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SOME RESULTS IN FUZZY SOFT α- CONTINUITY

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Abstract

α-continuous functions, fuzzy α-continuous functions and soft α-continuous functions have been already investigated by topologists. In this paper the concept of a fuzzy soft α-continuous function is introduced and its relationship with the existing concept in the literature of fuzzy soft topology is discussed.

1. Introduction

In the year 1985, Reilly and Vanamurthy [11] have been discussed the concept of α-continuity in topological spaces. In 2014, Akdag and Ozkan [2] introduced the concept of Soft $\propto$ open sets and soft $\propto$ continuous between soft topological spaces. In this paper we introduce the notion of fuzzy soft α-continuity and some results along with examples have been discussed. Throughout this paper X and Y denote the initial sets. E and K denote the parameter spaces.

Key Words : *Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft mapping, Fuzzy soft α-continuous.*

2. Preliminaries

Definition 2.1 : A pair (F, E) is called a soft set [8] over X where F is a mapping given by $F : E \to 2^X$ and 2^X is the power set of X.

Definition 2.2 : A fuzzy set [14] of on X is a mapping $f : X \to I^X$ where $I = [0, 1]$.

Definition 2.3 : A pair

$tilde\lambda = (\lambda, E)$ is called a fuzzy soft set [12] over (X, E) where $\lambda : E \to I^X$ is a mapping, I^X is the collection of all fuzzy subsets of X. $FS(X, E)$ denotes the collection of all fuzzy soft sets over (X, E). We denote $\tilde{\lambda}$ by $\tilde{\lambda} = \{(e, \lambda(e)) : e \in E\}$ where $\lambda(e)$ is a fuzzy subset of X for each e in E.

Definition 2.4 [12] : For any two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over a common universe X and a common parameter space E, $\tilde{\lambda}$ is a fuzzy soft subset of $\tilde{\mu}$ if $\lambda(e) \leq \mu(e)$ for all $e \in E$. If $\tilde{\lambda}$ is a fuzzy soft subset of $\tilde{\mu}$ then we write $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu}$ contains $\tilde{\lambda}$.

Two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X, E) are soft equal if $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu} \subseteq \tilde{\lambda}$. That is $\tilde{\lambda} = \tilde{\mu}$ if and only if $\lambda(e) = \mu(e)$ for all $e \in E$. We use the following notations:

$\overline{0}(x) = 0$, for all x in X and $\overline{1}(x) = 1$ for all x in X.

Definition 2.5 [12] : A fuzzy soft set $\tilde{\varphi}_X$ over (X, E) is said to be a null fuzzy soft set if for all $e \in E, \varphi_X(e) = \overline{0}$ and $\tilde{\varphi}_X = (\varphi_X, E)$.

Definition 2.6 [12] : A fuzzy soft set $\overline{1}_X$ over (X, E) is said to be absolute fuzzy soft set if for all $e \in E, 1_X(e) = \overline{1}$ and $\tilde{1}_X = (1_X, E)$.

Definition 2.7 [13] : The union of two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X, E) is defined as $\tilde{\lambda}\tilde{\cup}\tilde{\mu} = (\lambda\tilde{\cup}\mu, E)$ where $(\lambda\tilde{\cup}\mu)(e) = \lambda(e) \cup \mu(e) =$ the union of fuzzy sets $\lambda(e)$ and $\mu(e)$ for all $e \in E$.

Definition 2.8 [13] : The intersection of two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X, E) is defined as $\tilde{\lambda}\tilde{\cap}\tilde{\mu} = (\lambda\tilde{\cap}\mu, E)$ where $(\lambda\tilde{\cap}\mu)(e) = \lambda(e) \cap \mu(e)$ the intersection of fuzzy sets $\lambda(e)$ and $\mu(e)$ for all $e \in E$.

The arbitrary union and arbitrary intersection of fuzzy soft sets over (X, E) are defined as

$\tilde{\cup}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ and $\tilde{\cap}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ where $(\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \cup\{\lambda_\alpha(e) : \alpha \in \Delta\} =$ the union of fuzzy sets $\lambda_\alpha(e), \alpha \in \ D$ and $(\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \cap\{\lambda(e) : \alpha \in \Delta\} =$ the intersection of fuzzy sets $\lambda_\alpha(e), \alpha \in \Delta$, for all $e \in E$.

Definition 2.9 [13] : The complement of a fuzzy soft set (λ, E) over (X, E), denoted by $(\lambda, E)^C$ is defined as $(\lambda, E)^C = (\lambda^C, E)$ where $\lambda^C : E \to I^X$ is a mapping given by $\lambda^C(e) = 1 - \lambda(e)$ for every e in E.

Definition 2.10 [13] : A fuzzy soft topology $\tilde{\tau}$ on (X, E) is a family of fuzzy soft sets over (X, E) satisfying the following axioms.

(i) $\tilde{\varphi}_X, \tilde{1}_X$ belong to $\tilde{\tau}$,

(ii) Arbitrary union of fuzzy soft sets in $\tilde{\tau}$, belongs to $\tilde{\tau}$,

(iii) The intersection of any two fuzzy soft sets in $\tilde{\tau}$, belongs to $\tilde{\tau}$.

Members of $\tilde{\tau}$ are called fuzzy soft open sets in $(X, \tilde{\tau}, E)$. A fuzzy soft set $\tilde{\lambda}$ over (X, E) is fuzzy soft closed in $(X, \tilde{\tau}, E)$ if $(\tilde{\lambda})^C \in \tilde{\tau}$. The fuzzy soft interior of $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$ is the union of all fuzzy soft open sets $\tilde{\mu} \subseteq \tilde{\lambda}$ denoted by $\tilde{f}sint(\tilde{\lambda}) = \tilde{\cup}(\tilde{\mu} : \tilde{\mu} \subseteq \tilde{\lambda}, \tilde{\mu} \in \tilde{\tau}\}$. The fuzzy soft closure of $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$ is the intersection of all fuzzy soft closed sets $\tilde{\eta}, \tilde{\lambda} \subseteq \tilde{\eta}$ denoted by $\tilde{f}scl(\tilde{\lambda}) = \tilde{\cap}(\tilde{\eta} : \tilde{\lambda} \subseteq \tilde{\eta}, (\tilde{\eta})^C \in \tilde{\tau})\}$.

Definition 2.11 [1] : Let $(X, \tilde{\tau}, E)$ be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\tilde{\lambda}$ is fuzzy soft semi-open if $\tilde{\lambda} \subseteq \tilde{f}scl(\tilde{f}sint(\tilde{\lambda}))$ and fuzzy soft semi closed if $\tilde{f}s\ int(\tilde{f}scl(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.12 [1] : Let $(X, \tilde{\tau}, E)$ be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\tilde{\lambda}$ is fuzzy soft pre-open if $\tilde{\lambda} \subseteq \tilde{f}s\ int(\tilde{f}s\ cl(\tilde{\lambda}))$ and fuzzy soft pre-closed if $\tilde{f}s\ cl(\tilde{f}s\ Int(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.13 [1] : Let $(X, \tilde{\tau}, E)$ be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X, E). Then $\tilde{\lambda}$ is fuzzy soft α-open if $\tilde{\lambda} \subseteq \tilde{f}s\ int(\tilde{f}s\ CL(\tilde{f}s\ Int(\tilde{\lambda})))$ and fuzzy soft α-closed if $\tilde{\lambda} \supseteq \tilde{f}s\ Cl(\tilde{f}s\ Cl(\tilde{\lambda})))$

The classes of all fuzzy soft α-open, fuzzy soft pre-open, fuzzy soft semi-open and fuzzy soft semi-pre-open sets over (X, E) are denoted as $\tilde{FS}\ \alpha(X), \tilde{FS}\ SO(X), \tilde{FS}\ PO(X)$ and $\tilde{FS}\ SP(X)$ respectively.

The fuzzy soft pre-interior, fuzzy soft pre-closure, fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft α-interior,fuzzy soft α-closure, fuzzy soft semi-pre-interior, fuzzy soft semi-pre-closure of X are denoted by $\tilde{f}s\ PCl(\tilde{\lambda})$, $\tilde{f}s\ Plnt(\tilde{\lambda})$, $\tilde{f}s\ Slnt(\tilde{\lambda})$,$\tilde{f}s\ \alpha Cl(\tilde{\lambda})$, $\tilde{f}s\ \alpha\ Int(\tilde{\lambda})$, $\tilde{f}s\ SPInt(\tilde{\lambda})$, $\tilde{f}s\ SPCl(\tilde{\lambda})$ respectively.

Definition 2.14 [1] : Let $(X, \tilde{\tau}, E)$. be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X, E). Then its fuzzy soft pre-closure and fuzzy soft pre-interior are defined as:

$$\tilde{fs}\ PCl(\tilde{\lambda}) = \cap\{\tilde{\mu}|\tilde{\mu} \supseteq \tilde{\lambda}, \tilde{\mu} \in \tilde{FS}\ PC(X)\}.$$

$$\tilde{fs}\ PInt(\tilde{\lambda}) = \cup\{\tilde{\eta}|\tilde{\eta} \subseteq \tilde{\lambda}, \tilde{\eta} \in \tilde{FS}\ PO(X)\}.$$

The definitions for $\tilde{fs}\ SCl, \tilde{fs}\ SInt, \tilde{fs}\ \alpha cl$ and $\tilde{fs}\ \alpha Int$ are similar.

The following extension principle is used to define the mapping between the classes of fuzzy soft sets.

Definition 2.15 [13] : Let X and Y be any two non-empty sets. Let $g : X \to Y$ be a mapping. Let λ be a fuzzy subset of X and $\tilde{\mu}$ be a fuzzy subset of Y. Then $g(\lambda)$ is a fuzzy subset of Y and for y in Y

$$g(\lambda)(y) = \begin{cases} \sup\{\lambda(f(x)) : x \in g^{-1}\}, & g^{-1}(y) \neq \phi \\ 0 & \text{otherwise.} \end{cases}$$

$g^{-1}(\mu)$ is a fuzzy subset of X, defined by $g^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

Definition 2.16 [12] : Let $FS(X, E)$ and $FS(Y, K)$ be classes of fuzzy soft sets over (X, E) and (Y, K) respectively.

$\rho : X \to Y$ and $\psi : E \to K$ be any two mappings. Then a fuzzy soft mapping $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ is defined as follows:

For a fuzzy soft set $\tilde{\lambda}$ in $FS(X, E), g(\tilde{\lambda})$ is a fuzzy soft set in $FS(Y, K)$ obtained as follows:

$$g(\tilde{\lambda})(k) = \begin{cases} \bigcup\limits_{e \in \psi^{-1}(k)} \rho(\lambda(e)), & \psi^{-1}(k) \neq \phi \\ \overline{0}, & \text{otherwise.} \end{cases}$$

For every y in Y, where

$$\rho(\lambda(e))(y) = \begin{cases} \sup\{\lambda(e)(x) : x \in \rho^{-1}(y), & \rho^{-1}(y) \neq \phi \\ 0, & \text{otherewise} \end{cases}$$

That is

$$g(\tilde{\lambda})(k)(y) = \begin{cases} \sup\limits_{e \in \psi^{-1}(k)} \left\{ \sup\limits_{x \in \rho^{-1}(y)} \lambda(e)(x) \right\}, & \rho^{-1}(y) \neq \phi, \psi^{-1}(k) \neq \phi \\ 0, & \text{otherwise.} \end{cases}$$

$g(\tilde{\lambda})$ is the image of the fuzzy soft set $\tilde{\lambda}$ under the fuzzy mapping $g = (\rho, \psi)$. For a fuzzy soft set $\tilde{\mu}$ in $FS(Y, K)$, $g^{-1}(\tilde{\mu})$ is a fuzzy soft set in $FS(X, E)$ obtained as follows:
$g^{-1}(\tilde{\mu})(e)(x) = \rho^{-1}(\tilde{\mu}(\psi(e)))(x)$ for every x in X and $g^{-1}(\tilde{\mu})$ is the inverse image of the fuzzy soft set $\tilde{\mu}$.

Lemma 2.17 [9] : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be fuzzy soft topological spaces. Let $\rho : X \to Y$ and $\psi : E \to K$ be the two mappings and $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i \in FS(X, E)$ and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu}) \in FS(Y, K)$, where $i \in J$ is an index set.

1. If $\tilde{\lambda}_1 \subseteq \tilde{\lambda}_2$, then $g(\tilde{\lambda}_1) \subseteq g(\tilde{\lambda}_2)$.
2. If $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$, then $g^{-1}(\tilde{\mu}_1) \subseteq g^{-1}(\tilde{\mu}_2)$.
3. $\tilde{\lambda} \subseteq g^{-1}(g(\tilde{\lambda}))$, the equality holds if g is injective.
4. $g(g^{-1}(\tilde{\mu})) \subseteq \tilde{\mu}$, the equality holds if g is surjective.
5. $g^{-1}((\tilde{\mu})^C) = [g^{-1}(\tilde{\mu})]^C$.
6. $[g(\tilde{\lambda})]^C \subseteq g((\tilde{\lambda})^C)$.
7. $g^{-1}(\tilde{1}_K) = \tilde{1}_E, g^{-1}(\tilde{0}_K) = \tilde{0}_E$.
8. $g(\tilde{1}_g) = \tilde{1}_K$ if g is surjective.
9. $g(\tilde{0}_E) = \tilde{0}_K$.

Lemma 2.18 [9] : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be the two fuzzy soft topological spaces. Let $\rho : X.Y$ and $\psi : E \to K$ be the two mappings and $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i \in FS(X, E)$ and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu})_i \in FS(Y, K)$, where J is an index set.

1. $g(\bigcup_{i \in J} \tilde{\lambda}_i) = \bigcup_{i \in J} g(\tilde{\lambda}_i)$.
2. $g(\bigcap_{i \in j} \tilde{\lambda}_i) \subseteq \bigcap_{i \in j} g(\tilde{\lambda}_i)$, the equality holds if g is injective.
3. $g^{-1}(\bigcup_{i \in j} \tilde{\mu}_i) = \bigcup_{i \in j} g^{-1}(\tilde{\mu}_i)$.
4. $g^{-1}(\bigcap_{i \in j} \tilde{\mu}_i) = \bigcap_{i \in j} g^{-1}(\tilde{\mu}_i)$.

Definition 2.19 [13] : Fix $x \in X, 0 < \alpha < 1$. Then the fuzzy subset x^{α} of X is called fuzzy point if

$$x^{\alpha}(y) = \begin{cases} \alpha & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$$

Definition 2.20 [13] : Fix $x \in X$, $0 < \lambda < 1$, $e \in E$. The fuzzy soft set x_e^{α} over (X, E) is called fuzzy soft point if

$$x_e^{\alpha}(e_1) = \begin{cases} x^{\alpha} & \text{for } e_1 = e \\ \overline{0} & \text{otherwise.} \end{cases}$$

$$x_e^{\alpha}(e_1)(y) = \begin{cases} \alpha & \text{for } e_1 = e, y = x \\ \overline{0} & \text{otherwise.} \end{cases}$$

Definition 2.21 [4] : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be the fuzzy soft topological spaces. Let $\rho : X \to Y$ and $\psi : E \to K$ be the two mappings and $g = (\rho, \psi) : FS(X, E) \to FS(Y, K)$ be a fuzzy soft mapping. Then $g = (\rho, \psi)$ is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in $(Y, \tilde{\sigma}, K)$ is fuzzy soft open in $(X, \tilde{\tau}, E)$. That is $g^{-1}(\tilde{\mu}) \in \tilde{\tau}$, for all $\tilde{\mu} \in \tilde{\sigma}$.

3. Fuzzy Soft α-continuity

Definition 3.1 : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft α- continuous if for each fuzzy soft open set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$, the inverse image $g^{-1}(\tilde{\mu})$ is fuzzy soft α- open set in $(X, \tilde{\tau}, E)$.

Definition 3.2 : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft α- irresolute if for each fuzzy soft α- open set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$, the inverse image $g^{-1}(\tilde{\mu})$ is fuzzy soft α- open set in $(X, \tilde{\tau}, E)$.

Definition 3.3 : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft α- open mapping if for each fuzzy soft open set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$, the image $g(\tilde{\lambda})$ is fuzzy soft α- open set in $(Y, \tilde{\sigma}, K)$.

Definition 3.4 : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft α- closed if for each fuzzy soft closed set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$, the image $g(\tilde{\lambda})$ is fuzzy soft α- closed set in $(Y, \tilde{\sigma}, K)$.

Proposition 3.5 : For a fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$, the following are equivalent

(i) g is fuzzy soft α- continuous.

(ii) The inverse image of every fuzzy soft closed set in $(Y, \tilde{\sigma}, K)$ is fuzzy soft α-closed in $(X, \tilde{\tau}, E)$.

Proof : Suppose (i) holds. Let $\tilde{\mu}$ be a fuzzy soft closed in $(Y, \tilde{\sigma}, K)$. Then $(\tilde{\mu})^C$ is fuzzy soft open in $(Y, \tilde{\sigma}, K)$. Using definition 3.1, $g^{-1}((\tilde{\mu})^C)$ is fuzzy soft α-open. Since $g^{-1}((\tilde{\mu})^C) = [g^{-1}(\tilde{\mu})]^C$, $g^{-1}(\tilde{\mu})$ is fuzzy soft α-closed. This proves (i) . (ii).

Conversely we assume that (ii) holds. Let $\tilde{\mu}$ be fuzzy soft open in $(Y, \tilde{\sigma}, K)$. Therefore $(\tilde{\mu})^C$ is fuzzy soft closed set in $(Y, \tilde{\sigma}, K)$. Then by applying (ii), $[g6{-}1(\tilde{\mu})]^C$ is fuzzy soft α-closed in $(X, \tilde{\tau}, E)$. That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft α-open in $(X, \tilde{\tau}, E)$. This proves (ii) . (i).

Proposition 3.6 : For a fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$. If g is fuzzy soft α-irresolute then it is fuzzy soft α-continuous.

Proof : Suppose g is fuzzy soft α- irresolute. Let $\tilde{\mu}$ be a fuzzy soft open set in $(Y, \tilde{\sigma}, K)$. Since every fuzzy soft open set is fuzzy soft α- open and since g is fuzzy soft irresolute, by using Definition 3.2, $g^{-1}(\tilde{\mu})$ is fuzzy soft α-open. That implies g is fuzzy soft α-continuous.

Proposition 3.7 : A fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is fuzzy soft α- continuous iff $g^{-1}(\tilde{fs}\ Int\ \tilde{\mu}) \tilde{\subseteq} \tilde{fs}\ \alpha\ Int(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$.

Proof : Let $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be fuzzy soft α-continuous. Let $\tilde{\mu}$ be a fuzzy soft set in $(Y, \tilde{\sigma}, K)$. Then $\tilde{fs}\ Int(\tilde{\mu})$ is fuzzy soft open in Y. Since g is fuzzy soft α- continuous, by using Definition 3.1, $g^{-1}(\tilde{fs}\ Int(\tilde{\mu}))$ is fuzzy soft α- open in $(X, \tilde{\tau}, E)$. Then by using Lemma 2.18, $g^{-1}(\tilde{fs}\ Int(\tilde{\mu})) \subseteq g^{-1}(\tilde{\mu})$. This implies that $\tilde{fs}\ \alpha\ Int\ g^{-1}(\tilde{fs}\ Int(\tilde{\mu})) \subseteq \tilde{fs}\ \alpha\ int(g^{-1}(\tilde{\mu}))$.

Therefore $g^{-1}(\tilde{fs}\ Int\ \tilde{\mu}) \tilde{\subseteq} \tilde{fs}\ \alpha\ Int(g^{-1}(\tilde{\mu}))$.

Conversely we assume that, $g^{-1}(\tilde{fs}\ Int\ \tilde{\mu}) \tilde{\subseteq} \tilde{fs}\ \alpha\ Int(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$. In particular the above statement is true for fuzzy soft open sets in

$\tilde{\mu}$. If $\tilde{\mu}$ is fuzzy soft open sets in Y, $g^{-1}(\tilde{\mu})\tilde{\subseteq}\tilde{f}s\ \alpha\ Int(\tilde{\mu}))\tilde{\subseteq}g^{-1}(\tilde{\mu})$. That implies $g^{-1}(\tilde{\mu}) = \tilde{f}s\ \alpha\ Int(g^{-1}(\tilde{\mu}))$ is fuzzy soft α-open. Therefore g is fuzzy soft α- continuous.

Proposition 3.8 : A fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is fuzzy soft α- continuous iff $g(\tilde{f}s\ \alpha\ cl\ \tilde{\lambda}) \subseteq \tilde{f}s\ Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

Proof : Let $g : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be fuzzy soft α- continuous. Let $\tilde{\lambda}$ be fuzzy soft set in $(X, \tilde{\tau}, E)$. Then $g(\tilde{\lambda})$ is fuzzy soft set in $(Y, \tilde{\sigma}, K)$. Since g is fuzzy soft α-continuous, by using Definition 3.1, $g^{-1}(\tilde{f}s\ CLg(\tilde{\lambda}))$ is fuzzy soft α- closed in $(X, \tilde{\tau}, E)$. Since $g(\tilde{\lambda}) \subseteq (\tilde{f}s\ Clg(\tilde{\lambda}))$,

$$g^{-1}(g(\tilde{\lambda})) \subseteq g^{-1}(\tilde{f}s\ Clg(\tilde{\lambda})), \tilde{\lambda} \subseteq g^{-1}(g(\tilde{\lambda})) \subseteq g^{-1}(\tilde{f}s\ Clg(\tilde{\lambda})).$$

This implies that

$$(\tilde{f}s\ \alpha\ Cl\ \tilde{\lambda}) \subseteq \tilde{f}s\ \alpha\ Cl(g^{-1}(\tilde{f}s\ Clg(\tilde{\lambda}))) = g^{-1}(\tilde{f}s\ Clg(\lambda)).$$

Therefore $g(\tilde{f}s\ \alpha\ cl\ \tilde{\lambda}) \subseteq g(g^{-1}(\tilde{f}s\ Clg(\tilde{\lambda}))) \subseteq \tilde{f}s\ Clg(\tilde{\lambda})$.

Conversely we assume that, $g(\tilde{f}s\alpha cl\tilde{\lambda}) \subseteq \tilde{f}sCl(g(\lambda))$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

Let $\tilde{\mu}$ be a fuzzy soft closed in $(Y, \tilde{\sigma}, K)$. Let $\tilde{\lambda} = g^{-1}(\tilde{\mu})$. Since by our assumption,

$$g(\tilde{f}s\ \alpha\ cl\tilde{\lambda}) \subseteq \tilde{f}s\ Cl(g(\tilde{\lambda})), g(\tilde{f}s\ \alpha\ cl\ g^{-1}(\tilde{\mu})) \subseteq \tilde{f}s\ Clg(g^{-1}(\tilde{\mu})) \subseteq \tilde{f}s\ Cl\ \tilde{\mu}.$$

$g(\tilde{f}s\,\alpha\,cl\,g^{-1}(\tilde{\mu})) \subseteq \tilde{f}s\,cl\,\tilde{\mu} = \tilde{\mu}.\ g^{-1}(g(\tilde{f}s\,\alpha\,cl\,g^{-1}(\tilde{\mu})) \subseteq g^{-1}(\tilde{\mu}).\tilde{f}s\,\alpha\,cl\,g^{-1}(\tilde{\mu}) \subseteq g^{-1}(\tilde{\mu})$.

This implies that $g^{-1}(\tilde{\mu}) = \tilde{f}s\ \alpha\ cl\ g^{-1}(\tilde{\mu})$. Therefore $g^{-1}(\tilde{\mu})$ is fuzzy soft α-closed. Hence g is fuzzy soft α-continuous.

Proposition 3.9 : For a fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$. The following are equivalent.

(i) g is fuzzy soft α-continuous.

(ii) $g(\tilde{f}s\ \alpha\ cl(\tilde{\lambda}))\tilde{\subseteq}\tilde{f}s\ P\ cl\ g(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.

(iii) $g(\tilde{f}s\ \alpha\ cl(\tilde{\lambda}))\tilde{\subseteq}\tilde{f}s\ \alpha\ cl\ g(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$.

Proof : Assume (i) holds. By Proposition 3.8, $g(\tilde{f}s\ \alpha\ cl\ \tilde{\lambda}) \subseteq \tilde{f}s\ cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$. Since $\tilde{f}s\ cl\ (g(\tilde{\lambda})) = \tilde{f}s\ P\ clg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.

This proves (i) $\Rightarrow$ (ii).

Assume (ii) holds, $g(\tilde{fs}\ \alpha\ cl(\tilde{\lambda}))\tilde{\subseteq}\tilde{fs}\ P\ clg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$. Let $\tilde{\mu}$ be fuzzy soft closed set in $(Y, \tilde{\sigma}, K)$ and let $\tilde{\lambda} = g^{-1}(\tilde{\mu})$.

$$g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}\tilde{fs}\ P\ clg(g^{-1}(\tilde{\mu}))\tilde{\subseteq}\tilde{fs}\ P\ cl(\tilde{\mu}),$$

$$g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}\tilde{fs}\ Pcl(\tilde{\mu}) = \tilde{\mu},$$

$$g^{-1}(g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}g^{-1}(\tilde{\mu}),\quad g^{-1}(\tilde{\mu}) = \tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})).$$

That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft α-closed. Therefore g is fuzzy soft α-continuous. This proves (ii) $\Rightarrow$ (i).

Assume (i) holds. By Proposition 3.8, $g(\tilde{fs}\ \alpha\ cl\tilde{\lambda}) \subseteq \tilde{fs}\ cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$. Since $\tilde{fs}\ cl(g(\tilde{\lambda})) = \tilde{fs}\ \alpha\ clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$.

This proves (i) $\Rightarrow$ (iii).

Assume (iii) holds, $g(\tilde{fs}\ \alpha\ cl(\tilde{\lambda}))\tilde{\subseteq}\tilde{fs}\ \alpha\ clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$. Let $\tilde{\mu}$ be fuzzy soft closed set in $(Y, \tilde{\sigma}, K)$ and let $\tilde{\lambda} = g^{-1}(\tilde{\mu})$.

$$g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}\tilde{fs}\ \alpha\ clg(g^{-1}(\tilde{\mu}))\tilde{\subseteq}\tilde{fs}\ \alpha\ cl(\tilde{\mu}),$$

$$g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}\ \alpha\ cl(\tilde{\mu}) = \tilde{\mu},$$

$$g^{-1}(g(\tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})))\tilde{\subseteq}g^{-1}(\tilde{\mu}),$$

$$g^{-1}(\tilde{\mu}) = \tilde{fs}\ \alpha\ cl(g^{-1}(\tilde{\mu})).$$

That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft α-closed. Therefore g is fuzzy soft α-continuous. This proves (iii) $\Rightarrow$ (i).

Proposition 3.10 : A fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is fuzzy soft α-open iff $g(\tilde{fs}\ Int\ \tilde{\lambda}) \subseteq \tilde{fs}\ \alpha\ Intg(\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

Proof : Let $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be fuzzy soft $= al$-open. Let $\tilde{\lambda}$ be fuzzy soft open set in $(X, \tilde{\tau}, E)$. Then $\tilde{fs}\ Int(\tilde{\lambda})$ is fuzzy soft set in $(X, \tilde{\tau}, E)$. Since g is fuzzy soft α-open, by Definition 3.4, $g(\tilde{fs}\ Int(\tilde{\lambda}))$ is fuzzy soft α- open in $(Y, \tilde{\sigma}, K)$. Then by using Lemma 2.18,

$$g(\tilde{fs}\ Int(\tilde{\lambda})) \subseteq g(\tilde{\lambda}), \tilde{fs}\ \alpha\ Intg(\tilde{fs}\ Int(\tilde{\lambda})) \subseteq \tilde{fs}\ \alpha\ intg(\tilde{\lambda}).$$

Therefore $g(\tilde{fs}\ Int\ \tilde{\lambda}) \subseteq \tilde{fs}\ \alpha\ Intg(\tilde{\lambda})$.

Conversely we assume that $g(\tilde{fs}!Int\ \tilde{\lambda}) \subseteq \tilde{fs}\ \alpha\ Intg(\tilde{\lambda})$ for every fuzzy soft set λ in $(X, \tilde{\tau}, E)$.

In particular the above statement is true for fuzzy soft open sets in $\tilde{\lambda}$. If $\tilde{\lambda}$ is fuzzy soft open in $\tilde{\lambda}$, $g(\tilde{\lambda}) \subseteq \tilde{fs}\ \alpha\ Intg(\tilde{\lambda}) \subseteq g(\tilde{\lambda})$. That implies $g(\tilde{\lambda}) = \tilde{fs}\ \alpha\ Intg(\tilde{\lambda})$ is fuzzy soft α open. Therefore g is fuzzy soft α-continuous.

Proposition 3.11 : A fuzzy soft mapping $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is fuzzy soft α- closed iff $\tilde{fs}\ \alpha\ clg(\tilde{\lambda}) \subseteq g(\tilde{fs}\ Cl\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

Proof : Let $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be fuzzy soft α-closed. Let $\tilde{\lambda}$ be fuzzy soft set in $(X, \tilde{\tau}, E)$. Then $\tilde{fs}\ cl(\tilde{\lambda})$ is fuzzy soft closed set in $(X, \tilde{\tau}, E)$. Since g is fuzzy soft α- closed, by Definition 3.5, $g(\tilde{fs}\ cl(\tilde{\lambda}))$ is fuzzy soft α-closed in $Y, \tilde{\sigma}, K)$. Since $g(\tilde{\lambda}) \subseteq g(\tilde{fs}\ cl(\tilde{\lambda})), (\tilde{fs}\ \alpha\ cl(\tilde{\lambda}) \subseteq \tilde{fs}\ \alpha clg(\tilde{fs}\ cl(\tilde{\lambda})) = g(\tilde{fs}\ cl(\tilde{\lambda}))$. Therefore $\tilde{fs}\ \alpha\ clg(\tilde{\lambda}) \subseteq g(\tilde{fs}\ cl(\tilde{\lambda}))$.

Conversely we assume that, $\tilde{fs}\ \alpha\ clg(\tilde{\lambda}) \subseteq g(\tilde{fs}\ CL\ \tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

Let $\tilde{\lambda}$ be fuzzy soft closed in $(X, \tilde{\tau}, E)$. By our assumption, $\tilde{fs}\ \alpha\ clg(\tilde{\lambda}) \subseteq g(\tilde{fs}\ cl\ \tilde{\lambda}) = g(\tilde{\lambda}) \subseteq \tilde{fs}\ \alpha\ clg(\tilde{\lambda})$. Therefore $g(\tilde{\lambda}) = \tilde{fs}\ \alpha\ Clg(\tilde{\lambda})$. Therefore $g(\tilde{\lambda})$ is fuzzy soft α-closed.

Theorem 3.12 : Let $g = (\rho, \psi) : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be fuzzy soft mapping. Then the following are equivalent.

(i) g is fuzzy soft α-continuous.

(ii) The inverse image of every fuzzy soft closed set in $(Y, \tilde{\sigma}, K)$ is fuzzy soft α-closed in $(X, \tilde{\tau}, E)$.

(iii) $g^{-1}(\tilde{fs}\ Int\ \tilde{\mu}) \tilde{\subseteq} \tilde{fs}\ \alpha\ Int(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$.

(iv) $g(\tilde{fs}\ \alpha\ cl\ \tilde{\lambda}) \subseteq \tilde{fs}\ cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$.

(v) $g(\tilde{fs}\ \alpha\ cl(\tilde{\lambda})) \tilde{\subseteq} \tilde{fs}\ P\ lg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.

(vi) $g(\tilde{fs}\ \alpha\ cl(\tilde{\lambda})) \tilde{\subseteq} \tilde{fs}\ \alpha\ clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$.

Proof : Follows from Proposition 3.5, Proposition 3.7, Proposition 3.8, Proposition 3.9.

Remark 3.13 : The above discussions give the following implication diagram.

Fuzzy soft continuous mapping → Fuzzy soft α-continuous mapping.

4. Conclusion

Fuzzy soft α-continuous mappings have been characterized using recent concepts in the literature of fuzzy soft topology.

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