

THEORY OF INTUITIONISTIC FUZZY SOFT MATRIX AND ITS APPLICATION IN DECISION MAKING PROBLEM USING MEDICAL DIAGNOSIS

R. RATHIKA¹ AND S. SUBRAMANIAN²

¹ Research scholar, Department of Mathematics,
Prist University, Thanjavur, Tamil Nadu, India

² Department of Mathematics,
Prist University, Thanjavur, Tamil Nadu, India

Abstract

Soft set theory is a newly mathematical tool to deal with uncertain problems. It has a rich potential for application in solving practical problems in economics, social science, medical science etc. The concept of fuzzy soft sets extended fuzzy soft set to Intuitionistic fuzzy soft sets .In this paper we proposed intuitionistic fuzzy soft matrices and we define intuitionistic fuzzy soft set aggregative operator that allows constructing more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contain uncertainties.

1. Introduction

In the fuzzy set theory [15] there were no scopes to think about the hesitation in the membership degree, which arise in various real life situations. To overcome these

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situations Atanassov [1] introduced theory of intuitionistic fuzzy set in 1986 as a generalization of fuzzy set. Most of the problems in engineering, medical science, economics, environments etc have various uncertainties. Molodtsov [12] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. Research works on soft set theory are progressing rapidly. Maji et al. [8] defined several operations on soft set theory. Combining soft sets with fuzzy sets and intuitionistic fuzzy sets, Feng et al. [7] and Maji et al. [9,10] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. Matrices play an important role in the broad area of science and engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. In [14] Yang et al, initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. The concept of fuzzy soft matrix theory was studied by Borah et al. in [2]. In [5], Chetia et al. and in [13] Rajarajeswari et al. defined intuitionistic fuzzy soft matrix. Again it is well known that the matrices are important tools to model/study different mathematical problems specially in linear algebra. Due to huge applications of imprecise data in the above mentioned areas, hence are motivated to study the different matrices containing these data. Soft set is also one of the interesting and popular subject, where different types of decision making problem can be solved. So attempt has been made to study the decision making problem by using intuitionistic fuzzy soft aggregation operator. Das and Kar [6] proposed an algorithmic approach for group decision making based on IF soft set. The authors [6] have used cardinality of IF soft set as a novel concept for assigning confident weight to the set of experts. Cagman and Enginogh [3, 4] pioneered the concept of soft matrix to represent a soft set. Mao et al. [11] presented the concept of intuitionistic fuzzy soft matrix (IFSM) and applied it in group decision making problem.

In this paper, we define intuitionistic fuzzy soft set aggregative operator that allows constructing more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contain uncertainties.

2. Preliminaries and Definitions

In this section we briefly review some basic definitions related to fuzzy soft set, intuition-

istic fuzzy soft sets, soft sets and intuitionistic fuzzy soft matrix their generalizations, which will be used in the rest of the paper.

Definition 2.1 : Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power Set of U . Let $A \subseteq E$. A pair (F_A, E) is called a **soft set** over U , where F_A is a mapping given by : $E \rightarrow P(U)$ Such that $F_A(e) = \varphi$ if $e \notin A$. Here F_A is called approximate function of the soft set (F_A, E) . The set $F_A(e)$ is called e -approximate value set which consist of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Example 2.1 : Let $U = \{e_1, e_2, e_3, e_4\}$ be a set of four pens and $E = \{\{e_1, e_2, e_3, e_4\} = \{\text{black } (e_1), \text{red } (e_2), \text{blue } (e_3), \text{green } (e_4)\}\}$ be a set of parameters. If $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Let $F_A(e_1) = \{u_1, u_2, u_3, u_4\}$ and $F_A(e_2) = \{u_1, u_4\}$, $F_A(e_3) = \{u_1, u_3, u_4\}$, $F_A(e_4) = \{u_4\}$ then we write the soft set $(F_A, E) = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_4\}), (e_3, \{u_1, u_3, u_4\}), (e_4, \{u_4\})\}$ over U which describe “the colour of the pens” which Mr. A is going to buy. We may represent the fuzzy soft set in the following form :

U	e_1	e_2	e_3	e_4
u_1	1	1	1	0
u_2	1	0	0	0
u_3	1	0	1	0
u_4	1	1	1	1

Definition 2.2 : Let U be an initial universe, E be the set of all parameters and $A \subseteq E$. A pair (F_A, E) is called a **fuzzy soft set** over U where F_A is a mapping given by, $F_A : E \rightarrow P(U)$ such that $F_A(e) = \varphi$ if $e \notin A$, where φ is a null fuzzy set and $\tilde{P}(U)$ denotes the collection of all subsets of U .

Example 2.2 : Consider the Example 2.1, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval $[0, 1]$. Then $(F_A, E) = \{F_A(e_1) = \{(u_1, 0.8), (u_2, 0.6), (u_3, 0.5), (u_4, 0.2)\}, F_A(e_2) = \{(u_1, 0.5), (u_4, 0.2)\}, F_A(e_3) = \{(u_1, 0.6), (u_3, 0.4)(u_4, 0.8)\}, F_A(e_4) = \{(u_4, 0.4)\}$ is the fuzzy soft set representing the “colour of the pens” which Mr. A is going to buy. We

may represent the fuzzy soft set in the following form:

U	e_1	e_2	e_3	e_4
u_1	0.8	0.5	0.6	0
u_2	0.6	0	0	0
u_3	0.5	0.0	0.4	0
u_4	0.2	0.2	0.8	0.4

Definition 2.3 : Let (F_A, E) be fuzzy soft set over U . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$, which is called relation form of (F_A, E) . The characteristic function of R_A is written by $\mu_{R_A} : U \times E \rightarrow [0, 1]$, where $\mu_{R_A}(u, e) \in [0, 1]$ is the membership value of $u \in U$ for each $e \in E$. If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{pmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{pmatrix}$$

which is called an $m \times n$ soft matrix of the soft set (F_A, E) over U . Therefore we can say that a fuzzy soft set (F_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

Example 2.3 : Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters. If $A \subseteq E = \{e_1, e_2, e_3, e_4\}$ and $F_A(e_1) = \{(u_1, .7), (u_2, .6), (u_3, .8), (u_4, .2), (u_5, .7), (u_6, .8)\}$, $F_A(e_2) = \{(u_1, .5), (u_3, .8), (u_4, .1), (u_5, .2), (u_6, .9)\}$, $F_A(e_3) = \{(u_1, .5), (u_2, .7), (u_4, .5), (u_5, .6), (u_6, .7)\}$, $F_A(e_4) = \{(u_1, .9), (u_6, .1)\}$. Then the fuzzy soft set (F_A, E) is a parameterized family $\{F_A(e_1), F_A(e_2), F_A(e_3), F_A(e_4)\}$ of all fuzzy sets over U . Hence the fuzzy soft matrix

$[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} 0.7 & 0.5 & 0.5 & 0.9 \\ 0.6 & 0.0 & 0.7 & 0.0 \\ 0.8 & 0.8 & 0.0 & 0.0 \\ 0.2 & 0.1 & 0.5 & 0.0 \\ 0.7 & 0.2 & 0.6 & 0.0 \\ 0.8 & 0.9 & 0.7 & 0.1 \end{bmatrix}$$

Definition 2.4 : A fuzzy soft matrix of order $1 \times n$ i.e., with a single row is called a row-fuzzy soft Matrix.

Definition 2.5 : A fuzzy soft matrix of order $m \times 1$ i.e., with a single column is called a column]fuzzy soft matrix.

3. Intuitionistic Fuzzy Soft Matrix Theory

3.1 Intuitionistic Fuzzy Soft Set (IFSS)

Let U be an initial universe, E be the set of parameters and $A \subseteq E$. A pair (F_A, E) is called an intuitionistic fuzzy soft set (IFSS) over U , where F_A is a mapping given by $F_A : E \rightarrow I^U$, where I^U denotes the collection of all intuitionistic fuzzy subsets of U .

Example 3.1 : Suppose that $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{\text{white}(e_1), \text{blue}(e_2), \text{green}(e_3)\}$ be a set of parameters. If $A = \{e_1, e_2\} \in E$. Let $F_A(e_1) = \{(u_1, 0.3, 0.7), (u_2, 0.8, 0.1), (u_3, 0.4, 0.2), (u_4, 0.6, 0.2)\}$ $F_A(e_2) = \{(u_1, 0.8, 0.1), (u_2, 0.9, 0.1), (u_3, 0.4, 0.5), (u_4, 0.2, 0.3)\}$ then we write intuitionistic fuzzy soft set is

$$(F_A, E) = \{F_A(e_1) = \{(u_1, 0.3, 0.7), (u_2, 0.8, 0.1), (u_3, 0.4, 0.2), (u_4, 0.6, 0.2)\}$$

$$F_A(e_2) = \{(u_1, 0.8, 0.1), (u_2, 0.9, 0.1), (u_3, 0.4, 0.5), (u_4, 0.2, 0.3)\}\}.$$

We would represent this intuitionistic fuzzy soft set in matrix form as

$$\begin{bmatrix} (.3, .7) & (.8, .1) & (.0, .0) \\ (.8, .1) & (.9, .1) & (.0, .0) \\ (.4, .2) & (.4, .5) & (.0, .0) \\ (.6, .2) & (.2, .3) & (.0, .0) \end{bmatrix}.$$

3.2. Intuitionistic Fuzzy Soft Matrix (IFSM) [5]

Let U be an initial universe, E be the set of parameters and $A \subseteq E$. Let (F_A, E) be an intuitionistic fuzzy soft set (IFSS) over U . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$ which is called relation form of (F_A, E) . The membership and non-membership functions of are written by $\mu_{R_A} : U \times E \rightarrow [0, 1]$ and $\gamma_{R_A} : U \times E \rightarrow [0, 1]$ where $\mu_{R_A} : (u, e) \in [0, 1]$ and $\gamma_{R_A} : (u, e) \in [0, 1]$ are the membership value and nonmembership value of $u \in U$ for each $e \in E$. If $(u_{ij}, v_i) = (\mu_{R_A}(u_i, e_j), \gamma_{R_A}(u_i, e_j))$ we can define a

$$[(u_{ij}, v_{ij})] = \begin{pmatrix} (\mu_{11}, v_{11}) & (\mu_{12}, v_{12}) & \cdots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (\mu_{22}, v_{22}) & \cdots & (\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1}, v_{m1}) & (\mu_{m2}, v_{m2}) & \cdots & (\mu_{mn}, v_{mn}) \end{pmatrix}$$

which is called an $m \times n$ IFSM of the IFSS (F_A, E) over U . Therefore, we can say that IFSS (F_A, E) is uniquely characterized by the matrix $[(\mu_{ij}, v_{ij})]_{m \times n}$ and both concepts are interchangeable. The set of all $m \times n$ IFS matrices will be denoted by IFS $M_{m \times n}$.

Example 3.2 : Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of parameters. If $A = \{e_1, e_3, e_4, e_5\} \subseteq E$ and $F_A(e_1) = \{(u_1, .8, .4), (u_2, .8, .1), (u_3, .5, .5), (u_4, .5, .4), (u_5, .2, .1)\}$, $F_A(e_3) = \{(u_1, .4, .6), (u_3, .2, .2), (u_4, 1, 0), (u_5, .6, .2)\}$, $F_A(e_4) = \{(u_1, .6, .2), (u_2, 1, 0), (u_3, .8, .2), (u_4, .6, .3), (u_5, .7, .3)\}$, $F_A(e_5) = \{(u_1, .7, .8), (u_2, 1, 0), (u_3, .6, .5), (u_4, .5, .3), (u_5, .9, .2)\}$. Then the IFS set (F_A, E) is a parameterized family $\{F_A(e_1), F_A(e_2), F_A(e_3), F_A(e_4)\}$ of all IFS sets over U . Hence IFSM $[(\mu_{ij}, \gamma_{ij})]$ can be written as

$$[(\mu_{ij}, \gamma_{ij})] = \begin{bmatrix} (.8, .4) & 0, 0 & (.4, .6) & (.6, .2) & (.7, .8) \\ (.8, .1) & (0, 0) & (0, 0) & 1, 0 & (1, 0) \\ (.5, .5) & (0, 0) & (.2, .2) & (.8, .2) & (.5, .5) \\ (.5, .4) & 0, 0 & (1, 0) & (.6, .3) & (.5, .3) \\ (.2, .1) & (0, 0) & (.6, .2) & (.7, .3) & (.9, .2) \end{bmatrix}$$

3.3 Intuitionistic Fuzzy Soft Set Complement Matrix

Let $A = [a] = [a_{ij}]$ IFSM $_{m \times n}$, where $a_{ij} = (\mu_j(c_i), v_j(c_i))$ for all i, j . Then A^C IFSM is called a Intuitionistic Fuzzy Soft Complement Matrix if $A^C = [d_{ij}]_{m \times n}$, where $d_{ij} = (v_j(c_i), \mu_j(c_i))$ for all i, j .

3.4 Intuitionistic Fuzzy Soft Sub Matrix

Let $A = [a_{ij}]$ IFSM $_{m \times n}$, $B = [b_{ij}]$ IFSM $_{m \times n}$, then A is a intuitionistic fuzzy soft submatrix of B , denoted by $A \subseteq B$, if $\mu_A \leq \mu_B$ and $v_A \geq v_B \forall i, j$.

3.5 Intuitionistic Fuzzy Soft Null (Zero) Matrix

An intuitionistic fuzzy soft matrix of order $m \times n$ is called intuitionistic fuzzy soft null (zero) matrix. If all its elements are $(0, 1)$. It is denoted by Φ .

3.6 Intuitionistic Fuzzy Soft Universal Matrix

An intuitionistic fuzzy soft matrix of order $m \times n$ is called each intuitionistic fuzzy soft universal matrix if all its elements are $(1, 0)$. It is denoted by U .

3.7 Intuitionistic Fuzzy Soft Equal Matrix

$A = [a_{ij}]$ IFSM $_{m \times n}$, $B = [b_{ij}]$ IFSM $_{m \times n}$, then A is equal to B , denoted by $A = B$, if $\mu_A = \mu_B$ and $v_A = v_B \forall i, j$.

3.8 Intuitionistic Fuzzy Soft Transpose Matrix

Let $A = [a_{ij}]$ IFSM $_{m \times n}$, then A^T is a intuitionistic fuzzy soft transpose matrix of A if $A^T = [a_{ji}]$.

3.9 Intuitionistic Fuzzy Soft Rectangular Matrix

Let $A = [a_{ij}]$ IFSM $_{m \times n}$, where $a_{ij} = (\mu_j(c_i), v_j(c_i))$. Then A is called a Intuitionistic Fuzzy Soft rectangular Matrix if $m \neq n$.

3.10 Intuitionistic Fuzzy Soft Upper Triangular Matrix

Let $A = [a_{ij}]$ IFSM $_{m \times n}$, where $a_{ij} = (\mu_j(c_i), v_j(c_i))$. Then A is called a Intuitionistic Fuzzy Soft upper rectangular Matrix if $m = n$ and $a_{ij} = (0, 1) i > j$.

3.11 Intuitionistic Fuzzy Soft Lower Triangular Matrix

Let $A = [a_{ij}]$ IFSM $_{m \times n}$, where $a_{ij} = (\mu_j(c_i), v_j(c_i))$. Then A is called a Intuitionistic Fuzzy Soft lower rectangular Matrix if $m = n$ and $a_{ij} = (0, 1) i < j$.

4. Intuitionistic Fuzzy Soft Aggregation Operator

In this section we define an intuitionistic fuzzy soft aggregation operator that produces an aggregate intuitionistic fuzzy set from an intuitionistic fuzzy soft set and its cardinal set. The approximate functions of an IF Soft set are IF set. An IF Soft-set aggregation

operator on the IF sets is an operation by which several approximate functions of an IF Soft-set are combined to produce a single IF set which is the aggregate IF set of the IF Soft-set.

Definition 4.1 : Let $\Gamma_A \in IFS(U)$ then the cardinal set of Γ_A is denoted by $c\Gamma_A$, where $c\Gamma_A = \left\{ \frac{(\mu_{\lambda_A}(x), \vartheta_{\eta_A}(x))}{x} : X \in E, \lambda_A, \eta_A \in IFS(U) \right\}$ is an intuitionistic fuzzy soft set over E . $\mu_{c\lambda_A} : E \rightarrow [0, 1]$ and $\vartheta_{c\eta_A} = \frac{|\lambda_A(x)|}{|U|}, \vartheta_{\eta_A} : E \rightarrow [0, 1]$ and $\vartheta_{c\eta_A} = \frac{|\eta_A(x)|}{|U|}$, where $|U|$ is the cardinality of the universe U , $|\lambda_A(X)|$ and $|\eta_A(X)|$ are the scalar cardinality of the intuitionistic fuzzy soft sets $\lambda_A(X)$ and $\eta_A(x)$ respectively.

Definition 4.2 : Let $\Gamma_A \in IFSS(U)$ and $c\Gamma_A \in IFsS(U)$. Assume that $E = \{x_1, x_2, x_3, x_4\}$ and $A \subseteq E$ then $c\Gamma_A$ can be represented by the tabular form

E	x_1	x_2	\dots	x_n
$c\Gamma_A$	$(\mu_{\lambda_A}(x_1), \vartheta_{\eta_A}(x_1))$	$(\mu_{ \lambda_A}(x_2), \vartheta_{\eta_A}(x_1))$		$(\mu_{\lambda_A}(x_n), \vartheta_{\eta_A}(x_n))$

If $(a_{1j}, b_{1j}) = \mu_{\lambda_A}(x_j)$ for $j = 1, 2, 3, \dots, n$ then $c\Gamma_A$ is represented by a matrix given by $[a_{1j}, b_{1j}]_{1 \times n} = [(a_{11}, b_{11})(a_{12}, b_{12}) \dots (a_{1n}, b_{1n})]$ which is called the cardinal matrix of the cardinal set $c\Gamma_A$ over E .

Definition 4.4 : Let $\Gamma_A \in IFsS(U)$ and $c\Gamma_A \in IFsS(U)$. Then the intuitionistic fuzzy soft aggregation operator denoted by IFS_{agg} is defined by $IFS_{agg} : cIFSS(U) \times IFSS(U) \rightarrow IFSS(U)$. So $IFS_{agg}(c\Gamma_A, \Gamma_A) = \Gamma_{A^*}$ where

$$\Gamma_{A^*} = \left\{ \frac{(\mu_{\Gamma_{A^*}}(u))}{X} : u \in U \right\} = \left\{ \frac{(\mu_{\lambda_{A^*}}(u), \vartheta_{\eta_{A^*}}(u))}{X} : u \in U \right\}$$

is a IFSS over U . Γ_{A^*} is called aggregate IF set of the IFsoft set Γ_A . Then the membership and non-membership function of Γ_{A^*} is defined as $\mu_{\lambda_{A^*}} : U \rightarrow [0, 1]$.

$$\mu_{\lambda_{A^*}}(u) = \frac{1}{|E|} \sum_{X \in E} (\mu_{c\lambda_A}(x), \mu_{\lambda_A}(x))(u)$$

$$\vartheta_{\eta_{A^*}} : U \rightarrow [0, 1] \quad \text{and} \quad \vartheta_{\eta_{A^*}} = \frac{1}{|E|} \sum_{X \in E} (\vartheta_{c\eta_A}(x), \vartheta_{\eta_A}(x))(u)$$

where $|E|$ is called the cardinality of E .

Definition 4.3 : Let $\Gamma_A A \in IFsS(U)$ and Γ_{A^*} be its aggregate IF set. Assume that $U = \{u_1, u_2, \dots, u_m\}$ then $\Gamma_{A^*} A^*$ can be presented as

Γ_A	$\mu_{\Gamma_{A^*}}$
u_1	$\mu_{\Gamma_{A^*}}(u_1) = (\mu_{\lambda^*A}(u_1), \vartheta_{\eta^*A}(u_1))$
u_2	$\mu_{\Gamma_{A^*}}(u_2) = (\mu_{\lambda^*A}(u_2), \vartheta_{\eta^*A}(u_2))$
\vdots	\vdots
\vdots	\vdots
u_m	$\mu_{\Gamma_{A^*}}(u_m) = (\mu_{\lambda^*A}(u_m), \vartheta_{\eta^*A}(u_m))$

If $[a_{1j}, b_{1j}] = (\mu_{\Gamma^*(A)}(u_i))$ for $i = 1, 2, 3, \dots, m$ then Γ_{A^*} is uniquely characterized by the matrix

$$[a_{1j}, b_{1j}]_{m \times 1} = \begin{bmatrix} (a_{11}, b_{11}) \\ (a_{21}, b_{21}) \\ \vdots \\ \vdots \\ a_{m1}, b_{m1} \end{bmatrix}$$

which is called the aggregate matrix of Γ_{A^*} over U .

Theorem 1 : Let $\Gamma_A \in IFsS(U)$ and $A \subseteq E$. If $M_{\Gamma_A}, M_{c\Gamma_{A^*}}$ are represented matrices of $\Gamma_A, c\Gamma_A, \Gamma_{A^*}$ respectively then

$$|E| \times M_{\Gamma_{A^*}} = M_{\Gamma_A} \times M_{c\Gamma_A}^T,$$

where $M_{c\Gamma_A}^T$ is the transpose of $M_{c\Gamma_A}$ and $|E|$ is the cardinality of E .

Proof : It is sufficient to consider $|E| \times [a_{1j}, b_{1j}] = [a_{1j}, b_{1j}]_{m \times n} \times [a_{1j}, b_{1j}]_{1 \times n}$.

5. Algorithm

We have an aggregate IF set, now it is necessary to choose the best alternative form of this set. We can make a decision by the following algorithm.

- (1) Construct an intuitionistic fuzzy soft set Γ_A over U and choose the set of parameters.
- (2) Construct the intuitionistic fuzzy soft matrices for each of the parameters. Find the cardinal set $c\mu_{\lambda A}$ of i.e. $c\mu_{\lambda A}$ and $c\vartheta_{\eta A}$.

- (3) Find the aggregate Γ_{A^*} of Γ_A .
- (4) Find the matrix of the set that has the largest membership grade by $\max \mu_{\lambda A} * (u)$ and the smallest non membership grade by $\min \vartheta_{\eta A} * (u)$.

5.1 Case Study

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the set of five persons related with high blood sugar. The five persons have been went to a hospital for medical treatment and all the five persons were identified by high blood sugar and in this case all the person were given proper medical treatment and all of them have been cured, but particularly one person was cured in an extraordinary manner by getting maximum score point.

Let us consider $E = \{e_1, e_2, e_3, e_4, e_5\}$ as the set of parameter for choosing the service rendered to patients by the hospitals.

e_1 is the patients suffers from increased thirst.

e_2 is the patients suffers from headaches.

e_3 is patients suffers from weight loss.

e_4 is patients suffers from blurred vision.

And e_5 is patients suffers from fatigue (weak, feeling tried) respectively. Finally the expert committee applies the following steps.

Step 1 : The committee construct an intuitionistic fuzzy soft set $\Gamma_{A,A}$ over U IF soft set

$$\Gamma_A = \left\{ \left(e_1, \left(\frac{(0.3, 0.3)}{u_1}, \frac{(0.1, 0.2)}{u_2}, \frac{(0.3, 0.3)}{u_3}, \frac{(0.3, 0.3)}{u_4} \right) \right), \right. \\ \left(e_2, \left(\frac{(0.5, 0.4)}{u_1}, \frac{(0.3, 0.5)}{u_3}, \frac{(0.3, 0.6)}{u_4}, \frac{(0.1, 0.8)}{u_5} \right) \right), \\ \left(e_3, \left(\frac{(0.4, 0.6)}{u_2}, \frac{(0.4, 0.5)}{u_3}, \frac{(0.9, 0.1)}{u_4}, \frac{(0.3, 0.6)}{u_5} \right) \right), \\ \left(e_4, \left(\frac{(0.2, 0.6)}{u_1}, \frac{(0.5, 0.4)}{u_2}, \frac{(0.1, 0.4)}{u_3}, \frac{(0.7, 0.3)}{u_4}, \frac{(0.1, 0.6)}{u_5} \right) \right), \\ \left. \left(e_5, \left(\frac{(0.5, 0.4)}{u_1}, \frac{(0.1, 0.7)}{u_2}, \frac{(0.8, 0.2)}{u_3}, \frac{(0.7, 0.2)}{u_5} \right) \right) \right\}$$

Step 2 : The intuitionistic fuzzy soft matrices for each of the parameters

$$\begin{bmatrix} (0.3, 0.8) & (0.1, 0.2) & (0.1, 0.7) & (0.7, 0.2) & (0.0, 0.0) \\ (0.5, 0.4) & (0.0, 0.0) & (0.3, 0.5) & (0.3, 0.6) & (0.1, 0.8) \\ (0.0, 0.0) & (0.4, 0.6) & (0.4, 0.5) & (0.9, 0.1) & (0.3, 0.6) \\ (0.2, 0.6) & (0.5, 0.4) & (0.1, 0.4) & (0.7, 0.3) & (0.1, 0.6) \\ (0.5, 0.4) & (0.1, 0.7) & (0.8, 0.2) & (0.0, 0.0) & (0.7, 0.2) \end{bmatrix}$$

Compute cardinality

$$c\Gamma_A = \left\{ \frac{(0.24, 0.38)}{e_1}, \frac{(0.24, 0.46)}{e_2}, \frac{(0.4, 0.36)}{e_3}, \frac{(0.32, 0.44)}{e_3}, \frac{(0.32, 0.44)}{e_4}, \frac{(0.42, 0.3)}{e_5} \right\}$$

In the IFSM form is

$$\begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.24, & 0.28 \\ 0.24, & 0.46 \\ 0.40, & 0.36 \\ 0.32, & 0.44 \\ 0.42, & 0.3 \end{bmatrix} .$$

Step 3 : The aggregate IF set is obtained by using theorem 1

$$\begin{aligned}
 M_{\Gamma^*A} &= \frac{1}{5} \begin{bmatrix} (0.3, 0.8) & (0.1, 0.2) & (0.1, 0.7) & (0.7, 0.2) & (0.0, 0.0) \\ (0.5, 0.4) & (0.0, 0.0) & (0.3, 0.5) & (0.3, 0.6) & (0.1, 0.8) \\ (0.0, 0.0) & (0.4, 0.6) & (0.4, 0.5) & (0.9, 0.1) & (0.3, 0.6) \\ (0.2, 0.6) & (0.5, 0.4) & (0.1, 0.4) & (0.7, 0.3) & (0.1, 0.6) \\ (0.5, 0.4) & (0.1, 0.7) & (0.8, 0.2) & (0, 0, 0, 0) & (0.7, 0.2) \end{bmatrix} \\
 & \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.24, & 0.28 \\ 0.24, & 0.46 \\ 0.40, & 0.36 \\ 0.32, & 0.44 \\ 0.42, & 0.3 \end{bmatrix} . \\
 &= \left(\frac{1}{5} \begin{pmatrix} \begin{bmatrix} 0.3 & 0.1 & 0.1 & 0.7 & 0.0 \\ 0.5 & 0.0 & 0.3 & 0.3 & 0.1 \\ 0.0 & 0.4 & 0.4 & 0.9 & 0.3 \\ 0.2 & 0.5 & 0.1 & 0.7 & 0.1 \\ 0.5 & 0.1 & 0.8 & 0.0 & 0.7 \end{bmatrix} \begin{bmatrix} 0.24 \\ 0.24 \\ 0.40 \\ 0.32 \\ 0.42 \end{bmatrix} \\ * \frac{1}{5} \begin{pmatrix} \begin{bmatrix} 0.8 & 0.2 & 0.7 & 0.2 & 0.0 \\ 0.4 & 0.0 & 0.5 & 0.6 & 0.8 \\ 0.0 & 0.6 & 0.5 & 0.1 & 0.6 \\ 0.6 & 0.4 & 0.4 & 0.3 & 0.6 \\ 0.4 & 0.7 & 0.2 & 0.0 & 0.2 \end{bmatrix} \begin{bmatrix} 0.38 \\ 0.46 \\ 0.36 \\ 0.44 \\ 0.30 \end{bmatrix} \end{pmatrix} \right) \\
 &= \begin{bmatrix} 0.0720, & 0.1472 \\ 0.0756, & 0.1592 \\ 0.1340, & 0.1360 \\ 0.0948, & 0.1616 \\ 0.1516, & 0.1132 \end{bmatrix} .
 \end{aligned}$$

Step 4 : The patient u_5 has the largest membership grade i.e. 0.1516 and smallest non membership grade i.e. 0.1132 and that patient was highly cured and decision was in favour of patient u_5 who is observing as maximum curable person.

6. Conclusion

In this work we define intuitionistic fuzzy soft aggregation operator for intuitionistic fuzzy soft set and construct an algorithm using intuitionistic fuzzy soft aggregation operator for the decision making problem. Finally we apply the algorithm to solve a group decision making problem. Here we get better result by using intuitionistic fuzzy soft matrix than fuzzy soft matrix (as there are largest membership grade and smallest non membership grade).

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