International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 11 No. I (April, 2017), pp. 79-87

THE ORLICZ SPACE OF GENERALIZED ENTIRE SEQUENCES OF FUZZY NUMBERS

A. PANDIARANI
Department of Mathematics,
G.Venkataswamy Naidu College, Kovilpatti -628502,
Tamil Nadu, India

Abstract

In this paper the Orlicz space of generalized entire sequences of fuzzy numbers is introduced. Some properties of this space like completeness solidness, symmetricity, convergence free are studied. Some inclusion relations involving this sequence space are also obtained.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by zadeh [15] and subsequently several authors have discussed various aspects of theory and applications of fuzzy sets. Bounded and convergent sequences of fuzzy numbers was introduced by matloka [7]. Nandha[9] has studied the space of all absolutely p-summable convergent sequences of fuzzy numbers and shown that they are all complete metric spaces. Later on sequence of fuzzy numbers have been discussed by Dutta [2], Mursaleen [8]. Nuray and savas [10], Talo and Basar [11] and many others.

Key Words : *Fuzzy numbers, Entire sequence space, Completeness solidity, Symmetricity.*
AMS Subject Classification : 40A45, 40A35.

An Orlicz function is a function $M : [0, \infty] \rightarrow [0, \infty]$ which is continuous, non decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$. If the convexity of the Orlicz function is replaced by sub-additivity then this function is called modulus function. An Orlicz function satisfies the inequality $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda < 1$. Lindenstrauns and trafrirl [ ] investigated orlicz sequence space in more detail.

2. Definition and Preliminalies

In this section we give some required definitions and statements of theorems, propositions and lemmas. A fuzzy number is a fuzzy set on the real axis i.e, a mapping $u : R \rightarrow [0, 1]$ which satisfies the following four conditions

(i) u is normal i.e. there exist an x_0 in R such that $u(x_0) = 1$

(ii) u is fuzzy convex i.e $u[\lambda x + (1 - \lambda)y] \geq \min\{u(x), u(y)\}$ for all $x, y \in R$ and for all $\lambda \in [0, 1]$.

(iii) u is upper semi continuous

(iv) the set $[u]_o = \{x \in R : u(x) > 0\}$ is compact where $\{x \in R : u(x) > 0\}$ denotes the closure of the set $\{x \in R : u(x) > 0\}$ in the usual topology of R.

We denote the set of all fuzzy numbers on R by E and called it as the space of fuzzy numbers. The λ-level set $[u]\lambda$ of $u \in E'$ is defined by

$$[u]_\lambda = \left\{ \frac{\{t \in R : u(t)\lambda\}, (0 < \lambda \leq 1)}{\{t \in R : u(t) > \lambda} \right\}, \lambda = 0.$$

The set $[u]_\lambda$ is a closed bounded and non-empty interval for each $\lambda \in [0, 1]$ which is defined by $[u]_\lambda = [u^-\lambda, u^+\lambda]$ since each $r \in R$ can be regarded as a fuzzy number r defined by

$$r(x) = \begin{cases} 1, & x = r \\ \\ 0, & x \neq r \end{cases}$$

Let $u, v, w \in E$ and $k \in R$ the operations addition, scalar multiplication and product defined on 'E' by

$$u + v = w \Leftrightarrow [W]_d = [u]_\lambda + [v]_\lambda \text{ for all } \lambda \in [0, 1]$$

$$\Leftrightarrow [\lambda^{-} t\lambda = u^{-}(\lambda)v^{-}(\lambda)] \text{ and } w^{+}(\lambda) = [u^{+}(\lambda)v^{+}(\lambda)]$$

and for $\lambda \in [0,1]$ $[ku]_\lambda = K[u]_\lambda$ for all$\lambda \in [0,1]$ and $uv = w \Leftrightarrow [w]_\lambda = [u]_\lambda [v]_\lambda$ for all $\lambda \in [0,1]$ where it is immediate that

$$w^{-}(\lambda) = \min\{u^{-}(\lambda)v^{-}(\lambda), u^{-}(\lambda)v^{+}(\lambda)u^{+}(\lambda)v^{-}(\lambda), u^{+}(\lambda)v^{+}(\lambda)\}$$

and

$$w^{-}(\lambda) = \max\{u^{-}(\lambda)v^{-}(\lambda), u^{-}(\lambda)v^{+}(\lambda)u^{+}(\lambda)v^{-}(\lambda), u^{+}(\lambda)v^{+}(\lambda)\}.$$

$$u/v = w \Leftrightarrow [w]_\lambda = [u]_\lambda [v]_\lambda \text{ for all } \lambda \in [0,1].$$

Let w be the set of all closed and bounded intervals A of real numbers with end points A and A i.e. $A = [A, A]$ define the relation d on w by

$$d(A,B) = \max\{|A - B|, |A - B|\}.$$

Then it can be observed that d is a metric on W and (w, d) is a complete metric space. Now we can define the metric D on E' by means of a Hausdorff metric d as

$$D(u,v) = \sup d([u]_\lambda, [v]_\lambda) = \sup_{\lambda \in [0,1]} \max\{u^{-}(\lambda) - v^{+}(\lambda)|, (u^{+}(\lambda) - v^{+}(\lambda)|)\}.$$

$(E' - D)$ is a complete metric space one can extend the natural order relation on the real line to intervals as follows:

$$A \le B \text{ iff } \overline{A} \le \overline{B} \text{ and } \underline{A} \le \underline{B}.$$

The partial order relation on E' is defined as follows $u \le v \Leftrightarrow [u]_\lambda \Leftrightarrow u^{-}(\lambda) \le v^{-}(\lambda)$ and $u^{+}(\lambda) \le v^{+}(\lambda)$ an absolute value $|u|$ of a fuzzy number u is defined by

$$|u|(t) = \begin{cases} \max\{U^{-}(t), u^{+}(t)\}, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

λ level set $[|u|]_\lambda$ of the absolute value $|u|(t)$ is in the form $[|u|]_\lambda$ where

$$|u|^{-}(\lambda) = \max\{0, u^{-}(\lambda), -u^{+}(\lambda)\}$$

$$|u|^{+}(\lambda) = \max\{|u \in v|\}.$$

Definition 2.1 : A sequence $u = (u_k)$ of fuzzy numbers is a function u from the set N into the set E'. The fuzzy number u_k denotes the value of the function at $R \in N$ and is called the k-th term of the sequence. Let $w(F)$ denote the set of all fuzzy sequences.

Definition 2.2 : A sequence $(u_k) \in w(F)$ is called convergent with limit $u \in E'$ if and only if for every $\epsilon > 0$ there exist an no such that $d(x_k, x) < \epsilon$ for all $k \in n_o$.

Definition 2.3 : A sequence $(u_k) \in w(F)$ is called bounded if and only if the set of all fuzzy numbers consisting of the terms of the sequence (u_k) is a bounded set.

Definition 2.4 : Let $(u_k) \in w(F)$. Then the expression $\sum u_k$ is called a series of fuzzy numbers. Denote $S_n = \sum_{k=1}^{\infty} u_k$ for all $n \in N$, if the sequences (s_n) converges to a fuzzy number $u$then we say that the series $\sum_{k=1}^{\infty} u_k$ of fuzzy numbers converges to u and write $\sum u_k = u$.

Definition 2.5 : A fuzzy sequence space $E(F) \subset W(F)$ is said to be normal (or solid) if $(u_k) \in E(F)$ and (v_k) is such that $D(v_k, \overline{0}) \leq D(u_k, \overline{0})$ implies $(v_k) \in E(F)$.

Definition 2.6 : A sequence space $E(F)$ is said to be symmetric if $(u_{n(k)}) \in E(F)$ whenever $(u_k) \in E(F)$.

A sequence space $E(F)$ is monotone whenever it is solid Lidenstrauss and Tzafrir [ ] used the notion of orlicz function and introduced the sequence space

$$\ell_M = \left\{ (x_k) = \epsilon W) : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \ forsome \ \ \rho > 0 \right\}.$$

The space ℓ_M becomes a Banach space with the norm defined by

$$\|(x_k)\| = \inf \left\{ \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}$$

which is called an orlicz sequence space.

T. Balasubramanian and A. Pandiarani [1] introduced the space $G_\lambda(F)$. Let $\lambda = (\lambda_k)$ be a nonnegative sequence of fuzzy numbers such that $D(\lambda_k, 1)^{-1/k} \to 0$ as $k \to \infty \lambda_k \neq 1$. Let $u = (u_k)$ be a sequence of fuzzy numbers.

We define $Au_l = \lambda_k u_k$ then we introduce the space $G_\lambda(F)$as

$$G_\lambda(F) = \left\{ u = (u_k) \in W(F) \sum_{k=1}^{\infty} [D(\Lambda u_k, \overline{o})]^2 < \infty \right\}.$$

The space $m(\phi)$ introduced by Sargent [13] defined by

$$m(\phi) = \left\{ (x_k) \in w(F) : \sup_{s>1\sigma\in\wp}, \frac{1}{\phi_s} \sum_{k\in\sigma} |x_k| < \infty \right\}.$$

In this paper we introduce the space $G_\lambda(M, \phi, F)$. It is defined by

$$G_\lambda(M, \phi, F) = \left\{ u = (u_k) \in W(F) : \sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k, \overline{o})}{\rho} \right) \right]^2 < \infty \right\}$$

for same $\rho > 0$.

3. Main Results

Theorem 3.1 : The set $G_\lambda(M, \phi, F)$ is a complete metric space with the metric

$$g(u, v) = \inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k, \Lambda v_k)}{\delta} \right) \right]^2 < \infty \right\}$$

for some $\rho > 0$ and $u, v \in G_\lambda(M, \phi, F)$.

It is easy to show that $G_\lambda(M, \phi, F)$ is a metric space with the metric g. let $(u^{(i)})$ be a Cauchy sequence in $G_\lambda(M, \phi, F)$. Let $\epsilon > 0$ be given. For a fixed $x_0 > o$ choose $r > o$ such that $M\left(\frac{rx_0}{2}\right) \geq 1$. Then there exist a positive integer n_o such that

$$g(u^{(i)}, u^{(ij)}) < \frac{\epsilon}{r(x_0)} \quad \text{for all} \quad i, j \geq n_0.$$

By the definition of g we get

$$\inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)})}{\delta} \right) \right]^2 < 1 \right\} < \epsilon \text{ for all` } i, j \geq n_0 \quad (3.1)$$

which implies that

$$\inf \left\{ \sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)})}{\delta} \right) \right]^2 < 1 \right\} \text{ for all } i, j \geq n_0 \quad (3.2)$$

for $s = 1$ and σ varying over ρ, we get

$$\sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)})}{\delta} \right) \right]^2 < \phi\} \text{for all } i, j \geq n_0$$

which implies $M\left(\frac{D(\Lambda u_k^{(i)},\Lambda v_k^{(j)})}{\delta}\right) \leq \phi^{1/2} \leq M\left(\frac{rx_0}{2}\right)$ for all $i, j \geq n_0$. Using the continuity of M we get

$$D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)}) \leq \left(\frac{rx_0}{2}\right)\delta \quad \text{for all} \quad i, j \geq n_0.$$

Thus

$$D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)}) \leq \left(\frac{rx_0}{2}\right) \cdot \frac{\epsilon}{rx_0} = \frac{\epsilon}{2} \quad \text{for all} \quad i, j \geq n_0.$$

which implies that Λu_k is a Cauchy sequence in E'. Since E' is complete, it is convergent. Let $\lim_i \Lambda u_k^{(i)} = \Lambda u_k$. We have to prove $\lim_i \Lambda u^{(i)} = u$ and $u \in G_\lambda(M, \phi, F)$.
Using the continuity of M in (3.2) we get

$$\sup_{s\geq 1, \sigma\in\rho_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)})}{\delta}\right)\right]^2 \leq 1$$

for some $\rho > 0$ and $i \geq n_o$.
Now taking the infimum of such ρ's and using 3.1

$$\inf\left\{\rho > 0 : \sup_{s\geq 1, \sigma\in\rho_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k^{(i)}, \Lambda v_k^{(j)})}{\delta}\right)\right]^2 < 1\right\} < \epsilon \quad \text{for all} \quad i \geq n_0.$$

Hence we get $g(u^{(i)}, u) < \epsilon/2$ which implies that $\lim_i \Lambda u^{(i)} = u$.
Since $(u^{(i)})$ is a sequence in $G_\lambda(M, \phi, F)$ we get for given $\epsilon > 0$ $g(u^{(i)}, \overline{0}) < \epsilon/2 \ \forall \ i \geq n_1$.
Now $g(u^{(i)}, \overline{0}) \leq g(u^{(i)}, u) + g(u^{(i)}, \overline{0}) < \epsilon/2 + \epsilon/2 = \epsilon \ \ \forall \ \ i \geq N = \max\{n_0, n_1\}$. Hence $u \in G_\lambda(M, \phi, F)$.
Therefore $G_\lambda(M, \phi, F)$ is a complete metric space.
Theorem 3.2 : $G_\lambda(M, \phi, F) \subseteq G_\lambda(M, \psi, F)$ if and only if $\sup_{s\geq 1}\left[\frac{\phi_s}{\psi_s}\right] < \infty$ for the sequence (ϕ_s) and (ψ_s) of real numbers.
Proof : Suppose $\sup_{s\geq 1}\left[\frac{\phi_s}{\psi_s}\right] = k < \infty$. Then we have $\phi_s \leq k\psi_s$.
Now if $(u_k) \in G_\lambda(M, \phi, F)$ then

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \infty$$

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{k\psi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \infty.$$

Thus $(u_k) \in G_\lambda(M, \psi, F)$.

Therefore $G_\lambda(M,\phi,F) \subseteq G_\lambda(M,\psi,F)$.

Conversely, suppose that $G_\lambda(M,\phi,F) \subseteq G_\lambda(M,\psi,F)$.

Let $\eta_s = \left(\frac{\phi_s}{\psi_s}\right)$. Suppose $\sup_{s\geq 1}(\eta_s) = \infty$. Then there exists a subsequence (η_{si}) of (η_s) such that $\lim_{i\to\infty}(\eta_{si}) = \infty$ then for $(u_k) \in G_\lambda(M,\phi,F)$ we have

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 = \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\psi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2.$$

But $\lim_{i\to\infty}(\eta_{s_i}) = \infty$.

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\psi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 = \infty$$

which implies that $(u_k) \in G_\lambda(M,\psi,F)$, a contradicton. This completes the proof.

Theorem 3.3 : $G_\lambda(M,\phi,F)$ is a solid space.

Proof : Let (u_k) and (v_k) be two fuzzy real valued sequences such that $D(v_k,\overline{0}) \leq D(u_k,\overline{0})$ for all $k \in N$ and $(u_k) \in G_\lambda(M,\phi,F)$. Then

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \infty$$

We have $D(v_k,\overline{0}) \leq D(u_k,\overline{0}) \to D(v_k,\overline{0}) \leq D(u_k,\overline{0}) \leq D(v_k,\overline{0})$ which implies that

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 \leq \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \infty.$$

Therefore $(v_k) \in G_\lambda(M,\phi,F)$.

Theorem 3.4 : $G_\lambda(M,\phi,F)$ is a symmetric space.

Proof : Let (u_k) be a sequence fuzzy numbers in $G_\lambda(M,\phi,F)$. Then

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \infty.$$

For $\epsilon > 0$ there exist $k(\epsilon)$ such that

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 - \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < ep.$$

Let (v_k) be a rearrangement of (u_k). Let k be such that $u_k : k \leq k_0 \leq v_k : k \leq k_1$. Then

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 = \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2$$

$$\leq \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 - \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \epsilon.$$

$$\sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 < \epsilon.$$

Thus $(v_k) \in G_\lambda(M, \phi, F)$ which implies that $G_\lambda(M, \phi, F)$ is symmetric.

Definition 3.5 : Let $u = (u_k)$ be a fuzzy sequence let $(u^{(n)})$ denotes its nth section. In other words $u^{(n)} = (u_1, u_2, \cdots, u_n, \cdots, 0, 0, \cdots)$. If $E(F)$ is a fuzzy sequence space and if d is a metric for $E(F)$ we say that $E(F)$ has monotone metric if

(i) $d(u^{(n)}, \overline{0}) \leq d(u^{(m)}, \overline{0})$ whenever $n < m$

(ii) $\sup\limits_{(n)} d(u^{(n)}, \overline{0}) = d(u, \overline{0})$.

Theorem 3.6 : $G_\lambda(M, \phi, F)$ has monotone metric.

The metric on $G_\lambda(M, \phi, F)$ is given by

$$g(u,v) = \inf\left\{\rho > 0 : \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 \leq 1\right\}$$

Now

$$g(u^{(n)}, \overline{0}) = \left\{\rho > 0 : \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}^{n}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 \leq 1\right\}$$

$$g(u^{(m)}, \overline{0}) = \inf\left\{\rho > 0 : \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}^{m}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)\right]^2 \leq 1\right\}.$$

Hence if $n < m$, we get $g(u^{(n)}, \overline{0}) < g(u^{(m)}, \overline{0})$. Also $\lim\limits_{n\to\infty} g(u^{(n)}, \overline{0}) = g(u, 0)$ and since $g(u^{(n)}, \overline{0})$ is an increasing sequence we get $\sup\limits_{(n)} g(u^{(n)}, \overline{0}) = g(u, 0)$.

Thus $G_\lambda(M, \phi, F)$ is has monotone metric.

Theorem 3.7 : $G_\lambda(M, \phi, F) \subset \Gamma(M, \phi, F)$ where

$$\Gamma(M, \phi, F) = \left\{\rho > 0 : \sup_{s\geq 1,\sigma\in\wp_s} \frac{1}{\phi_s}\sum_{k\in\sigma}\left[M\left(\frac{D(\Lambda u_k \overline{0})}{\rho}\right)^{\frac{1}{k}}\right]\right\} \to 0 \text{ as } k \to \infty.$$

Let $u = (u_k) \in G_\lambda(M, \phi, F)$. Then we get

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left[M \left(\frac{D(\Lambda u_k \overline{0})}{\rho} \right) \right]^2 \leq \infty.$$

Then there exist $k = k_0(\epsilon)$ such that

$$D(\Lambda u_k, , \overline{0}) < \epsilon \text{ for } k \geq k_0(\epsilon)$$

Which implies that

$$D(u_k, \overline{0}) = D(\lambda_k u_k, \overline{0}) \dot{D} \left(\frac{1}{\lambda_k} \overline{0} \right).$$

References

[1] Balasubramanian T. and Pandiarani A., A new class of sequences of fuzzy numbers, Int. J. pure Appl. Sci. Technol., 8(1) (2012), 1-14.

[2] Hemen Dutta, On the sometric spaces of c_0^F, c^F and ℓ_∞^F, Acta Universitatis Apulensis, 19 (2009), 107-112.

[3] Kamthan P. K., Bases in a certain class of Frechet space, Tamkang. J. Math., 7 (1976), 41-49.

[4] Kavi Kumar J. Azme Bin Khamis and Kandasamy R., Fuzzy entire sequence spaces, International journal of mathematics and mathematical sciences, Article ID 58368, (2007).

[5] Maddox I. J., Elements of Functional Analysis, Cambridge University Press.

[6] Maddox I. J., Spaces of strongly summable sequences, Quart. J. Math. Oxford, 18(2) (1967), 345-355.

[7] Matloka M., Sequences of Fuzzy numbers, BUSEFAL, 28 (1986), 28-37.

[8] Mursaleen M., Basarir M., On some new sequence spaces of fuzzy numbers, Indian J. Pure Appl. Math., 34(9) (2003), 1351-1357.

[9] Nandha S., On Sequences of Fuzz numbers, Fuzzy Sets and Systems, 33 (1989), 123-126.

[10] Nuray F., Savas E., Statistical convergence of sequence of fuzzy realnumbers, Math. Slovaca, 45(3) (1995), 269-273.

[11] Ozer Talo, Feyzi Basar, Determination of the duals of classical sets of sequences of fuzzy numbers and related matrix transformations. Computers and Mathematics with Applications, 58 (2009), 717-733.

[12] Savas E., A note on sequences of Fuzzy numbers, Information sciences, 124 (2000), 297-300.

[13] Sargent W. L. C., Some sequence spaces related to the space ℓ^p, Jour. London Math. Soc., 35 (1960), 161-171.

[14] Tripathy B. C., A class of difference sequence spaces related to the p-normed spaces ℓ^p, Demonstratio Math., 36(4) (2003), 867-872.

[15] Zadeh L. A., Fuzzy sets, Inf. Control, 8 (1965), 338-353.