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A STUDY ON BLOTTING NUMBER OF GRAPHS

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Abstract

By the effacement of a vertex v of a graph G, we mean the removal of all edges incident with the vertex v in G. The blotting number of a graph G is the minimum number of vertices to be effaced so that the reduced graph becomes a null graph. In this paper, we discuss the blotting number of certain graph classes.

1. Introduction

For definitions and terminology in graph theory, which are not specifically mentioned in this paper, we refer to [3, 5, 6, 10, 12]. For more graph classes, see [4, 8]. All graphs we mention here are simple, finite, connected and undirected unless mentioned otherwise.

Key Words : *Effacement of vertices, Blotting number.*

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The degree of a vertex v in graph G, denoted $d(v)$ or $d_G(v)$, is the number of edges incident with v. The minimum degree of a graph G, denoted by $\delta(G)$, is the minimum among the degrees of all vertices of G and the maximum degree of G, denoted by $\Delta(G)$, is the maximum degree among the degrees of all vertices of G.

For a given a graph G, a graph modification problem is to find the optimal number of modifications that are needed in order to reduce G into a graph satisfying some property. Different types of graph modification problems can be found in [9].

In edge modification problem, we try to find the smallest number of edge deletions, which turns G into a graph satisfying some property. Graph modification problems are well extensively studied computational problems. These problems are found to have numerous applications in various fields (for example, see [2, 9]. Graphs can be used extensively to model experimental data, where graph modifications correspond to correcting errors in the data - adding an edge means correcting a false negative, while deleting an edge means correcting a false positive (see [9] for the terminology).

In this paper, we study a particular type of edge deletion problems and introduce a new concept namely effacement of vertices of a given graph and the notion of the blotting number of the graph G.

2. Effacement of Vertices and Blotting Number

By the term *effacements* of a vertex v of a graph G, we mean the deletion of all the edges that are incident with v. The *blotting number* of a graph G,denoted by $\mathfrak{b}$, is defined as the minimum number of effacements required in G so that the resulting graph becomes a null graph.

Different edge deletion techniques have proved to be efficient tools for many practical applications. See [1, 2, 9, 14] for more information in this area. The blotting number of a graph can also be viewed as such a parameter with some important applications in practical or real-life situations.

One of the major applications of this parameter is in the war-field. The minimum number of enemy destinations to be destroyed so that their supply and communication networks are completely disconnected can be found out by precisely modelling their locations in terms of graphical networks. The blotting number provides the required number in this case.

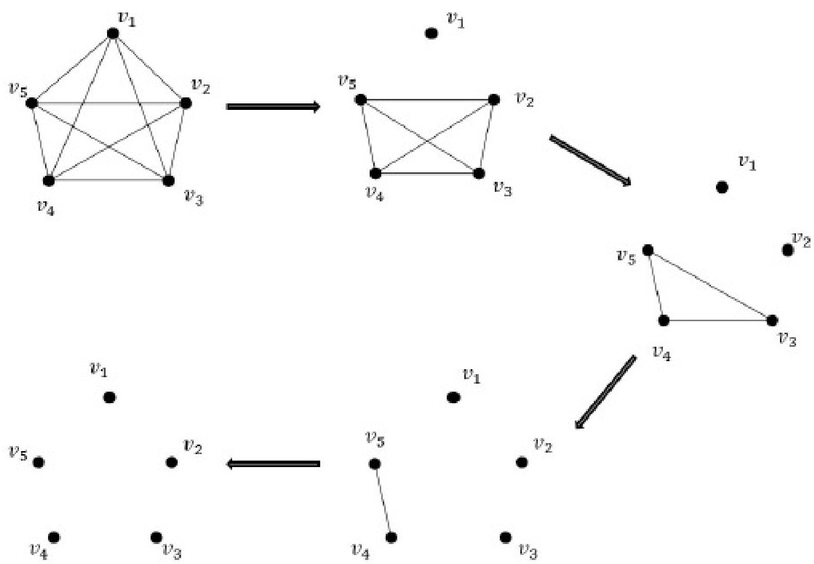

Figure 1

This parameter can effectively be applied by the police personnel to determine the optimal attacks or optimal locations to be attacked for suppressing the activities several anti-social gangs like drug and smuggling groups.

This parameter can also be used in communication problems such as in identifying the minimum number of signal towers to be established so that entire locations are properly connected in a required manner.

The concept of blotting number is also useful for the distribution and service networks to determine the optimum number of customer service points, storage places etc. required for the efficient functioning.

We can identify several real-life or practical applications for this area and this makes our studies interesting and significant.

2.1 New Results

Theorem 2.1 : For a path graph $P_n, n \geq 2$, $\mathfrak{b}(P_n) = \lfloor \frac{n}{2} \rfloor$.

Proof : Consider a path P_n. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n. Clearly $d(v_1) = d(v_n) = 1$ and all other $n-2$ vertices have degree 2. Here we have to consider two cases.

Case - 1: If n is odd, then starting from vertex v_2, effacing the vertices of degree 2 alternatively, that is, effacing the vertices $v_2, v_4, \ldots, v_{n-1}$, all the edges in P_n will be removed. Therefore $\frac{n-1}{2}$ effacements are required so that the reduced graph is a null graph. Therefore, the blotting number of P_n is $\mathfrak{b}(P_n) = \frac{n-1}{2}$.

Case-2: Let n be even. Then, as mentioned in the previous case, effacing the internal vertices alternatively and one pendant vertex (ie, effacing the vertices $v_1, v_3, \dots, v_{n-1}$ or $v_2, v_4, \dots, v_n$), all the edges of P_n will be removed. Therefore, $\frac{n}{2}$ effacements are required for the complete removal of edges of P_n. That is, in this case, the blotting number of P_n is $\frac{n}{2}$.

From the above cases, we can say that $\mathfrak{b}(P_n) = \lfloor \frac{n}{2} \rfloor$. □

Theorem 2.2 : For a cycle $C_n, n \geq 3$, $\mathfrak{b}(C_n) = \lceil \frac{n}{2} \rceil$.

Proof : Consider a cycle C_n. Let $v_1, v_2, \dots, v_n$ be the vertices of C_n. All the n vertices of C_n are of degree 2. Here we have to consider two cases.

Case - 1: If n is odd, then starting from vertex v_1, effacing the alternate vertices , (that is, effacing the vertices $v_1, v_3, \dots, v_n$), all the edges in C_n will be removed. That is, $\frac{n+1}{2}$ effacements are required for reducing C_n to a null graph. Therefore, the blotting number of C_n is $\mathfrak{b}(C_n) = \frac{n+1}{2}$.

Case-2: Let n be even. Then, as mentioned in the previous case, effacing the vertices $v_1, v_3, \dots, v_{n-1}$, all the edges of C_n will be removed. That is, $\frac{n}{2}$ effacements are required for the complete removal of edges of C_n. That is, in this case, the blotting number of C_n is $\frac{n}{2}$.

From the above cases, we can say that $\mathfrak{b}(C_n) = \lceil \frac{n}{2} \rceil$. □

Theorem 2.3: For a bipartite graph $B_{m,n}$, $\mathfrak{b}(B_{m,n}) = \min\{m, n\}$.

Proof : Let G be a bipartite graph with bipartition (X, Y). Effacing all edges incident with the vertices in X or all edges incident with the vertices in Y results in the formation of a null graph. Therefore, the blotting number of G is $\mathfrak{b}(B_{m,n}) = \min\{|X|, |Y|\}$. □

Theorem 2.4 : For a complete graph K_n, $\mathfrak{b}(K_n) = n - 1$.

Proof : Note that every of a complete graph K_n is incident with $n-1$ edges of K_n. Effacing any vertex of K_n, we get the disjoint union of an isolated vertex and a complete graph K_{n-1}. On generalising this, we see that effacing r vertices, where $1 \leq r \leq n$, we get a disjoint union of r isolated vertices and a complete graph K_{n-r}. This process can be repeated until $r = n - 1$. That is, $\mathfrak{b}(K_n) = n - 1$. □

Definition 2.5 [15] : The $n-$sunlet graph is the graph on $2n$ vertices obtained by attaching n pendant edges to a cycle graph C_n and is denoted by L_n.

Proposition 2.1 : For a sunlet graph L_n on $2n$ vertices, $n \geq 3$, $\mathfrak{b}(L_n) = n$.

Proof : For a sunlet graph L_n it can be noted that the graph becomes empty if and only if all vertices of C_n are effaced. Therefore $\mathfrak{b}(L_n) = n$. □

For $n \geq 3$, a *wheel graph*, denoted by W_{n+1}, is the graph $K_1 + C_n$ (see [10]). A wheel graph W_{n+1} has $n + 1$ vertices and $2n$ edges.

Theorem 2.6 : For a wheel graph W_{n+1},

$$\mathfrak{b}(W_{n+1}) = \begin{cases} \frac{n+3}{2} & \text{if } n \text{ is odd} \\ \frac{n+2}{2} & \text{if } n \text{ is even} \end{cases}$$

Proof : Consider a wheel graph W_{n+1}. Let $v_1, v_2, \dots, v_n$ be the vertices of the outer cycle C_n. All the n vertices of C_n are of degree 3. Here we have to consider two cases.

Case - 1: If n is odd, then starting effacing from the central vertex K_1, then continue from vertex v_1, effacing the alternate vertices, (that is, effacing the vertices $v_1, v_3, \dots, v_n$), all the edges of the outer cycle C_n will be removed. That is, $1 + \frac{n+1}{2}$ effacements are required for reducing W_{n+1} to a null graph. Therefore, the blotting number of W_{n+1} is $\mathfrak{b}(W_{n+1}) = 1 + \frac{n+1}{2} = \frac{n+3}{2}$.

Case-2: Let n be even. Then, as mentioned in the previous case, start effacing from the central vertex K_1, after that effacing the vertices $v_1, v_3, \dots, v_{n-1}$, all the edges of the outer cycle C_n will be removed. That is, $1 + \frac{n}{2}$ effacements are required for the complete removal of edges of W_{n+1}. That is, in this case, the blotting number of W_{n+1} is $1 + \frac{n}{2} = \frac{n+2}{2}$.

Hence, For a wheel graph W_{n+1},

$$\mathfrak{b}(W_{n+1}) = \begin{cases} \frac{n+3}{2} & \text{if } n \text{ is odd} \\ \frac{n+2}{2} & \text{if } n \text{ is even} \end{cases}$$

□

A helm graph is a graph obtained from a wheel by attaching one pendant edge to each vertex of the cycle (see [4, 8]). In the following theorem, we discuss the blotting number of helm graphs.

Theorem 2.7 : For a helm graph H_n, $n \geq 3$, $\mathfrak{b}(H_n) = n$.

Proof : For a helm graph H_n it can be noted that the graph becomes empty if and only if all vertices of C_n are effaced. Therefore $\mathfrak{b}(H_n) = n$. □

Another graph which attracts much interest in this context is an $n-$sun or a *trampoline* (see [4, 15]), denoted by S_n, which is defined to be a chordal graph on $2n$ vertices, where

$n \geq 3$, whose vertex set can be partitioned into two sets $U = \{u_1, u_2, u_3, \ldots, u_n\}$ and $W = \{w_1, w_2, w_3, \ldots, w_n\}$ such that

(i) U is an independent set of G,

(ii) u_i is adjacent to w_j if and only if $j = i$ or $j = i + 1 \,(\text{mod n})$.

A *complete sun* is a sun G where the induced subgraph $\langle U \rangle$ is complete. The following theorem discusses the blotting number of a complete sun graph.

Theorem 2.8 : For a complete sun $S_n, n \geq 3, \mathfrak{b}(S_n) = n$.

Proof : For a complete sun graph S_n it can be noted that it is consisting of a central complete graph K_n with an outer ring of n vertices, each of which is joined to both endpoints of the closest outer edge of the central core. The graph becomes empty if and only if all vertices of K_n are effaced. Therefore $\mathfrak{b}(S_n) = n$. □

3. An Algorithm for the Blotting Number of a Graph

The reduction of a given graph into a null graph by effacing its vertices is an iterative process. Hence, to determine the minimum number of effacing required in that process demands a systematic procedure in identifying the vertices to be effaced at each iteration. This makes an algorithmic study possible for finding out the blotting number of graphs. In this section, we propose an algorithm for this purpose.

Blotting Number Algorithm

Let G be the given graph with vertex set V.

Step-1 : Let $G_1 = G$, $S_1 = \emptyset$.
Let $i \to 1$ to n.

Step-2 : Choose a vertex v_i such that $d(v_i)a$ is maximum in G_i.

Step-3 : Let $G_{i+1} = G_i - v_i$ and $S_{i+1} = S_i \cup \{v_i\}$.

Step-4 : If $G_{i+1} \cong K_1$, then go to Step-5. If not, go to Step-3.

Step-5 : $\mathfrak{b}(G) = |S_i| + 1 = i + 1$.

4. Blotting Number of Graph Operations

Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. The union of G_1 and G_2, denoted by $G_1 \cup G_2$, is the graph with vertex set $V_1 \cup V_2$ and the edge set $E_1 \cup E_2$ (see [3]). The union $G_1 \cup G_2$ is said to be edge-disjoint if $E_1 \cap E_2 = \emptyset$ and $G_1 \cup G_2$ is said to be disjoint if $V_1 \cap V_2 = \emptyset$ and $E_1 \cap E_2 = \emptyset$.

If these graphs are disjoint, then it can be noted that $\mathfrak{b}(G_1 \cup G_2) = \mathfrak{b}(G_1) + \mathfrak{b}(G_2)$. If G_1 and G_2 have common elements, we have the following theorem.

Theorem 4.1 : Let G_1 and G_2 be two non-disjoint graphs. Then,

(i) if G_1 and G_2 are edge-disjoint, then $\mathfrak{b}(G_1 \cup G_2) = \mathfrak{b}(G_1) + \mathfrak{b}(G_2)$,

(ii) if G_1 and G_2 are not edge-disjoint, $\mathfrak{b}(G_1 \cup G_2) = \mathfrak{b}(G_1) + \mathfrak{b}(G_2) - \mathfrak{b}(G_1 \cup G_2)$.

If G_1 and G_2 are edge-disjoint, we note that the minimum number of effacements can be done without effacing the common vertex (or vertices) of G_1 and G_2. Moreover, if we efface the common vertex, it cannot be counted as the effacing of both G_1 and G_2 (see Figure 2 for illustration). Therefore, $\mathfrak{b}(G_1 \cup G_2) = \mathfrak{b}(G_1) + \mathfrak{b}(G_2)$.

If G_1 and G_2 have common edges, then one vertex corresponding to each common edge must be effaced in both G_1 and G_2 so that they become null graphs. Hence, effacing the common vertices in this cases count separately for G_1 and G_2 (see Figure 3 for illustration). Therefore, $\mathfrak{b}(G_1 \cup G_2) = \mathfrak{b}(G_1) + \mathfrak{b}(G_2) - \mathfrak{b}(G_1 \cup G_2)$. □

In the following figures, the vertices of graphs to be effaced are highlighted.

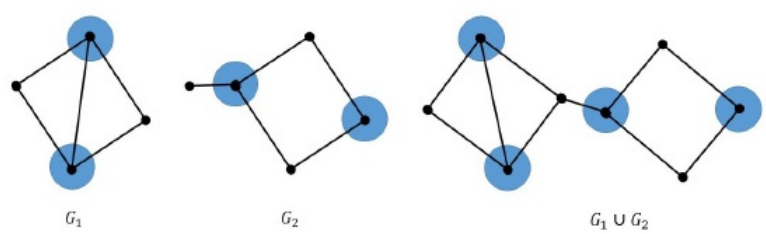

Figure 2

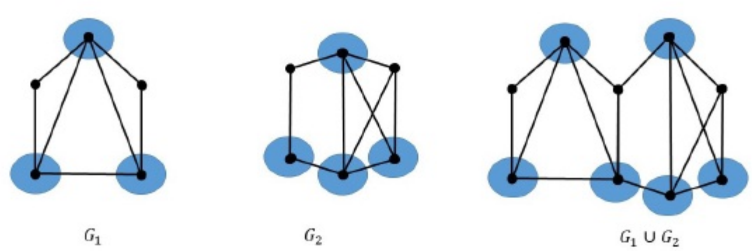

Figure 3

The *join* of two graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, denoted by $G_1 + G_2$, is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup E_{ij}$, where $E_{ij} = \{u_i v_j : u_i \in G_1, v_j \in G_2\}$. In the following theorem, we determine the blotting number of the join of two graphs.

Theorem 4.2 : The blotting number of the join of two graphs is the sum of the order of the graph having fewer vertices and the blotting number of the other. That is $\mathfrak{b}(G_1 + G_2) = \min\{V(G_1) + \mathfrak{b}(G_2), V(G_2) + \mathfrak{b}(G_1)\}$.

Proof : It is to be noted that initially we have to efface all the vertices of any one of the two given graphs. Otherwise the edges between G_1 and G_2 connecting the vertices which are not effaced will remain in the reduced graph.

If we efface all the vertices of G_1, all edges in G_1 as well as all edges connecting G_1 and G_2 will be removed. What remains now is to make G_2 empty. For this, we need to efface $\mathfrak{b}(G_2)$ vertices of G_2. Hence the total number of vertices effaced in this process is $|V(G_1)| + \mathfrak{b}(G_2)$.

If we efface the vertices of the graph G_2 instead of G_1 in the above case, then the total number of vertices effaced will be $|V(G_2)| + \mathfrak{b}(G_1)$. Therefore, $\mathfrak{b}(G_1 + G_2) = \min\{V(G_1) + \mathfrak{b}(G_2), V(G_2) + \mathfrak{b}(G_1)\}$. □

5. Blotting Number of Complements of Graphs

An interesting question that arises when we study the blotting number of various graphs associated with certain graph classes is about the blotting number of the complements of the given graphs. In the following result, we study the blotting number of the complement of paths.

Theorem 5.1 : For a path $P_n, n \geq 3, \mathfrak{b}(\overline{P_n}) = n - 2$.

Proof : Consider $\overline{P_n}$, the complement of a path P_n with n vertices $v_1, v_2, v_3, \ldots, v_n$. We start effacing with the first vertex v_1. If v_1 is effaced, $d(v_2)$ remains unchanged and degree of all the other vertices decreases by 1. At this stage, v_2 and v_n have the same degree. Now, efface the second vertex v_2. If v_2 is effaced the vertices v_3 and v_n have the same degree and the degree of all the other vertices decreases by 1. Next, efface v_3 then, $d(v_4)$ remains unchanged and $d(v_n)$ decreases by 1. At this stage, the vertices v_4 and v_n have the same degree. Proceeding like this, after $n - 3$ effaces, the edge $v_{n-2} - v_n$ remains. By effacing any of these vertices, the reduced graph becomes an empty graph.

Therefore, $\mathfrak{b}(\overline{P_n}) = n - 2$. □

Theorem 5.2 : For a cycle $C_n, n \geq 3, \mathfrak{b}(\overline{C_n}) = n - 2$.

Proof : Consider $\overline{C_n}$, the complement of a cycle C_n with n vertices $v_1, v_2, v_3, \ldots, v_n$. We start effacing with the first vertex v_1. If v_1 is effaced, then $d(v_2)$ and $d(v_n)$ do not change and the degree of all other vertices is reduced by 1. In the reduced graph the vertices v_2 and v_n have the maximum degree. Now, efface the second vertex v_2, then $d(v_3)$ will not be affected and $d(v_n)$ reduces by 1. At this stage, $d(v_3) = d(v_n)$ and v_3 and v_n have the maximum degree in this reduced graph. Now, efface v_3 and then $d(v_4)$ remains unchanged while $d(v_n)$ decreases by 1 such that $d(v_4) = d(v_n)$ and v_4 and v_n have the maximum degree in the reduced graph. Proceeding like this after $n-3$ effaces, the edge $v_{n-2}v_n$ remains. By effacing one of these vertices, the reduced graph becomes an empty graph. Therefore, $\mathfrak{b}(\overline{C_n}) = n - 2$. □

For $n \geq 3$, a *wheel graph*, denoted by W_{n+1}, is the graph $K_1 + C_n$ (see [10]). A wheel graph W_{n+1} has $n+1$ vertices and $2n$ edges.

Theorem 5.3 : For a wheel graph W_{n+1}, $\mathfrak{b}(\overline{W}_{n+1}) = n - 2$.

Proof : Let v be the central vertex and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the outer cycle in W_{n+1} and hence the vertices of $\overline{W}_{n+1}$ also. Since v is adjacent to all other vertices in W_{n+1}, v is an isolated vertex in $\overline{W}_{n+1}$. Hence, we need not efface the vertex v in this case.

All the vertices except the central vertex of the complement $\overline{W}_{n+1}$ of a wheel graph W_{n+1} having degree $n-3$. We start effacing with the first vertex v_1. If v_1 is effaced, then $d(v_2)$ and $d(v_n)$ do not change and the degree of all other vertices is reduced by 1. In the reduced graph the vertices v_2 and v_n have the maximum degree. Now, efface the second vertex v_2, then $d(v_3)$ will not be affected and $d(v_n)$ reduces by 1. At this stage, $d(v_3) = d(v_n)$ and v_3 and v_n have the maximum degree in this reduced graph. Now, efface v_3, then $d(v_4)$ remains unchanged while $d(v_n)$ decreases by 1 such that $d(v_4) = d(v_n)$ and v_4 and v_n have the maximum degree in the reduced graph. Proceeding like this after $n-3$ effaces, the edge $v_{n-2} - v_n$ remains. By effacing any of these vertices, the reduced graph becomes an empty graph. Therefore, $\mathfrak{b}(\overline{W}_{n+1}) = n-2$.
□

Theorem 5.4 : For a complete bipartite graph $K_{m,n}$, $\mathfrak{b}(\overline{K}_{m,n}) = m + n - 2$

Proof : The complement of a complete bipartite graph $K_{m,n}$ is the disjoint union of two complete graphs K_m and K_n. That is, $\overline{K}_{m,n} = K_m \cup K_n$. Also, note that the blotting number of a complete graph K_n is $n-1$ (see Theorem 2.4). Then, by Theorem 4.1, we have $\mathfrak{b}(\overline{K}_{m,n}) = \mathfrak{b}(K_m \cup K_n) = m - 1 + n - 1 = m + n - 2$.

□

6. Blotting Number of Some Other Graph Classes

A *split graph* is a graph in which the vertices can be partitioned into a clique K_r and an independent set I. We denote a split graph by $S_{r,s}$, where $s = |I|$. The blotting numbers of split graphs and their complements are determined in the following theorem.

Theorem 6.1 : Let $S_{r,s}$ be a split graph with clique K_r and independent set I, where $|I| = s$. Then,

$$\mathfrak{b}(S_{r,s}) = \begin{cases} r; & \text{if } N(I) = V(K_r) \\ r-1; & \text{otherwise.} \end{cases}$$

Proof :

(i) Let K_r be the clique and I be the independent set of the split graph $S_{r,s}$. Now, assume that $N(I) = V(K_r)$. In order to make $V(K_r)$ independent, we need to efface $r-1$ vertices of K_r. Efface $r-1$ vertices of K_r in a sequential manner from v_1 to v_{r-1}. At this stage, $V(K_r)$ is independent, but the edges from v_r to some vertices of I remains in the reduced graph. Now, one more effacing - precisely, effacing of the vertex v_r - is required to get the reduced graph empty. Therefore, $\mathfrak{b}(S_{r,s}) = r$.

(ii) Next, assume that $N(I) \neq V(K_r)$. Here, some vertices of K_r are not adjacent to any vertex in I. As mentioned in the above case, we need to efface $r-1$ vertices of K_r, so that $V(K_r)$ becomes independent. Efface the vertices of K_r iteratively in such a way that the vertices in $N(I)$ are effaced first. After $r-1$ effacing, the uneffaced vertex v_r is neither adjacent to any vertex of K_r nor a vertex in I. Therefore, the reduced graph is a null graph. That is, $\mathfrak{b}(S_{r,s}) = r-1$. □

Theorem 6.2 : Let $S_{r,s}$ be a split graph with clique K_r and independent set I, where $|I| = s$. Then,

$$\mathfrak{b}(\overline{S}_{r,s}) = \begin{cases} s & \text{if } N(V(K_r)) = I, \\ s-1 & \text{otherwise.} \end{cases}$$

Proof : The complement of a split graph $S_{r,s}$ is another split graph $S_{s,r}$ with clique K_s and independent set I' where $|I'| = r$. Therefore, the result follows from the above theorem. □

An $n-$*sun* or a *trampoline*, denoted by S_n, is a chordal graph on $2n$ vertices, where $n \geq 3$, whose vertex set can be partitioned into two sets $U = \{u_1, u_2, u_3, \dots, u_n\}$ and $W = \{w_1, w_2, w_3, \dots, w_n\}$ such that U is an independent set of G and u_i is adjacent to w_j if and only if $j = i$ or $j = i + 1 (\mod n)$ (see [4, 15]). A *complete sun* is a sun G where the induced subgraph $\langle U \rangle$ is complete.

The following result discusses the blotting number of complete sun graphs and their complements.

Proposition 6.1 : For a complete sun graph S_n, $\mathfrak{b}(S_n) = \mathfrak{b}(\overline{S_n}) = n$.

Proof : Any complete sun graph S_n is a split graph $S_{n,n}$. Also, note that $N(I) = V(K_n)$. Therefore, by Theorem 6.1, $\mathfrak{b}(S_n) = n$.

The complement of a complete sun graph S_n is also a split graph $S_{n,n}$ with $N(I) = V(K_n)$. Therefore, $\mathfrak{b}(\overline{S_n}) = n$.

□

7. Blotting Number of Arbitrary Graphs

Motivated by the above results on split graphs and sun graphs, a general theorem for the blotting number of an arbitrary graph is described below.

Theorem 7.1 : Let G be a graph on n vertices and $\alpha(G)$ be its independence number. Then, $\mathfrak{b}(G) = n - \alpha(G)$.

Proof : Let I be the maximal independent set of the graph G. Then, $\alpha(G) = |I|$. If we efface a vertex v in $V - I$, then the adjacency of v within $V - I$ as well as its adjacency with the vertices in I will be removed. Efface a sufficient number of vertices in $V - I$ so as to make the induced graph $\langle V - I \rangle$ empty. Now, in the reduced graph both $V - I$ and I are independent sets such that the only remaining edges are between $V - I$ and I. At this stage, we have to efface the remaining vertices in $V - I$ also to make the graph G a null graph. Therefore, the blotting number of G is $\mathfrak{b}(G) = |V - I| = |V| - |I| = n - \alpha(G)$.

□

8. Conclusion

In this paper, we have introduced a new graph parameter called the blotting number of a graph and have discussed the blotting number of certain fundamental graph classes and certain graph operations. We have also proposed an algorithm to find out the blotting number of a given graph.

More problems regarding the blotting number of certain other graph classes, graph operations, graph products and graph powers are still to be settled. Establishing relationships between the blotting number of graphs with certain other parameters like colouring number, matching number, covering number etc. is also an open area for further investigation.

In the application point of view, we have mentioned some well-known applications while introducing the concept of blotting number of a graph. The investigations regarding the applicability of this parameter to various other fields also seem to be promising. All these facts highlight a wide scope for further investigations in this area.

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