda_{3_2} t}[\frac{\lambda_{3_1}}{\lambda_{3_1} - \lambda_{3_2}} \frac{\lambda_{3_3}}{\lambda_{3_3} - \lambda_{3_2}}] \\ &\quad + e^{-\lambda_{3_3} t}[\frac{\lambda_{3_1}}{\lambda_{3_1} - \lambda_{3_3}} \frac{\lambda_{3_2}}{\lambda_{3_2} - \lambda_{3_3}}]] * [e^{-\lambda_{2_1} t}[\frac{\lambda_{2_2}}{\lambda_{2_2} - \lambda_{2_1}}] + e^{-\lambda_{2_2} t}[\frac{\lambda_{2_1}}{\lambda_{2_1} - \lambda_{2_2}}]] * e^{-\lambda_{1_1} t} \\ &= [\sum_{j=1}^{n_3=3} e^{-\lambda_{3_j} t} \prod_{\alpha \neq j} \frac{\lambda_{3_\alpha}}{\lambda_{3_\alpha} - \lambda_{3_j}}] * [\sum_{j=1}^{n_2=2} e^{-\lambda_{2_j} t} \prod_{\alpha \neq j} \frac{\lambda_{2_\alpha}}{\lambda_{2_\alpha} - \lambda_{2_j}}] * e^{-\lambda_{1_1} t} \\ &= \prod_{i=1}^{3} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}} \end{aligned}$$

Therefore in general for m subsystem and n_i components in sequential series - parallel system, the system reliability is

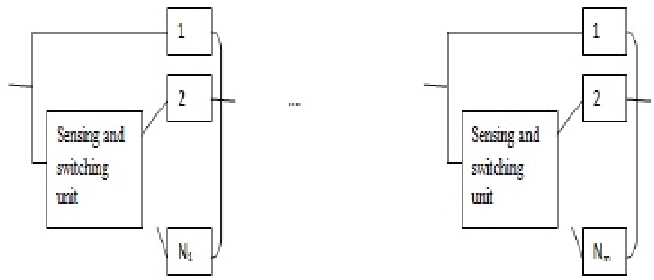

Figure 3: Sequential series - parallel system standby system

$$R_s t = \prod_{i=1}^{m} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}} \tag{2.3}$$

where $i = 1, 2, 3, ..., m,; \; j = 1, 2, 3, ..., n_i; \; \alpha \neq j; \alpha \in j.$

2.4 Optimization of the Model

The optimization problem can be formulated for series parallel system as:

$$\text{minimize} \quad C(x) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} C_{i_j} * (x_{i_j})$$

subject to the constraints

$$R_s(t) = \prod_{i=1}^{m} [1 - \prod_{j=1}^{n_i} (1 - R_{i_j})^{x_{i_j}}] \geq R_0$$

$$x \in X = (x | 0 \leq x_{i_j} \leq u_{i_j}, i = 1, 2, ..., m, j = 1, 2, .., n_i) \tag{2.5}$$

For sequential series - parallel standby system

$$\text{minimize} \;\; C(x) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} C_{i_j} * (x_{i_j})$$

subject to the constraints

$$R_s(t) = \prod_{i=1}^{m} \sum_{j=1}^{n_i} (R_{i_j})^{x_{i_j}} (\prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}}) \geq R_0$$

$$x \in X = (x | 0 \leq x_{i_j} \leq u_{i_j}, i = 1, 2, ..., m, j = 1, 2, .., n_i) \tag{2.6}$$

3. Numerical Results and Interpretations

(i). If $m = 1$ and $n = 1$ and $c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3$
then the graph given by equation(1) and (2) is shown in figure(4).

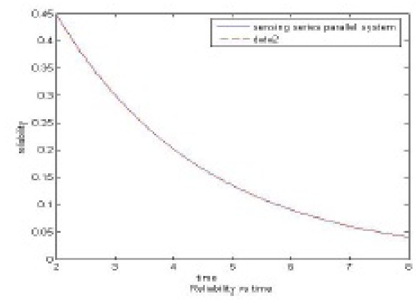

Figure 4: graph of reliability vs time

Graph explores that for single component reliability remain the same for both the structures, sequential series - parallel system and series-parallel configurations which is quite natural.

(ii). If $m = 3, n_1 = 1, n_2 = 2$ and $n_3 = 3$ and $c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3$
using the same above equations(1)and (2), the graph is shown in figure (5).

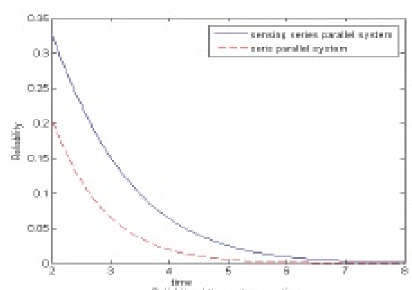

Figure 5: graph of reliability vs time

Graph explains that sequential series - parallel system has higher reliability than that of ordinary series-parallel system.

(iii). For general series-parallel system

$c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3; u_{1_1} = 5; u_{2_1} = 6; u_{2_2} = 7; u_{3_1} = 8; u_{3_2} = 9; u_{3_3} = 10; R0 = 0.9;$

ov	X11	X21	X22	X31	X32	X33	
46.2611	3.7617	0.2198	0.5203	0.1125	9.5213	1.4416	Cv1/cv2
38.3979	1.5775	0.8557	1.7006	2.6964	1.1927	3.3805	Cv1/cv3
40.8899	3.9996	1.4555	2.4431	3.5837	2.0076	1.1123	Cv1/cv4
37.7292	6.3822	0.0023	3.3563	2.7510	0.4453	0.9390	Cv1/cv5
40.0898	2.6590	0.6108	3.3277	2.9242	4.0248	0.3411	Cv1/cv6
34.1067	1.2377	0.9608	5.5147	0.0652	1.2569	0.5578	Cv1/cv7
34.4025	3.1647	1.9242	2.3317	2.1984	0.7944	1.2387	Cv2/cv3
37.5462	1.7079	2.0323	0.9468	0.3867	3.9336	2.3344	Cv2/cv4
42.3928	0.8843	5.8209	0.2110	0.0472	5.5409	1.1201	Cv2/cv5
51.1166	0.4176	0.5704	0.5902	4.7350	3.8403	5.0437	Cv2/cv6
46.0402	4.0704	1.9048	2.4611	9.3165	1.0679	0.1008	Cv2/cv7
27.2893	3.1290	1.7399	1.0116	0.2902	3.6055	0.0737	Cv3/cv4
44.1769	1.0804	1.1860	1.4662	1.4251	5.3696	2.7269	Cv3/cv5
27.1657	5.3686	1.7138	0.0331	3.4728	1.3012	0.0611	Cv3/cv6
40.5406	4.8424	0.2733	2.1576	1.4775	0.8291	3.1926	Cv3/cv7
47.5564	4.4598	5.5917	3.3391	3.2874	0.0054	0.3828	Cv4/cv5
30.8343	5.2670	0.0722	1.6998	0.0272	3.0167	0.8361	Cv4/cv6
20.4360	1.3419	0.1245	1.2953	1.6686	2.6025	0.2105	Cv4/cv7
42.9488	0.5704	4.9795	2.9370	0.6128	2.1027	1.5175	Cv5/cv6
19.0703	1.0095	1.1185	0.1306	4.0371	1.2874	0.2474	Cv5/cv7
52.1194	2.0040	5.2186	1.3688	4.5438	0.4352	3.3174	Cv6/cv7

Figure 6: Optimal solution table with objective functions of series- parallel system

(iv). For sequential series - parallel system

$c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3; u_{1_1} = 5; u_{2_1} = 6; u_{2_2} = 7; u_{3_1} = 8; u_{3_2} = 9; u_{3_3} = 10; R0 = 0.9;$

cv	X11	X21	X22	X31	X32	X33	
8.1558	0.3889	0.1641	0.0229	3.1007	0.0717	0.0755	Cv1/cv2
11.3478	0.4906	1.0873	0.6850	1.4988	0.2449	0.1233	Cv1/cv3
14.6098	1.4958	0.4493	0.3177	1.6581	1.7241	0.1022	Cv1/cv4
10.6537	1.0357	0.4009	0.2860	1.7173	0.3350	0.3592	Cv1/cv5
10.6715	0.2266	1.3577	0.7145	0.1680	0.8285	0.0932	Cv1/cv6
12.1885	2.0001	0.5052	0.2966	0.0729	1.5002	0.1680	Cv1/cv7
13.0242	0.2522	0.0901	1.7690	0.1277	1.6049	0.0206	Cv2/cv3
5.0559	0.5886	0.0699	0.1021	0.5434	0.6559	0.0412	Cv2/cv4
13.6288	0.2156	0.1835	0.4183	2.5252	1.3396	0.3810	Cv2/cv5
13.2558	2.1561	0.5467	0.2973	0.4978	0.3182	0.8329	Cv2/cv6
11.2164	1.1155	0.1659	0.1778	1.4078	0.4829	0.7024	Cv2/cv7
11.2295	0.1041	0.9148	0.1123	2.1518	0.1840	0.5945	Cv3/cv4
14.4785	2.5259	0.2604	0.3037	1.0557	0.7967	0.5858	Cv3/cv5
12.6107	0.8015	0.8817	0.7735	0.0596	0.6123	0.6025	Cv3/cv6
12.9255	2.2910	0.4698	1.3831	0.3175	0.2346	0.0125	Cv3/cv7
15.8497	3.0951	0.1918	1.2136	0.1206	0.4876	0.5051	Cv4/cv5
10.8146	0.1277	0.2246	1.8961	0.2371	0.4207	0.1129	Cv4/cv6
11.0046	0.5807	0.4845	1.0514	0.4270	0.1057	0.6026	Cv4/cv7
10.5409	0.1386	0.3781	0.9163	1.6681	0.6721	0.0223	Cv5/cv6
12.2999	0.5240	0.8184	0.3654	0.8148	0.2172	1.0108	Cv5/cv7
9.9156	0.5668	0.5174	0.8592	0.2593	0.7524	0.2035	Cv6/cv7

Figure 7: Optimal solution table with objective functions of sequencing series-parallel system

For the same objective function of series-parallel reliability system when we consider first and second constraints the minimum cost comes out to be 46.2181 for the number of redundant components $x_{1_1} = 4$, $x_{2_1} = 0$, $x_{2_2} = 1$, $x_{3_1} = 0$, $x_{3_2} = 10$, $x_{3_3} = 1$. Further more minimum costs are obtained by taking two constraints among many constraints at a time in the alteration . Among them least minimum cost is 19.07703 corresponding to redundant components $x_{1_1} = 1$, $x_{2_1} = 1$, $x_{2_2} = 0$, $x_{3_1} = 4$, $x_{3_2} = 1$, $x_{3_3} = 0$. Where as in sequential series - parallel system by the same alteration of two constraints the least minimum cost value is 8.1558 corresponding to redundant components $x_{1_1} = 0$, $x_{2_1} = 0$, $x_{2_2} = 0$, $x_{3_1} = 3$, $x_{3_2} = 0$, $x_{3_3} = 0$.

Thus we come to conclusion that optimal solution in sequential series - parallel system have least cost with compared to ordinary series-parallel system obtained by Vargas et al. [18]. Optimization the tables show that cost is minimum in every corresponding constraints taken in sequential series - parallel system compared to ordinary series-parallel system which also agrees with the graphical illustrations of reliability that the system has maximum reliability in sequential series-parallel system compared to ordinary series-parallel system.

4. Discussion and Conclusion

We have derived the explicit formula for sequential series - parallel system and we have also obtained the optimal number of redundant components of general series - parallel system and sequencing series - parallel system under the consideration of cost minimization. The comparative study of both type of reliability- configuration has also been carried out from the cost point of view. Sequencing series- parallel system works more efficiently than ordinary series-parallel system not only by point of view of to reliability but also from the decrease of cost of the operations of the system. The reliability and optimization of series-parallel system and sequencing series-parallel system may have ubiquitous applications in nuclear power plants, aero space industries, telecommunications, complex manufacturing system, textiles industries and machinery systems.

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