International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 11 No. I (April, 2017), pp. 133-141

ON MULTIPLE LAGUERRE TRANSFORM IN TWO VARIABLES

T. G. THANGE
Department of Mathematics,
Yogeshwari Mahavidylaya Ambajogai, Dist Beed, India

Abstract

In the present paper Multiple Laguerre Transform in Two Variables is introduced. Also studied some basic operational properties. Inversion formula is obtained. Finally we have given application for maple implementation [2].

1. Introduction

N. N. Lebedev [5], E. D. Rainville [6] studied different kinds of special functions. D. W Lee [1] defined the multiple Laguerre I polynomials $L_{\vec{n}}^{(\overline{|al},\beta)}(x)$ by

$$L_{\vec{n}}^{(\vec{\alpha},\beta)}(x) = w_r^{r-1}(w_r w_{r-1}^{-1} x^{n_r}(\cdots(w_1 x^{n_1})^{(n_2)\cdots})^{(n_{r-1})})^{n_r} \tag{1.1}$$

where $w_i = x^{\alpha_i} e^{\beta x}, \alpha_i > -1, 1 \leq i \leq r$, are the Laguerre weights with $\beta < 0$ and $\alpha_i - \alpha_j$ for $i \neq j$.

Similarly the multiple Laguerre II polynomials $L_{\tilde{n}}^{(\alpha,\overline{\beta})}(x)$ by

$$L_{\vec{n}}^{(\alpha,\vec{\beta})}(x) = w_r^{-1}(w_r w_{r-1}^{-1}(\cdots(w_2 w_1^{-1}(w_1 x^{|\vec{n}|})^{(n_1)})^{(n_2)\cdots})^{(n_{r-1})})^{n_r} \tag{1.2}$$

Key Words : *Multiple Laguerre polynomials, Laguerre Transform.*

AMS Subject Classification : 46 F12.

where $w_i = x^{\beta} e^{\beta_i x}, \beta_i < 0, 1 \leq i \leq r$ are the Laguerre weights with $\alpha > -1$ and $\beta_i \neq \beta_j$ for $i \neq j$.
Here r is a positive integer, $\vec{n} = (n_1, n_2, \cdots, n_r) \in \mathbb{N}^r$ with

$$|\vec{n}| = n_1 + n_2 + \cdots n_r, \vec{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_r) \in \mathbb{R}^r \quad \text{and}$$

$$\vec{\beta} = (\beta_1, \beta_2, \cdots, \beta_r) \in \mathbb{R}^r$$

these polynomials have some orthogonality with respect to r weight functions, from which the multiple orthogonal polynomials are named. More precisely, the multiple Laguerre polynomials $L_{\vec{n}}^{(\vec{\alpha},\beta)}(x)$ satisfy

$$\int_0^{\infty} L_{\vec{n}}^{(\vec{\alpha},\beta)}(x) x^k w_i(x) dx = 0, \quad k = 0, 1, \cdots, n_i - 1, i = 1, 2, \cdots, r$$

where w_i is the Laguerre I weight in equation (1.1) and the multiple Laguerre II polynomials $L_{\vec{n}}^{(\alpha,\vec{\beta})}(x)$ satisfy

$$\int_0^{\infty} L_{\vec{n}}^{(\alpha,\vec{\beta})}(x) x^k w_i(x) dx = 0, \quad k = 0, 1, \cdots, n_i - 1, i = 1, 2, \cdots, r$$

where w_i is the Laguerre II weight in equation (1.2).

2. Multiple Laguerre Polynomials of Two Variables

M.S. Chaudhary and T. G. Thange [3,4] studied generalized Laguerre Transform and differential operator. Now we define multiple Laguerre I Polynomials as $L_{\vec{n}}^{(\vec{\alpha},\beta)}(x)$

$$L_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) = \frac{\Gamma(\alpha+n+1)\Gamma(\beta+n+1)}{n!} \sum_{k=0}^{n} \frac{L_{\vec{n}-\vec{k}}^{(\vec{\alpha},\beta)}(x,y)(-y)^k}{k!\Gamma(\alpha+n-k+1)\Gamma(\beta+k+1)}. \tag{2.1}$$

Similarly we can define $L_{\vec{n}}^{(\alpha,\vec{\beta})}(x,y)$.
In the present paper , we define the multiple Laguerre Transform $F_{\vec{n}}(\vec{\alpha},\beta)$ of $f(x,y)$ as,

$$F_{\vec{n}}(\vec{\alpha},\beta) = \int_0^{\infty} \int_0^{\infty} e^{-(x+y)} x_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) f(x,y) dx dy \tag{2.2}$$

where, $f(x,y)$ - be a Riemann integrable function defined on set $S = R^+ \times R^+, \alpha > -1, \beta > -1$, n is a non negative integer, here $R^+ = (0m, \infty)$ is the set of positive real numbers and

$$K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) = \sum_{r=0}^{n} \frac{(-xy)^r}{r!(-n)r} L_{\vec{n}-\vec{r}}^{(\vec{\alpha}+r,\beta+r)}(x,y). \tag{2.3}$$

The Pochhammer symbol, for $(x)_{\vec{n}}$ defined as,

$$(x)_{\vec{n}} = \frac{\Gamma(x+\vec{n})}{\Gamma(x)}, \quad (-x)_{\vec{n}} = (-1)^n (x-\vec{n}+1)_{\vec{n}}.$$

Shukla 02- gives

$$L_{\vec{n}}^{\vec{\alpha}}(x) L_{\vec{n}}^{\beta}(y) = \sum_{t=0}^{n} \frac{(-xy)^t}{t!(-n)_t} L_{\vec{n}-t}^{(\vec{\alpha}+t,\beta+t)}(x,y). \tag{2.4}$$

Using (2.3) and (2.4) we get,

$$K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) = L_{\vec{n}}^{\vec{\alpha}}(x) L_{\vec{n}}^{\beta}(y). \tag{2.5}$$

Therefore the equivalent definition for the multiple Laguerre transform of $f(x,y)$ is

$$F_{\vec{n}}(\vec{\alpha},\beta) = \int_0^\infty \int_0^\infty e^{-(x+y)} x^{\vec{\alpha}}, y^{\beta} L_{\vec{n}}^{\vec{\alpha}}(x) L_{\vec{n}}^{\beta}(y) f(x,y) dxdy \tag{2.6}$$

D. W. Lee [1] studied following differential Properties for multiple Laguerre polynomials.

$$\frac{d}{dx} L_{\vec{n}}^{(\vec{\alpha},\beta)}(x) = \beta [L_{\vec{n}}^{(\vec{\alpha}+1,\beta)}(x) - L_{\vec{n}}^{(\vec{\alpha},\beta)}(x)] \tag{2.7}$$

$$\begin{aligned} n_i L_{\vec{n}+e_i}^{(\vec{\alpha},\beta)}(x) &= (\beta x + \alpha_i + 1) L_{\vec{n}}^{(\vec{\alpha}+e_i,\beta)}(x) + x \tfrac{d}{dx} L_{\vec{n}}^{(\vec{\alpha}+e_i,\beta)}(x) \\ &= (\alpha_i + 1) L_{\vec{n}}^{(\vec{\alpha}+e_i)}(x) + \beta x L_{\vec{n}}^{(\vec{\alpha}+e_i,\beta)}(x) \end{aligned} \tag{2.8}$$

$$\frac{d}{dx} L_{\vec{n}}^{(\alpha,\vec{\beta})}(x) = \sum_{i=1}^{r} \beta_i L_{\vec{n}+e_i}^{(\alpha-1,\vec{\beta})}(x) \tag{2.9}$$

$$\frac{d}{dx} L_{\vec{n}}^{(\alpha,\vec{\beta})}(x) - \sum_{i=1}^{r} \frac{d}{dx} L_{\vec{n}-e_i}^{(\alpha,\vec{\beta})}(x) = \sum_{i=1}^{r} \beta_i L_{\vec{n}-e_i}^{(\alpha,\vec{\beta})}(x) \tag{2.10}$$

$$n_i L_{\vec{n}-e_i}^{(\alpha,\vec{\beta})}(x) - \sum_{i=1}^{r} n_i L_{\vec{n}-e_i+e_i}^{(\alpha,\vec{\beta})}(x) = (\beta_i x + \alpha + 1) L_{\vec{n}}^{(\alpha,\vec{\beta})}(x) + x \frac{d}{dx} L_{\vec{n}}^{(\alpha,\vec{\beta})}(x). \tag{2.11}$$

$$\frac{\partial}{\partial x} K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) = K_{\vec{n}}^{(\vec{\alpha}+1,\beta)}(x,y) \tag{2.12}$$

$$x \frac{\partial}{\partial x} K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y) = (n+\vec{\alpha}) K_{\vec{n}}^{(\vec{\alpha}-1,\beta)}(x,y) - \vec{\alpha} K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y). \tag{2.13}$$

Motivated from above work we obtained following basic Properties on Multiple Laguerre Transform.

3. Basic Properties

Theorem 3.1 : Linear Property: If $F\ f_1(x,y)$ and $F\ f_2(x,y)$ exist then

$$F\ c_1 f_1(x,y) + c_2 f_2(x,y) = c_1 F\ f_1(x,y) + c_2 F\ f_2(x,y) \tag{3.1}$$

where $f_1(x,y)$ and $f_2(x,y)$ are Riemann integrable functions on $S = R^+ \times R^+$ and c_1 and c_2 are constants.

We can easily prove Theorem 3.1.

Theorem 3.2 : If $F_{\vec{n}}(\vec{\alpha},\beta)$ is multiple Laguerre transform of $f(x,y)$, n is non negative integer then

$$L\frac{\partial f}{\partial x} = F_{\vec{n}}(\vec{\alpha},\beta) - (n+\vec{\alpha})F_{\vec{n}}(\vec{\alpha}-1,\beta) \text{ where } \alpha > 0, \beta > -1 \tag{3.2}$$

$$L\left\{\frac{\partial J}{\partial y}\right\} = F_{\vec{n}}(\alpha,\vec{\beta}) - (n+\vec{\beta})F_{\vec{n}}(\alpha,\vec{\beta}-1), \text{ where } \alpha > -1, \beta > 0 \tag{3.3}$$

$$L\left\{\frac{\partial^2 J}{\partial x}\right\} = F_{\vec{n}}(\vec{\alpha},\beta) - 2(n+\vec{\alpha})F_{\vec{n}}(\vec{\alpha}-1,\beta) + (n+\vec{\alpha})(n+\vec{\alpha}-1)F_{\vec{n}}(\vec{\alpha}-2,\beta), \tag{3.4}$$

$$L\left\{\frac{\partial^2 J}{\partial x^2}\right\} = F_{\vec{n}}(\vec{\alpha},\beta) - 2(n+\vec{\alpha})F_{\vec{n}}(\vec{\alpha}-1,\beta) + (n+\vec{\alpha})(n+\vec{\alpha}-1)F_{\vec{n}}(\vec{\alpha}-2,\beta), \tag{3.5}$$

$$L\left\{\frac{\partial^2 J}{\partial y^2}\right\} = F_{\vec{n}}(\alpha,\vec{\beta}) - 2(n+\vec{\beta})F_{\vec{n}}(\alpha,\vec{\beta}-1), +(n+\vec{\beta})(n+\vec{\beta}-1)F_{\vec{n}}(\alpha,\vec{\beta}-2), \tag{3.6}$$

where $\alpha > -1, \beta > 1$.

$$Lx\frac{\partial f}{\partial x} = F_{\vec{n}}(\vec{\alpha}+1,\beta) - (\vec{\alpha}+1)F_{\vec{n}}(\vec{\alpha},\beta), \text{ where } \alpha > 0, \beta > -1 \tag{3.7}$$

$$L\left\{y\frac{\partial J}{\partial y}\right\} = F_{\vec{n}}(\alpha,\vec{\beta}+1) - (\vec{\beta}+1)F_{\vec{n}}(\alpha,\vec{\beta}), \text{ where } \alpha > -1, \beta > 0 \tag{3.8}$$

$$L\left\{x\frac{\partial J}{\partial y} + y\frac{\partial J}{\partial y}\right\} = F_{\vec{n}}(\vec{\alpha}+1,\vec{\beta}) + F_{\vec{n}}(\vec{\alpha},\vec{\beta}+1) - (\vec{\alpha}+\vec{\beta}+2)F_{\vec{n}}(\vec{\alpha},\vec{\beta}), \text{ where } \alpha > 0, \beta > 0 \tag{3.9}$$

$$\begin{aligned} L\left\{x^2\tfrac{\partial^2 f}{\partial x^2}|2xy\tfrac{\partial^2 f}{\partial x \partial y} + y^2\tfrac{\partial^2 J}{\partial y^2}\right\} \;=\; &= F_{\vec{n}}(\vec{\alpha},\vec{\beta}+2)F_{\vec{n}}(\vec{\alpha}+2,\vec{\beta}) + 2F_{\vec{n}}(\vec{\alpha}+1,\vec{\beta}+1) \\ &-(2\vec{\alpha}+2\vec{\beta}+5)F_{\vec{n}}(\vec{\alpha}+1,\vec{\beta}) + F_{\vec{n}}(\vec{\alpha},\vec{\beta}+1) \\ &+(\vec{\alpha}+\vec{\beta}+2)^2 F_{\vec{n}}(\vec{\alpha},\vec{\beta}), \text{ where } \alpha > 1, \beta > -1. \end{aligned} \tag{3.10}$$

Here $f(x,y)$ and its first and second order partial derivatives are Riemann integrable functions defined on $S=R^{+}\times R^{+}$.

Proof : Using definition

$$L\left[\frac{\partial f}{\partial x}\right]=\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)\frac{\partial f}{\partial x}dxdy$$

this can be written as

$$L\left[\frac{df}{dx}\right]=\int_{0}^{\infty}\left[e^{-x}x^{\vec{\alpha}}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)\frac{\partial f}{\partial x}dx\right]e^{-x}y^{\beta}dy.$$

Integrating by parts we get

$$L\left[\frac{df}{dx}\right]=\int_{0}^{\infty}\left[\int_{0}^{\infty}\frac{\partial}{\partial x}(e^{-x}x^{\vec{\alpha}}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)f(x,y)dx\right]e^{-x}y^{\beta}dy.$$

On solving $\frac{\partial}{\partial x}\{e^{-x}x^{\vec{\alpha}}K_{\vec{n}}^{\vec{\alpha},\beta}(x,y)\}$ in inner integral, we obtain

$$L\left[\frac{df}{dx}\right]=\int_{0}^{\infty}\int_{0}^{\infty}e^{(-x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)f(x,y)dxdy.$$

$$-\alpha\int_{0}^{\infty}\int_{0}^{\infty}e^{(-x+y)}x^{\vec{\alpha-1}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)f(x,y)dxdy.$$

$$-(x+\vec{\alpha})\int_{0}^{\infty}\int_{0}^{\infty}e^{(-x+y)}x^{\vec{\alpha}-1}y^{\beta}K_{\vec{n}}^{(\vec{\alpha}-1,\beta)}(x,y)f(x,y)dxdy.$$

Now using the definition of $F_{\vec{n}}(\vec{\alpha},\beta)$ in the above equation, we get equation (3.2). Similarity we can prove equation (3.3).

Proof of (19) : Using equation (3.2) we have

$$L\left[\frac{\partial^{2}f}{\partial x^{2}}\right]=L\left\{\frac{\partial f}{\partial x},n,\vec{\alpha},\beta\right\}-(n+\vec{\alpha})L\left\{\frac{\partial f}{\partial x},n,\vec{\alpha}-1,\beta\right\}$$

again using (3.4) above equation reduces to

$$\begin{aligned}L\left[\frac{\partial^{2}f}{\partial x^{2}}\right] &= F_{\vec{n}}(\vec{\alpha},\beta)-(n+\vec{\alpha})F_{\vec{n}}(\vec{\alpha}-1,\beta)\\ &\quad -(n+\vec{\alpha})\left[F_{\vec{n}}(\vec{\alpha}-1,\beta n)-(n+\vec{\alpha}-1)F_{\vec{n}}(\vec{\alpha}-2,\beta)\right].\end{aligned}$$

Further simplification gives equation (3.6).

Similarly using equation (3.3) we can obtain equation (3.5).

Proof of (21) : Using equation previous equation we have

$$L\left[x\frac{\partial f}{\partial x}\right]=\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x+y)}x^{-\vec{\alpha}+1}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y)\frac{\partial f}{\partial x}dxdy.$$

Simplifying the above equation we obtain

$$L\left[x\frac{\partial f}{\partial x}\right]=-\int_0^\infty\left[\int_0^\infty\frac{\partial}{\partial x}(e^{-x}x^{\vec{\alpha}+1}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y))f(x,y)dx\right]e^{-y}y^{\beta}dy$$

and using equation (2.12) we have

$$\begin{aligned}L\left[x\frac{\partial f}{\partial x}\right] &= -(\vec{\alpha}+1)\int_0^\infty\int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y))f(x,y)dx\\ &\quad+\int_0^\infty\int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\beta)}(x,y))f(x,y)dxdy.\end{aligned}$$

Now using the definition (2.6) we get equation (3.4) similarly we can prove equation (3.7). Adding equation (3.6) and (3.7) we get equation (3.8)

Proof of 24 : Let

$$L\left\{x^2\frac{\partial^2}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}\right\}=L\left\{\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)\left(\frac{\partial f}{\partial n}+y\frac{\partial f}{\partial y}\right)\right\}.$$

Applying equation (1.2) we get

$$\begin{aligned}&L\left\{x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y},n,\vec{\alpha}+1,\beta\right\}+L\left\{x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y},n,\vec{\alpha},\beta-1\right\}\\ &-(\alpha+\beta+2)L\left\{x\frac{\partial f}{\partial x}+y\frac{\partial y}{\partial y},n,\vec{\alpha},\beta\right\}.\end{aligned}$$

Again using equation (3.8) and further simplification we get equation (3.9).

4. Inversion

To obtain inverse transform, we use the following result for $K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)$ which is defined by A. K. Shukla [2].

Theorem 4.1 : If $K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)$ is defined as in [1] then

$$\int_0^\infty\int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)dxdy=\delta_{\vec{n}}\delta_{\vec{m}} \qquad (4.1)$$

where $\delta_{\vec{n}\vec{m}}$ (Kronecker delta symbol) is defined as $\delta_{\vec{n}\vec{m}}=\begin{matrix}0, & \vec{m}\neq\vec{n}\\ 1 & \vec{m}=\vec{n}\end{matrix}$

$$\delta_{\vec{n}}=\frac{\Gamma(n+\vec{\alpha}+1)\Gamma(n+\beta+1)}{(n!)^2},\text{`}\ \alpha>-1\quad\text{and}\quad\beta>-1.$$

Theorem 4.2 : Suppose that $f(x,y)$ is Riemann integrable function defined on $S = R^+ \times R^+$ let $F_{\vec{n}}(\vec{\alpha},\beta)$ be Multiple Laguerre transform of $f(x,y), \alpha > -1, \beta > -1$, n is a nonnegative integer and

$$\delta_{\vec{n}} = \Gamma(\vec{n}+\vec{\alpha}+1)\Gamma(\vec{n}+\vec{\beta}+1)(n!)^2$$

then

$$f(x,y) = L^{-1}\{F_{\vec{n}}(\vec{\alpha},\vec{\beta})\} = \sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)F_{\vec{n}}(\vec{\alpha},\beta). \tag{4.2}$$

Proof :

$$g(x,y) = \sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)F_{\vec{n}}(\vec{\alpha},\beta). \tag{4.3}$$

Consider

$$\int_0^\infty \int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)(g(x,y)dxdy$$
$$= \int_0^\infty \int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)(g(x,y)$$
$$\sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)F_{\vec{n}}(\vec{\alpha},\beta)dxdy.$$

On changing the order of summation and integration, we obtain

$$= \sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}F_{\vec{n}}(\alpha,\beta)\int_0^\infty \int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)dxdy.$$

Using (4.1) we get

$$\int_0^\infty \int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)g(xy)dxdy = \sum_{n=0}^{\infty}F_{\vec{n}}(\vec{\alpha},\beta)\delta_{\vec{m}\vec{n}}.$$

By definition of $\delta_{\vec{m}\vec{n}}$, we have

$$\int_0^\infty \int_0^\infty e^{-(x+y)}x^{\vec{\alpha}}y^{\beta}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)g(xy)dxdy = F_{\vec{m}}(\vec{\alpha},\beta).$$

Using the equation (4.3)

$$f(x,y) = \sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)F_{\vec{n}}(\vec{\alpha},\beta).$$

Hence,

$$f(x,y) = L^{-1}\{F_{\vec{n}}(\vec{\alpha},\beta)\} = \sum_{n=0}^{\infty}(\delta_{\vec{n}})^{-1}K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)F_{\vec{n}}(\vec{\alpha},\beta).$$

5. Application

In this section we give a application of above defined transform for maple implementation [2]. A Maple is scientific-technical computing system used for to obtain the aforementioned transform of given function $f(x,y)$. To find $Lf(x,y)$ by using Maple, first we start new MAPLE windows in '**Worksheet Mode**', with default '**Typesetting Rules**', MAPLE an intrinsic command called '**assume**' for defining conditions like real numbers, nonnegative numbers. Now, the corresponding Maple code [2] for or > -1, 1 and n is nonnegative integer is

```
> restart;
> assume (alpha > - 1);
> assume (beta > - 1) ;
> assume (n >= 0);
> additionally (n: integer);
```

For defining $K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)$, we used definition (2.5). In MAPLE library the Laguerre $[\vec{n},\alpha,x]$ function computes the nth degree generalized Laguerre polynomial with parameter evaluated at x. Hence, to define $K_{\vec{n}}^{(\vec{\alpha},\vec{\beta})}(x,y)$, we can use the following MAPLE code [2]:

```
> K : = proc (alpha, beta, n, x, y) options operator arrow;
Laguerre L(n, alpha, x) * Laguerre L (n, beta , y) end proc;
```

Next, we defined the operator L by using the following MAPLE code,

```
> L = proc (alpha, beta, n, f) options operator, arrow;
int (int f * K ( alpha, beta, n , x, y ) * exp (- x -y ) * x ˆ alpha * y ˆ beta,
x = 0 . . infinity), y = 0 . . infinity ) end proc;
```

Using the above-mentioned MAPLE codes, [2] we can compute Multiple Laguerre transform of the given $f(x,y)$.

Similarly, to find the inverse transform of $F_{\vec{n}}(\vec{\alpha}, \beta)$ first define the aforesaid conditions and following MAPLE code, [2]

```
> Z : = proc (x, y, alpha, beta, F) options operator, arrow;
Sum (F*K (alpha, beta, n, x, y) * factorial (n) * factorial (n) / (GAMMA
(n + alpha + 1) * GAMMA (n + beta + 1) ), n = 0 . . infinity) end proc;
```

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