International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 11 No. I (April, 2017), pp. 47-55

EQUITABLE DOMINATION IN INTERVAL VALUED FUZZY GRAPHS

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Abstract

Let $G = (A, B)$ be an interval valued fuzzy graph on V and $x, y \in V$. We say x dominates y if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$. A subset S of V is called a fuzzy equitable dominating set if every $v \in V - S$ there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$ where $deg(u)$ denotes the degree of vertex u and $deg(v)$ denote the degree of vertex v and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{fe}. We introduce equitable domination in interval valued fuzzy graphs and obtain some interesting results for this new parameter in interval valued fuzzy graphs.

1. Introduction

L. A. Zadeh (1965) introduced the concepts of fuzzy subset of a set as a way for representing uncertainty. The earliest idea of domination occurred in the game of chess

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Key Words : *Dominating set, Domination in interval valued fuzzy graphs, Equitable domination in interval valued fuzzy graphs.*

where the problem was to place minimum number of chess pieces so as to dominate all the squares of the chess board. Mathematical research on the theory of domination for crisp graphs was initiated by Ore [1]. Domination in fuzzy graphs was introduced and studied by Somasundaram A. and Somasundaram S. in [2] Zadeh[3] introduced the notion interval valued fuzzy sets as an extension of fuzzy sets [4] which gives a more precise tool to model uncertainty in real life situations. Interval valued fuzzy-sets have been widely used in many areas of science and engineering. Example in approximate reasoning, medical diagnosis, multivalued logic, intelligent control, topological spaces etc. Hongmei and Lianhua introduced the definition of interval-Valued fuzzy graphs in [5].

In this paper, our aim to introduce and study the theory of equitable domination in the setting of interval valued fuzzy graphs.

2. Preliminaries

Throughout this paper a graph will denote a graph without loops. For graph theoretic notations and terminologies, the readers are referred to Hararay [6]. First we collect some definitions to be used in this paper.

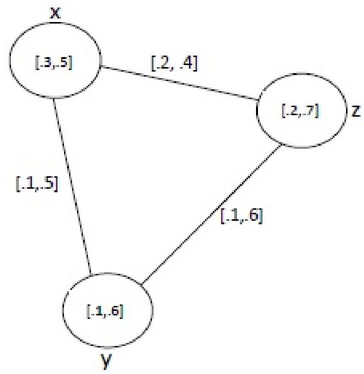
Definition 2.1 : An interval-valued fuzzy set A on a set V is defined by $A = \{x, \lfloor \mu_A^-(x), \mu_A^+(x) \rfloor : x \in V\}$ where μ_A^- and μ_A^+ are fuzzy subsets of V, such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$.

If $G^* = (V, E)$ is a crisp graph, then by an interval-valued fuzzy relation B on V. We mean an interval-valued fuzzy set on E such that $\mu_B^-(xy) \leq \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all $xy \in E$ and we write $B = \{xy, \lfloor \mu_B^-(xy), \mu_B^+(xy) \rfloor : xy : E\}$.

Definition 2.2 : An interval valued fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (A, B)$ where $A = \lfloor \mu_A^-, \mu_A^+ \rfloor$ is an interval valued fuzzy set on V and $B = \lfloor \mu_B^-, \mu_B^+ \rfloor$ is an interval-valued fuzzy relation on V.

Example 2.3 : Consider the graph $G^* = (V, E)$ where $V = \{x, y, z\}$ and $E = \{xy, yz, zx\}$. Let A be an interval-valued fuzzy set on V and let B be an interval valued fuzzy set on $E \subseteq V \times V$ defined by

$$A = \left\langle \left(\frac{x}{.3}, \frac{y}{.1}, \frac{z}{.2} \right), \left(\frac{x}{.5}, \frac{y}{.6}, \frac{z}{.7} \right) \right\rangle \quad B = \left\langle \left(\frac{xy}{.1}, \frac{yz}{.1}, \frac{zx}{.2} \right), \left(\frac{xy}{.5}, \frac{yz}{.6}, \frac{zx}{.4} \right) \right\rangle$$



Then $G = (A, B)$ is an interval valued fuzzy graph $G^* = (V, E)$.

Definition: 2.4 : The order p and size q of an interval valued fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ are defined to be $p = \sum_{v \in V} \frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$ and
$q = \sum_{xy \in E} \frac{1+\mu_B^+(xy)-\mu_B^-(xy)}{2}$.

Definition 2.5 : Let $G = (A, B)$ be an interval valued fuzzy graph $G^* = (V, E)$ and $S \subseteq V$. Then the cardinality of S is defined to be $\sum_{v \in S} \frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$.

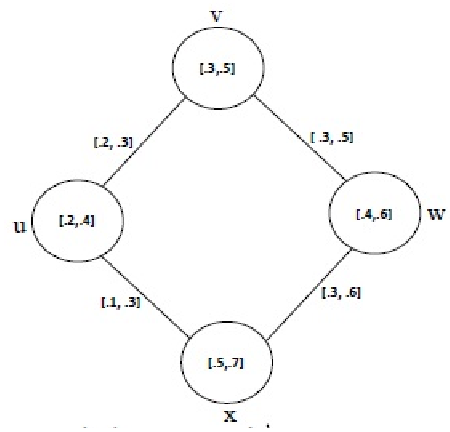
Definition 2.6 : An interval valued fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is said to be complete if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all $xy \in E$ and is denoted by $k_{\mu A}$.

Definition 2.7 : The complement of an Interval valued fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is the interval valued fuzzy graph $\overline{G} = (\overline{A}, \overline{B})$ where $\overline{A} = [\mu_A^-, \mu_A^+]$ and $\overline{B} = \lfloor \overline{\mu_B^-}, \overline{\mu_B^+} \rfloor$ is defined by $\overline{\mu_B^-}(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\} - \mu_B^-(xy), \overline{\mu_B^+}(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\} - \mu_B^+(xy)$ for all $xy \in E$.

Definition 2.8 : An interval valued fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu_B^-(xy) = 0$ and $\mu_B^+(xy) = 0$ if $x, y \in V_1$ or $x, y \in V_2$. Further if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all $x \in V_1$ and $y \in V_2$ then G is called a complete bipartite graph and is denoted by $K_{\mu_A}^-, K_{\mu_A}^+$ where μ_A^- and μ_A^+ are restrictions of μ_A^- and μ_A^+ on V_1 and V_2 respectively.

Definition 2.9 : An edge $e = xy$ of an interval valued fuzzy graph G is called an effective edge if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$. In this case the vertex x is called a neighbor of y and conversely. $N(x) = \{y \in V : y$ is a neighbor of $x\}$ is called the neighborhood of x.

Example 2.10 : Consider the graph $G^* = (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$. Let A be an interval valued fuzzy set on V and let B be an interval valued fuzzy set on $E \subseteq V \times V$ defined by



$$A = \left\langle \left(\frac{u}{.2}, \frac{v}{.3}, \frac{w}{.4}, \frac{x}{.5} \right), \quad \left(\frac{u}{.4}, \frac{v}{.5}, \frac{w}{.6}, \frac{x}{.7} \right) \right\rangle$$
$$B = \left\langle \left(\frac{uv}{.2}, \frac{vw}{.3}, \frac{wx}{.3}, \frac{xu}{.1} \right), \quad \left(\frac{uv}{.3}, \frac{vw}{.5}, \frac{wx}{.6}, \frac{xu}{.3} \right) \right\rangle.$$

Then $G = (A, B)$ is an interval valued fuzzy graph of $G^* = (V, E)$.

In this example ux and uv are effective edges. Also $N(u) = \{v, x\}$ and $N(v) = u, N(x) = u$, $N(w) = \phi$ (the empty set).

Definition 2.11 : Let $G = (A, B)$ be an interval valued fuzzy graph on V and $x, y \in V$. We say x dominates y if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$.

A subset S of V is called a dominating set in G if for every $v \notin S$ there exists $u \in S$ such that u dominates v.

The minimum cardinality of a dominating in G is called the domination number of G and is denoted by $\gamma(G)$.

Remark 2.12 :

(i) For any $x, y \in V$, if x dominates y then y dominates x and as such domination is a symmetric relation.

(ii) If $\mu_B^-(xy) < \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) < \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all $x, y \in V$ then the only dominating set in G is V.

Remark 2.13 : Since $\{v\}$ is a dominating set of $K_{\mu A}$ for each $v \in V$, we have

(i) $\gamma(k_{\mu A}) = \min\limits_{v \in V} \frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$

(ii) $\gamma(\overline{k_{\mu A}}) = p$

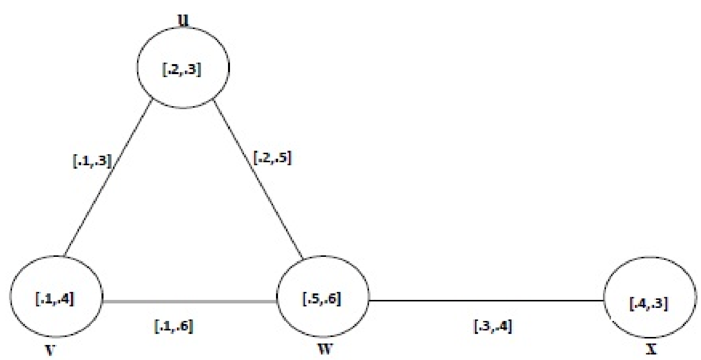
(iii) $\gamma(k_{\mu A^1, \mu A^2}) = \min\limits_{x \in V_1} \frac{1+\mu_A^+(x)-\mu_A^-(x)}{2} + \min\limits_{y \in V_2} \frac{1+\mu_A^+(y)-\mu_A^-(y)}{2}$.

Example 2.14 : Consider the graph $G^* = (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, uw, wx\}$. Let A be an interval valued fuzzy set on V and let B be an interval valued fuzzy set on $E \subseteq V \times V$ defined by

$$A = \left\langle \left(\frac{u}{.2}, \frac{v}{.1}, \frac{w}{.5}, \frac{x}{.4}\right), \quad \left(\frac{u}{.4}, \frac{v}{.3}, \frac{w}{.5}, \frac{x}{.3}\right)\right\rangle$$

$$B = \left\langle \left(\frac{uv}{.1}, \frac{vw}{.1}, \frac{uw}{.2}, \frac{wx}{.3}\right), \quad \left(\frac{uv}{.3}, \frac{vw}{.6}, \frac{uw}{.5}, \frac{wx}{.4}\right)\right\rangle.$$

Let $G = (A, B)$ is an interval valued fuzzy graph $G^* = (V, E)$.



Since $\{w\}$ is a dominating set of G for each $w \in V$ and we have the domination number of interval valued fuzzy graph G is

$$\gamma(G) = \min_{w \in V} \frac{1 + \mu_A^+(w) - \mu_A^-(w)}{2} = \frac{1 + 0.6 - 0.5}{2} = 0.55.$$

3. Equitable Domination in Interval Valued Fuzzy Graph

In the section, we introduce equitable domination in interval valued fuzzy graph.

Definition 3.1 : Let $G = (A, B)$ be an interval valued fuzzy graph on V and $x, y \in V$. A dominating set S is a equitable dominating set of interval valued fuzzy graph G if for every $v \in V - S$ there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$.

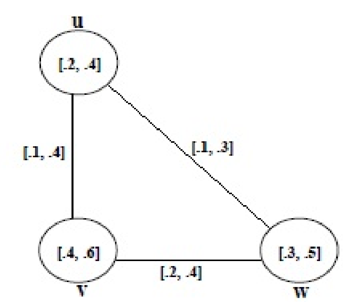
The Minimum cardinality taken over all equitable dominating set of interval valued fuzzy graph is called equitable domination number of G and is denoted by $\gamma_e(G)$.

The degree of a vertex of an interval valued fuzzy graph is defined below.

Definition 3.2 : Let $G = (A, B)$ be an interval valued fuzzy graph where $A = \lfloor \mu_A^-, \mu_A^+ \rfloor$ and $B = \lfloor \mu_B^-, \mu_B^+ \rfloor$ be two interval valued fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$ respectively. The positive degree of a vertex $u \in G$ is $d^+(u) = \sum\limits_{uv \in E} \mu_B^+(uv)$. Similarly negative degree of a vertex $u \in G$ is $d^-(u) = \sum\limits_{uv \in E} \mu_B^-(uv)$.

The degree of vertex u is $d(u) = [d^-(u), d^+(u)]$. If $d^+(u) = k_1, d^-(u) = k_2$ for all $u \in V$. k_1, k_2 are two real numbers, then the graph is called $[k_1, k_2]$-regular interval valued fuzzy graph.

Example 3.3 : We consider an interval valued fuzzy graph $G = (A, B)$.



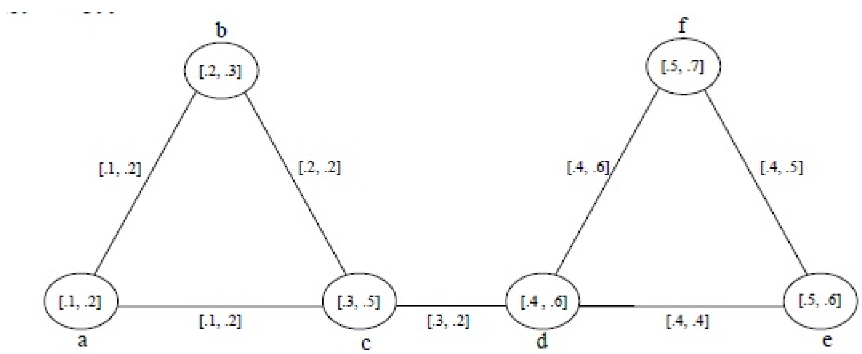
$$d^-(u) = .1 + .1 = .2, d^+(u) = .3 + .4 = .7, deg(u) = (.2, .7).$$

We have

$$d^-(v) = .1 + .2 = .3, d^+(v) = .4 + .4 = .8, deg(v) = (.3, .8)$$

$$d^-(w) = .2 + .1 = .3, d^+(w) = .4 + .3 = .7, deg(w) = (.3, .7)$$

Example: 3.4 :



$S = \{c, d\}$ $V = \{a, b, c, d, e, f\}$
$V - S = \{a, b, e, f\}$.
Since $\{c, d\}$ is a equitable dominating set of G.

$$\gamma_{fe}(G) = \min_{c \in V} \frac{1 + \mu_A^+(c) - \mu_A^-(c)}{2} + \min_{d \in V} \frac{1 + \mu_A^+(d) - \mu_A^-(d)}{2} = 0.6 + 0.6 = 1.2.$$

Theorem 3.5 : Let S be a minimal equitable dominating set of interval valued fuzzy graph G. Then for each $u \in S$ of following holds

(i) $N(u) \cap S = \phi$

(ii) There is a vertex $v \in V - S$ such that $N(v) \cap S = \{u\}$.

Proof : Let S be a minimal equitable dominating set of G. Then for every vertex $u \in S, S - \{u\}$ is not a equitable dominating set and so there exists $v \in V - \{S - \{u\}\}$ which is not dominated by any vertex in $S - \{u\}$. If $v = u$ then (i) holds. If $v \neq u$ then v is not dominated by $S - \{u\}$, but is dominated by S. If v is dominated by u in S. Hence $N(v) \cap S = \{u\}$.
Conversely, let S be a equitable dominating set and for each vertex $u \in S$, one of the two conditions holds. Suppose S be not a minimal equitable dominating set. Then there exists a vertex $u \in S$ such that $S - \{u\}$ is a equitable dominating set. Thus u is dominated by atleast one vertex in $S - \{u\}$ and so, the condition (i) does not holds. Again if $S - \{u\}$ is a equitable dominating set, then every vertex in $V - S$ is dominated by atleast one vertex in $S - \{u\}$ which implies that the condition (ii) does not hold. This leads to a contradiction.
Hence S must be a minimal equitable dominating set.

Definition 3.6 : The maximum and minimum fuzzy equitable degree of a vertex in interval valued fuzzy graph G are denoted by $\Delta_{fe}(G)$ and $\delta_{fe}(G)$. That is $\Delta_{fe}(G) = \max_{u \in V(G)} |N_{fe}(u)|$ and $\delta_{fe}(G) = \min_{u \in V(G)} |N_{fe}(u)|$.

Definition 3.7 : Let $G = (A, B)$ be an interval valued fuzzy graph where $A = \lfloor \mu_A^-, \mu_A^+ \rfloor$ and $B = \lfloor \mu_B^-, \mu_B^+ \rfloor$ be two interval valued fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$ respectively. The total degree of a vertex $u \in V$ is denoted by $td(u)$ and is defined as $td(u) = [td^+(u), td^-(u)]$ where

$$td^+(u) = \sum_{uv \in E} \mu_B^+(uv) + \mu_A^+(u), td^-(u) = \sum_{uv \in E} \mu_B^-(uv) + \mu_A^-(u).$$

If the total degrees of all vertices of an interval valued fuzzy graph are equal, then the graph is said to be totally regular interval valued fuzzy graph.

Definition 3.8 : A vertex u of an Interval valued fuzzy graph G is said to be an isolated vertex if $\mu_B^-(uv) < \min\{\mu_A^-(u), \mu_A^-(v)\}$ and $\mu_B^+(uv) < \max\{\mu_A^+(u), \mu_A^+(v)\}$ for all $v \in V - \{u\}$ such that there is an edge between u and v. i.e., $N(u) = \phi$.

Remark 3.9 : An isolated vertex does not dominate any other vertex in G.

Theorem 3.10 : Let G be an interval valued fuzzy graph without isolated vertices. Let S be a minimal equitable dominating set of G. Then $V - S$ is a equitable dominating set of G.

Proof : Let S be a minimal equitable dominating set and $a \in S$. Since G has no isolated vertices, there is a vertex $x \in N(a)$. By the Theorem 3.5 we get $x \in V - S$. Thus every element of S is equitable dominated by some element of $V - S$ and consequently $V - S$ is a dominating set.

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