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GEOMETRIC MEAN LABELING OF SOME CYCLE RELATED GRAPHS

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Abstract

A Graph with p vertices and q edges is said to be Geometric Mean Graph if it is possible to label vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \cdots, q+1$ in such a way that when edge $e = uv$ is labelled with $f^*(uv) = \left\lceil \sqrt{(f(u)f(v))} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Geometric Mean Labeling of G. In this paper, we prove Triple Triangular snake $T(Tn)$, Alternate Double Triangular snake $A(D(Tn))$ and Tadpole $T(n,l)$ are Geometric Mean Graphs.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph $G = (V, E)$ with $p = |V(G)|$ vertices and $q = |E(G)|$ edges. The concept of Geometric Mean Labeling was introduced by S. Somasundaram, R. Ponraj and P. Vidhyarani. We will

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provide a brief summary of definitions necessary to investigate the Geometric Mean Labeling of some cycle related graphs.

Definition 1.1 : A Geometric Mean Labeling of a graph G with p vertices and q edges is an injective function $f : V(G) \to \{1, 2, \cdots, q+1\}$ such that the induced edge labeling $f^* : E(G) \to \{1, 2, \cdots, q\}$ defined as $f^*(uv) = \left\lceil \sqrt{(f(u)f(v))} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ is bijective.

Definition 1.2 : A Triple triangular snake $T(Tn)$ consists of three triangular snakes that have a common path. That is, a Triple triangular snake is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2, \cdots, n-1$ and to a new vertex w_i for $i = 1, 2, \cdots, n-1$ and also to a new vertex z_i for $i = 1, 2, \cdots, n-1$.

Definition 1.3 : An Alternate Double triangular snake $A(D(Tn))$ consists of two alternate triangular snakes that have a common path. That is, an Alternate Double triangular snake is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i for $i = 1, 2, \cdots, n-1$ and also to a new vertex w_i for $i = 1, 2, \cdots, n-1$.

Definition 1.4 : Tadpole $T(n, l)$ is a graph in which Path P_l is attached to any one vertex of cycle C_n.

2. Results

Theorem 2.1 : Triple triangular snake $T(Tn)$ is a Geometric Mean graph.

Proof : Let $G = T(Tn)$ be a Triple triangular snake with p vertices and q edges.
Let, $V(G) = \{v_i, w_i, z_i : 1 \le i \le n-1; u_i : 1 \le i \le n\}$.

$$E(G) = \{u_iv_i, u_iw_i, u_iz_i, u_{i+1}v_i, u_{i+1}w_i, u_{i+1}z_i : 1 \le i \le n-1\}.$$

Then,

$$|V(G)| = p = 4n - 3, \quad |E(G)| = q = 7n - 7.$$

Define a function $f : V(G) \to \{1, \cdots, q+1\}$ as follows:

$$f(u_i) = \begin{cases} 7i - 4 & i = 1 \\ 7i - 7 & 2 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 7i - 6 & i = 1 \\ 7i - 3 & 2 \le i \le n-1 \end{cases}$$

$$f(w_i) = \begin{cases} 7i-2 & i=1 \\ 7i-1 & 2 \le i \le n-1 \end{cases}$$

$$f(z_i) = 7i+1 \quad 1 \le i \le n-1.$$

Define a function $f^* : E(G) \to \{1, 2, \cdots, q\}$ as follows :

$$\begin{aligned} f^*(u_1u_2) &= 7i-3 & i=1 \\ f^*(u_iu_{i+1}) &= 7i-4 & 2 \le i \le n-1 \\ f^*(u_iv_i) &= 7i-6 & 1 \le i \le n-1 \\ f^*(u_1w_1) &= 7i-4 & i=1 \\ f^*(v_1u_2) &= 7i-5 & i=1 \\ f^*(v_iu_{i+1}) &= 7i-2 & 2 \le i \le n-1 \\ f^*(w_iu_{i+1}) &= 7i-1 & 1 \le i \le n-1 \\ f^*(z_iu_{i+1}) &= 7i & 1 \le i \le n-1. \end{aligned}$$

Thus, we get, Set of edge labels

$$= \{7i; 7i-1, 7i-2, 7i-3, 7i-4, 7i-5, 7i-6 : 1 \le i \le n-1\}.$$

For, $1 \le i \le n$, we observe that,

$$|V(G)| = 4n-3 = p$$

$$|E(G)| = 7n-7 = q.$$

Hence f is a Geometric Mean Labeling of G.

Example 2.2 : Geometric Mean Labeling of $T(T_5)$ is shown in Figure 1.

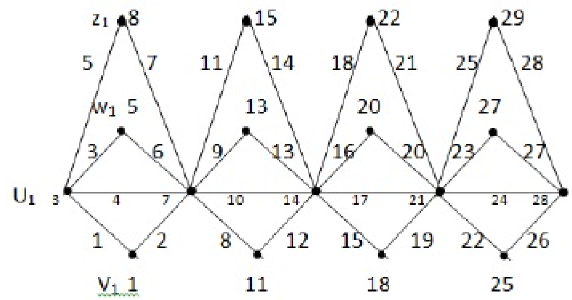

Figure – 1

Theorem 2.3 : Alternate Double Triangular snake $A(D(Tn))$ is Geometric Mean Graph.

Proof : Let $G = A(D(Tn))$ be Alternate Double Triangular snake with p vertices and q edges.

Let $V(G)$ and $E(G)$ denote the set of vertices and edges respectively.

We have the following two cases:

Case 1 : n is even.

Subcase 11 : G starts from u_1 the first vertex of the path $u_1, u_2, \cdots, u_n$. Let,

$$V(G) = \{u_i : 1 \leq i \leq n; v_i, w_i : 1 \leq i \leq \frac{n}{2}\}$$

$$E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1; u_{2i-1}v_i, u_{2i-1}w_i, u_{2i}v_i, u_{2i}w_i : 1 \leq i \leq \frac{n}{2}\}$$

Then,

$$|V(G)| = p = 2n, \quad |E(G)| = q = 3n - 1.$$

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ as follows :

$$\begin{aligned} f(u_{2i}) &= 6i - 1 \quad 1 \leq i \leq \tfrac{n}{2} \\ f(u_{2i-1}) &= 6i - 3 \quad 1 \leq i \leq \tfrac{n}{2} \\ f(w_i) &= 6i \qquad\;\; 1 \leq i \leq \tfrac{n}{2} \\ f(v_i) &= 6i - 5 \quad 1 \leq i \leq \tfrac{n}{2} \end{aligned}$$

Define a function $f^* : E(G) \to \{1, 2, \cdots, q\}$ as follows :

$$\begin{aligned}
f^*(u_{2i-1}v_i) &= 6i-5 \quad 1 \le i \le \tfrac{n}{2} \\
f^*(u_{2i}v_i) &= 6i-4 \quad 1 \le i \le \tfrac{n}{2} \\
f^*(u_{2i-1}w_i) &= 6i-2 \quad 1 \le i \le \tfrac{n}{2} \\
f^*(u_{2i}w_i) &= 6i-1 \quad 1 \le i \le \tfrac{n}{2} \\
f^*(u_{2i-1}u_{2i}) &= 6i-3 \quad 1 \le i \le \tfrac{n}{2} \\
f^*(u_{2i}u_{2i+1}) &= 6i \qquad 1 \le i \le \tfrac{n-2}{2}.
\end{aligned}$$

For, $1 \le i \le n$, we observe that

$$|V(G)| = 2n = p$$

$$|E(G)| = 3n - 1 = q.$$

Thus, f is a Geometric Mean Labeling of G.

Subcase 12 : G starts from u_2 the second vertex of the path $u_1, u_2, \cdots, u_n$. Let, $V(G) = \{u_i : 1 \le i \le n, v_i, w_i : 1 \le i \le \frac{n-2}{2}\}$.

$$E(G) = \{u_iu_{i+1} : 1 \le i \le n-1; u_{2i}v_i, u_{2i}w_i, u_{2i+1}v_i, u_{2i+1}w_i : 1 \le i \le \frac{n-2}{2}\}.$$

Then;

$$|V(G)| = p = 2n - 2$$

$$|E(G)| = q = 3n - 5.$$

Define a functionf : $V(G) \to \{1, 2, \cdots, q+1\}$ as follows :

$$\begin{aligned}
f(u_{2i}) &= 6i-4 \quad 1 \le i \le \tfrac{n}{2} \\
f(u_{2i-1}) &= 6i-5 \quad 1 \le i \le \tfrac{n}{2} \\
f(w_i) &= 6i-3 \quad 1 \le i \le \tfrac{n-2}{2} \\
f(v_i) &= 6i-1 \quad 1 \le i \le \tfrac{n-2}{2}
\end{aligned} .$$

Define a function $f^* : E(G) \to \{1, 2, \cdots, q\}$ as follows :

$$\begin{array}{lcll} f^*(u_{2i+1}v_i) & = & 6i & 1 \le i \le \frac{n-2}{2} \\ f^*(u_{2i}v_i) & = & 6i-3 & 1 \le i \le \frac{n-2}{2} \\ f^*(u_{2i+1}w_i) & = & 6i-1 & 1 \le i \le \frac{n-2}{2} \\ f^*(u_{2i}w_i) & = & 6i-4 & 1 \le i \le \frac{n-2}{2} \\ f^*(u_{2i-1}u_{2i}) & = & 6i-5 & 1 \le i \le \frac{n}{2} \\ f^*(u_{2i}u_{2i+1}) & = & 6i-2 & 1 \le i \le \frac{n-2}{2} \end{array}.$$

For, $1 \le i \le n$, we observe that

$$|V(G)| = 2n - 2 = p$$

$$|E(G)| = 3n - 5 = q.$$

Thus, f is a Geometric Mean Labeling of G.

Case 2 : n is odd.

Subcase 21 : G starts from u_1 the first vertex of the path $u_1, u_2, \cdots, u_n$. Let, $V(G) = \{u_i : 1 \le i \le n : v_i, w_i : 1 \le i \le \frac{n-1}{2}\}$.

$$E(G) = \{u_i u_{i+1} : 1 \le i \le n-1; u_{2i-1}v_i, u_{2i-1}w_i, u_{2i}v_i, u_{2i}w_i : 1 \le i \le \frac{n-1}{2}\}$$

Then,

$$|V(G)| = p = 2n - 1; \quad |E(G)| = q = 3n - 3.$$

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ as follows :

$$\begin{array}{lcll} f(u_{2i}) & = & 6i & 1 \le i \le \frac{n-1}{2} \\ f(u_{2i-1}) & = & 6i-5 & 1 \le i \le \frac{n+1}{2} \\ f(w_i) & = & 6i-4 & 1 \le i \le \frac{n-1}{2} \\ f(v_i) & = & 6i-3 & 1 \le i \le \frac{n-1}{2}. \end{array}$$

Define a function $f^* : E(G) \to \{1, 2, \cdots, q\}$ as follows :

$$\begin{aligned} f^*(u_{2i-1}v_i) &= 6i-4 \quad 1 \le i \le \tfrac{n-1}{2} \\ f^*(u_{2i}v_i) &= 6i-1 \quad 1 \le i \le \tfrac{n-1}{2} \\ f^*(u_{2i-1}w_i) &= 6i-5 \quad 1 \le i \le \tfrac{n-1}{2} \\ f^*(u_{2i}w_i) &= 6i-2 \quad 1 \le i \le \tfrac{n-1}{2} \\ f^*(u_{2i-1}u_{2i}) &= 6i-3 \quad 1 \le i \le \tfrac{n-1}{2} \\ f^*(u_{2i}u_{2i+1}) &= 6i \qquad 1 \le i \le \tfrac{n-1}{2}. \end{aligned}$$

For, $1 \le i \le n$, we observe that

$$|V(G)| = 2n - 1 = p$$

$$|E(G)| = 3n - 3 = q.$$

Thus, f is a Geometric Mean Labeling of G.

Subcase 22 : G starts from u_2 the second vertex of the path $u_1, u_2, \cdots, u_n$. Let, $V(G) = \{u_i : 1 \le i \le n; v_i, w_i : 1 \le i \le \frac{n-1}{2}\}$

$$E(G) = \{u_iu_{i+1} : 1 \le i \le n-1; u_{2i+1}v_i, u_{2i+1}w_i, u_{2i}v_i, u_{2i}w_i : 1 \le i \le \frac{n-1}{2}\}.$$

Then,

$$|V(G)| = p = 2n - 1; \quad |E(G)| = q = 3n - 3.$$

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ as follows :

$$\begin{aligned} f(u_{2i}) &= 6i-4 \quad 1 \le i \le \tfrac{n-1}{2} \\ f(u_{2i-1}) &= 6i-5 \quad 1 \le i \le \tfrac{n+1}{2} \\ f(w_i) &= 6i-3 \quad 1 \le i \le \tfrac{n-1}{2} \\ f(v_i) &= 6i-1 \quad 1 \le i \le \tfrac{n-1}{2}. \end{aligned}$$

Define a function $f^* : E(G) \rightarrow \{1, 2, \cdots, q\}$ as follows :

$$
\begin{aligned}
f^*(u_{2i+1}v_i) &= 6i & 1 \leq i \leq \tfrac{n-1}{2} \\
f^*(u_{2i}v_i) &= 6i - 3 & 1 \leq i \leq \tfrac{n-1}{2} \\
f^*(u_{2i+1}w_i) &= 6i - 1 & 1 \leq i \leq \tfrac{n-1}{2} \\
f^*(u_{2i}w_i) &= 6i - 4 & 1 \leq i \leq \tfrac{n-1}{2} \\
f^*(u_{2i-1}u_{2i}) &= 6i - 5 & 1 \leq i \leq \tfrac{n-1}{2} \\
f^*(u_{2i}u_{2i+1}) &= 6i - 2 & 1 \leq i \leq \tfrac{n-1}{2}.
\end{aligned}
$$

For, $1 \leq i \leq n$, we observe that

$$|V(G)| = 2n - 1 = p$$

$$|E(G)| = 3n - 3 = q.$$

Thus, f is a Geometric Mean Labeling of G.

Example 2.4 : Geometric Mean Labeling of $A(D(T_6))$ is shown in the following figures.

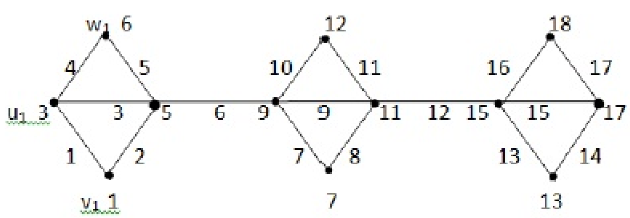

Figure - 2

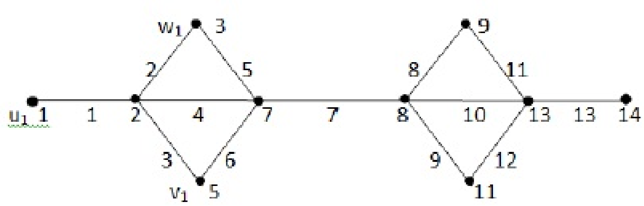

Figure – 3

Example 2.5 : Geometric Mean Labeling of $A(D(T_7))$ is shown in the following figures.

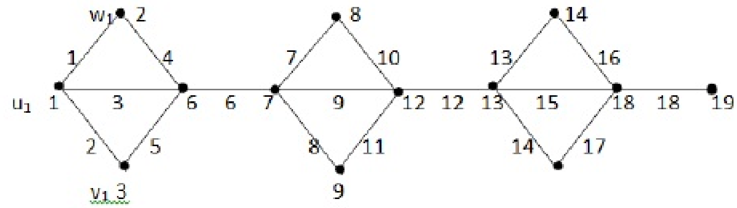

Figure – 4

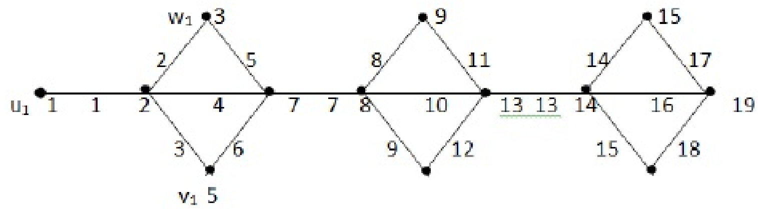

Figure - 5

Theorem 2.6 : Tadpole $T(n,l)$ is a Geometric Mean Graph.

Proof : Let $G = T(n,l)$ be a graph with p vertices and q edges.

Let $v_1, v_2, \cdots, v_n$ be the vertices of cycle C_n and let $u_1, u_2, \cdots, u_l$ be the vertices of Path P_l where $v_n = u_1$.

Let $V(G)$ and $E(G)$ denote the set of vertices and edges respectively.

Let $V(G) = \{v_i : 1 \le i \le n; u_i : 2 \le i \le l\}$.

$$E(G) = \{v_i v_{i+1} : 1 \le i \le n-1; \quad u_i u_{i+1} : 1 \le i \le l-1; v_n v_1\}.$$

Then

$$|V(G)| = p = n + l - p$$

$$|E(G)| = n + l - 1 = q.$$

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ as follows :

$$\begin{aligned} f(v_i) &= i & 1 \le i \le n \\ f(u_i) &= n + i - 1 & 2 \le i \le l \end{aligned}.$$

Let $t = \lfloor \sqrt{n} \rfloor$.

Define $f^* : G \to f_1, 2m \cdots, q\}$ as follows :

$$\begin{aligned} f^*(v_i v_{i+1}) &= i & 1 \leq i \leq t-1 \\ f^*(v_i v_{i+1}) &= i+1 & t \leq i \leq n-1 \\ f^*(v_i v_1) &= t & i = n \\ f^*(u_j u_{j+1}) &= n+j & 1 \leq j \leq l-1. \end{aligned}$$

For, $1 \leq i \leq n; 1 \leq j \leq l-1$, we observe that,

$$|V(G)| = n + l - 1 = p$$

$$|E(G)| = n + l - 1 = q.$$

Hence f is a Geometric Mean Labeling of G.

Example 2.7 : Geometric Mean Labeling of $T(6, 5)$ is shown in figure 6.

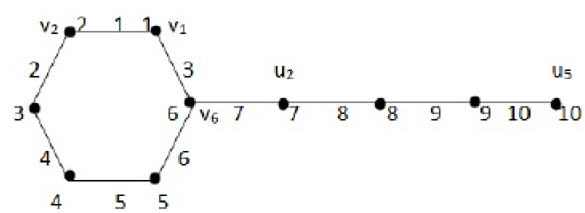

Figure- 6

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