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LOCAL AND GLOBAL STABILITY OF IMPULSIVE PEST MANAGEMENT MODEL WITH BIOLOGICAL HYBRID CONTROL

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Abstract

The aim of this paper is to analyze the dynamics of plant-pest-virus and natural enemy food chain model. The impulsive releasing of virus and natural enemy are considered as control input. Using this model, we study two periodic solutions namely, plant-pest extinction and pest extinction periodic solutions. By using Floquet theory of impulsive differential equations and small amplitude perturbation technique, we study pest control through the local stability of both the periodic solutions. The sufficient condition for global attractivity of pest-extinction periodic solution is obtained by using comparison principle of impulsive differential equations. At last numerical simulation is given to support the theoretical results.

1. Introduction

From thousands of years, the pest attacking on crops has been a serious economic

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Key Words : *Pest Management, Impulsive Differential Equations, Boundedness, Local and Global Stability.*

and ecological concern. People of different era associated with agriculture used their own technology and methodology to control the pests. With the passage of time and development of agriculture science and technology, a number of pesticides has been formulated which can kill the pests rapidly and are exhaustively used by farmers. But from last few decades, it has been observed that the pollution caused by pesticides is dangerous and has desperately damaged the health of human being and other creatures of the world. Moreover recent study shows that with regular use of pesticides, a number of pests have become resistant to some pesticides [9]. This also leads to farmers' loss and forces them to use strong pesticides.

As a result various different techniques were adopted. One of the most important techniques is biological technique which includes infusion of predators and/or some disease in pests through some virus or infected pest. Various mathematicians have studied these cases through modeling and have obtained various useful results [3, 10]. Some other food chain models describing different aspects are discussed in [4, 8, 12]. After that some mathematicians have also studied hybrid approach which include the use of biological as well as chemical control [2, 6, 7, 11, 10]

In this paper, we modified a food chain model developed by Wang et. al. [2] by incorporating virus (manufactured in laboratory) and making clear assumptions that pests are attacked or controlled by virus which makes them infected and predators consume them directly. This is also a type of hybrid approach in which two biological techniques are simultaneously applied to control the pest population.

The present paper is organized as follows. In section 2, we develop a pest control model in which viruses and predators are released impulsively. Some important lemmas and boundedness of the system are established in section 3. Using Floquet's theory, small amplitude perturbation technique and comparison principles, sufficient conditions for local stability and global attractivity of pest eradication periodic solutions are obtained in section 4. Finally numerical simulations and discussions are done in last section.

Mathematical Model

Before proposing the mathematical model describing the complex behavior of plant-pest-virus and natural enemy, we make following assumptions:

(A1) Susceptible pest attacks plant.

(A2) Virus attacks susceptible pest and make them infected.

(A3) Infected pest when dies release virus.

(A4) Natural enemy attacks susceptible pest and consumes them directly.

(A5) Virus and natural enemy are released periodically.

With these assumptions, the model proposed by Wang et. al. [2] is modified and following mathematical model is proposed:

$$\left\{\begin{aligned} \frac{dx(t)}{dt} &= x(t)(1-x(t)) - cx(t)s(t), \\ \frac{ds(t)}{dt} &= cx(t)s(t) - \alpha s(t)v(t) - \beta s(t)z(t), \\ \frac{dI(t)}{dt} &= \alpha s(t)v(t) - d_1 I(t), \\ \frac{dv(t)}{dt} &= \mu d_2 I(t) - d_2 v(t), \\ \frac{dz(t)}{dt} &= \beta s(t)z(t) - d_3 z(t), \end{aligned}\right\} \quad t \neq nT, \tag{2.1}$$

$$\left\{\begin{aligned} x(t^+) &= x(t), \\ s(t^+) &= s(t), \\ I(t^+) &= I(t), \\ v(t^+) &= v(t) + \theta_1, \\ z(t^+) &= z(t) + \theta_2, \end{aligned}\right\} t = nT,$$

where $x(t), s(t), I(t), v(t)$ and $z(t)$ are densities of plant, suspectable pest, infected pest, virus particles and natural enemy respectively, c is predation rate of plant by suspectable pest, α is conversion rate of suspectable pest to infected pest, β is rate of predation by natural enemy, μ is production rate of virus from infected pest, d_1, d_2 and d_3 are natural death rates of infected pests, virus particles and natural enemies respectively, θ_1 and θ_2 are pulse releasing amount of virus particles and natural enemies at $t = nT, n = 1, 2, ...$ and T is the period of impulsive effect.

3. Preliminaries

The solution of system (2.1) is denoted by $Y(t) = (x(t), s(t), I(t), v(t), z(t))'$ and is a piecewise continuous function $Y : R_+ \to R_+^5$, that is, $Y(t)$ is continuous in the interval $(nT, (n+1)T], n \in Z_+$ and $Y(nT^+) = lim_{t \to nT^+} Y(t)$ exists. The smoothness properties of variables guarantee the global existence and uniqueness of a solution of the system (2.1), for details, see [5].

Before proving the main results, we firstly state and establish some lemmas which are useful in coming section.

Lemma 3.1 [5] : Let the function $m \in PC^{'}[R^+, R]$ and $m(t)$ be left-continues at $t_k, k = 1, 2, ...$ satisfy the inequalities

$$\begin{cases} m^{'}(t) \leq p(t)m(t) + q(t),\ t \geq t_0,\ t \neq t_k, \\ m(t_k^+) \leq d_k m(t_k) + b_k,\ t = t_k, k = 1, 2, ... \end{cases} \tag{3.1}$$

where $p, q \in PC[R^+, R]$ and $d_k \geq 0, b_k$ are constants, then

$$m(t) \leq m(t_0) \prod_{t_0<t_k<t} d_k exp\big(\int_{t_0}^{t} p(s)ds\big) + \sum_{t_0<t_k<t} \big(\prod_{t_k<t_j<t} d_j exp\big(\int_{t_0}^{t} p(s)ds\big)\big) b_k$$

$$+ \int_{t_0}^{t} \prod_{s<t_k<t} d_k exp\big(\int_{s}^{t} p(\sigma)d\sigma\big) q(s)ds,\ t \geq t_0. \tag{3.2}$$

If all the directions of the inequalities in (3.1) are reversed, then (3.2) also holds true for the reversed inequality.

Lemma 3.2 : There exists a constant $L > 0$, such that $x(t) \leq L, s(t) \leq L, I(t) \leq L, z(t) \leq L$ and $v(t) \leq L$ for all solutions $Y(t) = (x(t), s(t), I(t), v(t), z(t))$ of system (2.1) with t large enough.

Proof : Define $U(t) = x(t) + s(t) + I(t) + z(t)$ and let $0 < \bar{d} < \min\{d_1, d_3\}$.
Then for $t \neq nT$, we obtain that $D^+U(t) + \bar{d}U(t) \leq (1 + \bar{d})x(t) - x^2(t) \leq M_0$, where $M_0 = (1 + \bar{d})^2/4$.
When $t = nT, U(t^+) \leq U(t) + \theta_2$. By lemma 3.1 for $t \in (nT, (n + 1)T]$, we have

$$\begin{aligned} U(t) &\leq U(0)exp(-\bar{d}t) + \int_0^t M_0\theta_2 exp(-\bar{d}(t-s))ds + \sum_{0<nT<t} \theta_2 exp(-\bar{d}(t-nT)) \\ &\rightarrow \frac{M_0\theta_2}{\bar{d}} + \theta_2 exp(-\bar{d}T)/(exp(\bar{d}t) - 1), \text{as } t \rightarrow \infty. \end{aligned}$$

Thus $U(t)$ is uniformly bounded. Hence, by the definition of $U(t)$, there exists a constant $L_1 := \frac{M_0\theta_2}{\bar{d}} + \theta_2 exp(-\bar{d}T)/(exp(\bar{d}t) - 1)$ such that $x(t) \leq L_1, s(t) \leq L_1, I(t) \leq L_1, z(t) \leq L_1$ for all t large enough.
Now taking subsystem of (2.1),

$$\begin{cases} v^{'}(t) = \gamma I(t) - d_2 v(t) \leq \gamma L_1 - d_2 v(t), t \neq nT \\ v(t^+) = v(t) + \theta_1, t = nT \end{cases}$$

Again using lemma 3.1, we get

$$\begin{aligned} v(t) &\leq v(0)\exp(-\theta t) + \int_0^t \gamma L_1 exp(-d_2(t-s))ds + \sum_{D<kT<t} \theta_1 exp(-d_2(t-kT)) \\ &\rightarrow \frac{\gamma L_1}{d_2} + \frac{\theta_1 exp(d_2 T)}{\exp(d_2 T)-1} \equiv L_2 \text{ (say)}. \end{aligned}$$

Choosing $L = \max\{L_1, L_2\}$, we get the required result.

Lemma 3.3 [6] : Consider the following impulsive system

$$\begin{cases} u^{'}(t) = c - du(t), \ t \neq nT, \\ u(t^+) = u(t) + \mu, \ t = nT, n = 1, 2, 3.... \end{cases} \tag{3.3}$$

Then system (3.3) has a positive periodic solution $u^*(t)$ and for every solution $u(t)$ of (3.3), we have

$\mid u(t) - u^*(t) \mid \rightarrow 0$ as $t \rightarrow \infty$, where, for $t \in (nT, (n+1)T]$,

$$u^*(t) = \frac{c}{d} + \frac{\mu exp(-d(t-nT))}{1-exp(-dt)} \text{ with } \mathrm{u}^*(0^+) = \frac{\mathrm{c}}{\mathrm{d}} + \frac{\mu}{1-\exp(-\mathrm{dt})}.$$

Now we proceed to find pest extinction periodic solutions for the model (2.1). For the case of pest-extinction, we obtain the following impulsive system

$$\left.\begin{cases} \frac{dx(t)}{dt} = x(t)(1-x(t)), \\ \frac{dv(t)}{dt} = -d_2 v(t), \\ \frac{dz(t)}{dt} = -d_3 z(t), \end{cases}\right\} t \neq nT, \tag{3.4}$$

$$\left.\begin{cases} v(t^+) = v(t) + \theta_1, \\ z(t^+) = z(t) + \theta_2, \end{cases}\right\} t = nT.$$

Considering the first equation of above subsystem, which is independent from the rest of the equations, we get two equilibrium points, namely, $x(t) = 0$ and $x(t) = 1$.

For rest of the system (3.4), using Lemma 3.3, we obtain that $v^*(t) = \frac{\theta_1 exp(-d_2(t-nT))}{1-exp(-d_2 T)}$ and $z^*(t) = \frac{\theta_2 exp(-d_3(t-nT))}{1-exp(-d_3 T)}$ is a positive solution of the subsystem, which is globally asymptotically stable.

4. Stability Analysis

Theorem 4.1 : Let $(x(t), s(t), I(t), v(t), z(t))$ be any solution of the system (2.1), then
(i) The plant-pest eradication periodic solution $(0, 0, 0, v^*(t), z^*(t))$ is unstable.
(ii) The pest eradication periodic solution $(1, 0, 0, v^*(t), z^*(t))$ is locally asymptotically stable iff $T \leq T_{max}$, where $T_{max} = \frac{1}{c}\left(\frac{\alpha\theta_1}{d_2} + \frac{\beta\theta_2}{d_3}\right)$.

Proof : (i) For the local stability of periodic solution $(0, 0, 0, v^*(t), z^*(t))$, we define $x(t) = \phi_1(t), s(t) = \phi_2(t), I(t) = \phi_3(t), v(t) = v^*(t) + \phi_4(t), z(t) = z^*(t) + \phi_5(t)$, where $\phi_i(t), i = 1, 2, ..., 5$ are small amplitude perturbation of the solution respectively, then the system (2.1) can be expanded in the following linearized form:

$$\left\{\begin{aligned} \frac{d\phi_1(t)}{dt} &= \phi_1(t), \\ \frac{d\phi_2(t)}{dt} &= -(\alpha v^*(t) + \beta z^*(t))\phi_2(t), \\ \frac{d\phi_3(t)}{dt} &= \alpha\phi_2(t)v^*(t) - d_1\phi_3(t), \\ \frac{d\phi_4(t)}{dt} &= \mu d_1\phi_3(t) - d_2(\phi_4(t) + v^*(t)), \\ \frac{d\phi_5(t)}{dt} &= \beta\phi_2(t)z^*(t) - d_3(\phi_5(t) + z^*(t)), \end{aligned}\right\} \quad t \neq nT, \tag{4.1}$$

$$\left\{\begin{aligned} \phi_1(t^+) &= \phi_1(t), \\ \phi_2(t^+) &= \phi_2(t), \\ \phi_3(t^+) &= \phi_3(t), \\ \phi_4(t^+) &= \phi_4(t), \\ \phi_5(t^+) &= \phi_5(t). \end{aligned}\right\} t = nT,$$

Let $\Phi(t)$ be the fundamental matrix of (4.1), it must satisfy

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -(\alpha v^*(t) + \beta z^*(t)) & 0 & 0 & 0 \\ 0 & \alpha v^*(t) & -d_1 & 0 & 0 \\ 0 & 0 & \mu d_1 & -d_2 & 0 \\ 0 & \beta z^*(t) & 0 & 0 & -d_3 \end{pmatrix} \Phi(t) = A\Phi(t). \tag{4.2}$$

The linearization of impulsive conditions of (2.1) i.e. equations sixth to tenth of (2.1) becomes

$$\begin{pmatrix} \phi_1(t^+) \\ \phi_2(t^+) \\ \phi_3(t^+) \\ \phi_4(t^+) \\ \phi_5(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1(t) \\ \phi_2(t) \\ \phi_3(t) \\ \phi_4(t) \\ \phi_5(t) \end{pmatrix}.$$

Thus the monodromy matrix of (4.1) is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \Phi(T).$$

From (4.2), we obtain that $\Phi(T) = \Phi(0)exp(\int_0^T Adt)$, where $\Phi(0)$ is identity matrix. Then the eigen values of the monodromy matrix M are

$$\begin{aligned} \lambda_1 &= exp(T) > 1, \\ \lambda_2 &= exp\big(-\int_0^T (\alpha v^*(t) + \beta z^*(t))\big) < 1, \\ \lambda_3 &= exp(-d_1 T) < 1, \\ \lambda_4 &= exp(-d_2 T) < 1, \\ \lambda_5 &= exp(-d_3 T) < 1. \end{aligned}$$

Clearly, since $\mid \lambda_1 \mid > 1$, therefore Floquet theory of impulsive differential equations implies that the plant-pest extinction periodic solution of the system (2.1) is unstable.

(ii) In order to discuss the stability of $(1, 0, 0, v^*(t), z^*(t))$, we proceed as in previous case. Let us define $x(t) = 1 + \phi_1(t),\ s(t) = \phi_2(t),\ I(t) = \phi_3(t),\ v(t) = v^*(t) + \phi_4(t)$ and $z(t) = z^*(t) + \phi_5(t)$, the system (2.1) can be expressed in the following linearized form

$$\left.\begin{cases} \frac{d\phi_1(t)}{dt} = -\phi_1(t) - c\phi_2(t), \\ \frac{d\phi_2(t)}{dt} = -(-c + \alpha v^*(t) + \beta z^*(t))\phi_2(t), \\ \frac{d\phi_3(t)}{dt} = \alpha\phi_2(t)v^*(t) - d_1\phi_3(t), \\ \frac{d\phi_4(t)}{dt} = \mu d_1\phi_3(t) - d_2(\phi_4(t) + v^*(t)), \\ \frac{d\phi_5(t)}{dt} = \beta\phi_2(t)z^*(t) - d_3(\phi_5(t) + z^*(t)), \end{cases}\right\} t \neq nT, \tag{4.3}$$

$$\left.\begin{cases} \phi_1(t^+) = \phi_1(t), \\ \phi_2(t^+) = \phi_2(t), \\ \phi_3(t^+) = \phi_3(t), \\ \phi_4(t^+) = \phi_4(t), \\ \phi_5(t^+) = \phi_5(t). \end{cases}\right\} t = nT.$$

Let $\Phi(t)$ be the fundamental matrix of (4.3), it must satisfy

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} -1 & -c & 0 & 0 & 0 \\ 0 & -(-c+\alpha v^*(t)+\beta z^*(t)) & 0 & 0 & 0 \\ 0 & \alpha v^*(t) & -d_1 & 0 & 0 \\ 0 & 0 & \mu d_1 & -d_2 & 0 \\ 0 & \beta z^*(t) & 0 & 0 & -d_3 \end{pmatrix} \Phi(t) = A\Phi(t). \tag{4.4}$$

The linearization of impulsive conditions of (2.1) i.e. equations sixth to tenth of (2.1) becomes

$$\begin{pmatrix} \phi_1(t^+) \\ \phi_2(t^+) \\ \phi_3(t^+) \\ \phi_4(t^+) \\ \phi_5(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1(t) \\ \phi_2(t) \\ \phi_3(t) \\ \phi_4(t) \\ \phi_5(t) \end{pmatrix}.$$

Thus the monodromy matrix of (4.3) is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \Phi(T).$$

From (4.4), we obtain that $\Phi(T) = \Phi(0)exp(\int_0^T Adt)$, where $\Phi(0)$ is identity matrix. Then the eigen values of the monodromy matrix M are

$$\begin{aligned} \lambda_1 &= exp(-T) < 1, \\ \lambda_2 &= exp\Big(\int_0^T (c - \alpha v^*(t) - \beta z^*(t))dt\Big), \\ \lambda_3 &= exp(-d_1 T) < 1, \\ \lambda_4 &= exp(-d_2 T) < 1, \\ \lambda_5 &= exp(-d_3 T) < 1. \end{aligned}$$

Thus Floquet theory of impulsive differential equations implies that the pest extinction periodic solution of the system (2.1) is locally asymptotically stable if and only if $|\lambda_2| \leq 1$, that is $T \leq T_{max}$. Hence the result.

Theorem 4.2 : Let $(x(t), s(t), I(t), v(t), z(t))$ be any solution of (2.1). Then the pest eradication periodic solution $(1, 0, 0, v^*(t), z^*(t))$ of (2.1) is globally attractive provided $T < T_{max}$.

Proof : If $(x(t), s(t), I(t), v(t), z(t))$ be any solution of (2.1), then the first equation of the system (2.1) can be expressed as

$$\frac{dx(t)}{dt} \leq x(t)(1 - x(t)),$$

which implies that $\lim_{t\to+\infty} \sup\ x(t) = 1$, thus there exists an integer $k_1 > 0$ such that for $t \geq k_1$, we have $x(t) < 1 + \epsilon_0$. From the fourth and eighth equations of the system (2.1), we have

$$\begin{cases} \frac{dv(t)}{dt} \geq -d_2 v(t),\ t \neq nT, \\ v(t^+) = v(t) + \theta_1,\ t = nT. \end{cases} \tag{4.5}$$

Consider the following comparison system

$$\begin{cases} \frac{dw_1(t)}{dt} = -d_2 w_1(t),\ t \neq nT, \\ w_1(t^+) = w_1(t) + \theta_1,\ t = nT. \end{cases} \tag{4.6}$$

Using the Lemma 3.3, we obtain that the system (4.6) has a periodic solution

$$w_1^*(t) = \frac{\theta_1 exp(-d_2(t - nT))}{1 - exp(-d_2 T)},\ nT < t \leq (n+1)T,\ n \in Z_+,$$

which is globally asymptotically stable. In view of Lemma 3.3 and the comparison theorem of the impulsive differential equations we have $v(t) \geq w_1(t)$ and $w_1(t) \to {w_1}^*(t)$ as $t \to \infty$. Then $\exists\, k_2 > k_1,\ t > k_2$ such that

$$v(t) \geq w_1(t) > v^*(t) - \epsilon_0,\ nT < t \leq (n+1)T,\ n > k_2. \tag{4.7}$$

Now from fifth and tenth equations of (2.1), we obtain the following subsystem

$$\begin{cases} \frac{dz(t)}{dt} \geq -d_3 z(t), t \neq nT, \\ z(t^+) = z(t) + \theta_2, t = nT. \end{cases} \tag{4.8}$$

As in the previous manner, we obtain that the system (4.8) has a periodic solution

$$z(t) \geq z^*(t) - \epsilon_0, nT \leq t \leq (n+1)T,\ t \geq k_3. \tag{4.9}$$

Now second equation of system (2.1) can be written as

$$\frac{ds(t)}{dt} \quad \leq \quad s(t)(c(1+\epsilon_0) - \alpha(v^*(t) - \epsilon_0) - \beta(z^*(t) - \epsilon_0)).$$

Integrating the above equation between the pulses, we get

$$s(t) \leq s(nT^+)exp\big(\int_{nT}^{(n+1)T}[c(1+\epsilon_0)-\alpha(v^*(t)-\epsilon_0)-\beta(z^*(t)-\epsilon_0)]dt\big).$$

After the successive pulse, we can obtain the following stroboscopic map

$$\begin{aligned} s((n+1)T^+) &\leq s(nT^+)exp\big(\int_{nT}^{(n+1)T}[c(1+\epsilon_0)-\alpha(v^*(t)-\epsilon_0)-\beta(z^*(t)-\epsilon_0)]dt\big) \\ &= s(nT^+)q, \end{aligned}$$

where $q = exp\big(\int_{nT}^{(n+1)T}[c(1+\epsilon_0)-\alpha(v^*(t)-\epsilon_0)-\beta(z^*(t)-\epsilon_0)]dt\big) < 1$, as $T < T_{max}$. Thus $s(nT^+) \leq s(0^+)q^n$ and so $s(nT^+) \to 0$ as $n \to \infty$. Therefore $s(t) \to 0$ as $n \to \infty$. Therefore there exists an $\epsilon > 0$, small enough, such that $0 < s(t) < \epsilon_1$. From system (2.1) we have $\frac{dI(t)}{dt} \leq \alpha L\epsilon_1 - d_1 I(t)$, so $\lim_{t\to\infty}\inf I(t) \leq \lim_{t\to\infty}\sup I(t) \leq \frac{\alpha L\epsilon_1}{d_1}$.
Again from fourth and ninth equation of system (2.1), we have

$$\begin{cases} \frac{dv(t)}{dt} \leq \frac{\gamma\alpha\epsilon_1}{d_1} - d_2 v(t),\ t \neq nT, \\ \\ v(t^+) = v(t) + \theta_1,\ t = nT. \end{cases} \tag{4.9}$$

Again using the comparison theorem for impulsive equations and Lemma 3.3, for t large enough, we get $v(t) \leq v_1^*(t) + \epsilon_1$, where $v_1^*(t) = \frac{\gamma\alpha\epsilon_1}{d_1 d_2} + \frac{\theta_1 \exp(-d_2(t-nT))}{1-\exp(-d_2 T)}$.
Similarly $z(t) \leq z_1^*(t) + \epsilon_1$, where $z_1^*(t) = \frac{\theta_2 \exp(-(d_3-\beta\epsilon_1)(t-nT))}{1-\exp(-d_3-\beta\epsilon_1)T}$.
If $\epsilon_1 \to 0$, we have $I^*(t) \to 0, v_1^*(t) \to v^*(t), z_1^*(t) \to z^*(t)$. Next when $t \to \infty$, we have $I^*(t) \to 0, v(t) \to v^*(t), z(t) \to z^*(t)$. Hence the proof.

5. Numerical Simulation and Discussion

A pest control model is proposed and analyzed with virus and natural enemies being control variables and are released impulsively. The purpose of this section is to verify the theoretical findings numerically. For this the values of various parameters of the system (2.1) are chosen per week as given in the Table 1, with $x(0^+) = 1, s(0^+) = 1, I(0^+) = 1, v(0^+) = 1$ and $z(0^+) = 1$. As established by theorems 4.1 and 4.2, the threshold limit T_{max} for the impulsive period is calculated as $T_{max} = 10$. For $T = 8 < T_{max} = 10$ it has been verified that the pest free periodic solution $(1, 0, 0, v^*(t), z^*(t))$ is locally and globally asymptotically stable (see Fig.1) and hence theorems 4.1 and 4.2 are verified. Further if we choose $T = 12 > T_{max} = 10$, from figure 2, it has been seen that all the populations co-exist and we can say that the system becomes permanent. Thus numerical simulation verifies that the impulse releasing amount of natural enemies, virus and the impulsive period T all are responsible for extinction or control of pest population.

Parametric values chosen for simulation

Parameter	Description	Value per week
c	Predation rate of plant	0.7
d_1	Natural death rate of infected pest population	0.1
d_2	Natural death rate of virus particles	0.2
d_3	Natural death rate of natural enemy	0.2
μ	Production rate of virus from infected pest	0.5
α	Conversion rate of plant to pest	0.5
β	Conversion rate of pest to natural enemy	0.2
θ_1	Impulsive releasing amount of virus particle	2
θ_2	Impulsive releasing amount of natural enemies	2

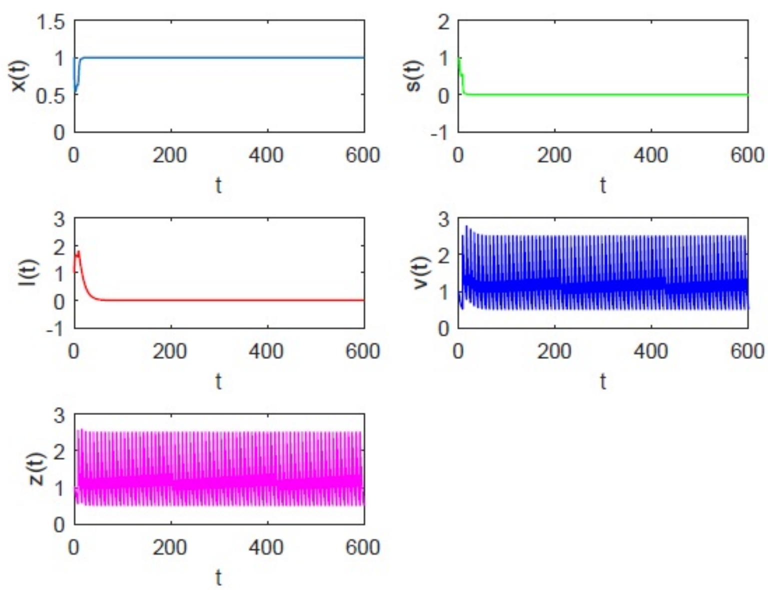

Figure 1: The pest extinction periodic solution $(1, 0, 0, v^*(t), z^*(t))$ for $T = 8 < T_{max} = 10$

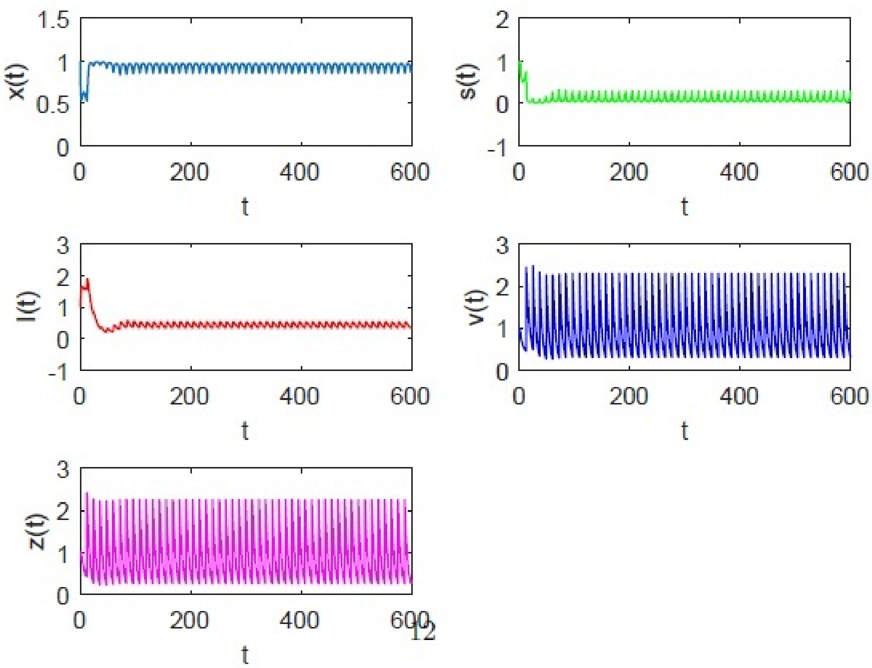

Figure 2: The periodic solution $(x(t), s(t), I(t), v(t), z(t))$ for $T = 12 > T_{max} = 10$

References

[1] Dhar J., Jatav K. S., Mathematical analysis of a delayed stage-structured predator prey model with impulsive diffusion between two predators territories, Ecol. Complex., 16 (2013), 59-67.

[2] Wang D. J., Dong Y. X., Yang S. H., Dynamics of a three-population food chain model with impulsive effect, Information Science and Engineering (ICISE) IEEE, (2009), 3768-3771.

[3] Ding Y., Gao S., Liu Y., Lan Y., A pest management epidemic model with time delay and stage-structure, Appl. Math., 1 (2010), 215-221.

[4] Wang W. D., Chen L. S., A predator prey system with stage structure for predator, Comput. Math. Appl., 33(8) (1997), 83-91.

[5] Lakshmikantham V., Bainov D. D., Simeonov P. S. , Theory of Impulsive Differential Equations, World Scientific, Singapore, (1989).

[6] Jatav K. S., Dhar J., Hybrid approach for pest control with impulsive releasing of natural enemies and chemical pesticides: A plant-pest-natural ememy model, Nonlinear Analysis: Hybrid Systems, 12 (2014), 79-92.

[7] Jatav K. S., Dhar J., Nagar A. K., Mathematical study of stage-structured pest control through impulsively released natural enemies with discrete and distributed delays, Appl. Math. Comput., 238 (2014), 511-526.

[8] Zhang S., Chen L. S., A Holling II functional response food chain model with impulsive perturbations, Chaos Solitons Fractals, 24(5) (2005), 1269-1278.

[9] Kotchen M. J., Incorporating resistance in pesticide management: A dynamic regional approach, in: Regional Sustainability, Springer-Verlag, New York, (1999).

[10] Zhang J., Xiang Z., Study on the mathematical model of virus infection in pest management, Control and Decision Conference, (2014), IEEE, 1616-1619.

[11] Shi R., Jiang X., Chen L., A predator prey model with disease in the prey and two impulses for integrated pest management, Applied Mathematical Modelling, 31, 33(5) (2009), 2248-56.

[12] Baek H., A food chain system with Holling type IV functional response and impulsive perturbations, Computers and Mathematics with Applications, 30; 60(5) (2010), 1152-63.