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# INVERSE DOMINATION NUMBER OF CIRCULANT GRAPH 

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#### Abstract

A set $D$ of vertices in a graph G , is a dominating set, if every vertices in $V \backslash D$ is adjacent to atleast one vertex in $D$. A dominating set is called a minimum dominating set, if $D$ consist of minimum number of vertices among all the dominating set. If $V \backslash D$ contains a dominating set $D^{\prime}$ of $G$ then $D^{\prime}$ is called an inverse dominating set with respect to D . An inverse dominating set $D^{\prime}$ is called a minimum inverse dominating set, if $D^{\prime}$ consist of minimum number of vertices among all the inverse dominating set. The number of vertices in a minimum inverse dominating set is defined as inverse domination number of a graph $G$ and it is denoted by $\gamma^{-1}(G)$. In this paper we investigate the inverse domination number of a Circulant Graph $G(n, \pm\{1,2\})$.


## 1. Introduction

The study of domination in graph have an immense growth in the recent years. The concept of domination was introduced by S. T. Hedetniemi and P.J.Slater [6]. In the last two decades, domination plays vital role in graph theory. More than 75 variations of domination parameter in [7].

Key Words : Dominating set, Inverse dominating set, Inverse domination number, Circulant Graph.

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C. Berge [2] in 1958 and O. Ore [13] in 1962 started the formal study on theory of dominating sets. Domination has various other applications in real world. It includes social networks, land surveying, communication networks, radio stations, interconnection networks, etc. The concept of inverse domination number was introduced by V. R. Kulli and C. Sigarkanti [12].
A set $D$ of vertices in a graph $G$, is a dominating set, if every vertices in $V \backslash D$ is adjacent to atleast one vertex in $D$. A dominating set is called a minimum dominating set, if $D$ consists of minimum number of vertices among all the dominating set. The number of vertices in a minimum dominating set is defined as the domination number of a graph G and it is denoted by $\gamma(G)$.
If $V \backslash D$ contains a dominating set $D^{\prime}$ of $G$, then $D^{\prime}$ is called an inverse dominating set with respect to $D$. An inverse dominating set $D^{\prime}$ is called a minimum inverse dominating set, if $D^{\prime}$ of minimum number of vertices among all the inverse dominating sets. The number of vertices in a minimum inverse dominating set is defined as inverse domination number of a graph $G$ and is denoted by $\gamma^{-1}(G)$.

A Circulant Graph denoted by $G(n ; \pm\{1,2 \ldots j\}), 1 \leq j \leq\lfloor n / 2\rfloor, n \geq 3$, is a graph with vertex set $V=\{0,1,2 \ldots n-1\}$ and the edge set $E=\{(i, j):|j-i| \equiv s(\bmod n), s \in$ $\{1,2 \ldots j\}\}$.

The Circulant Graph was originally discussed by Elspas and Turner [17]. The Circulant is a natural generalization of the double loop network and was first considered by Wong and Coppersmith [16]. Every Circulant Graph is a vertex trasitive graph and a cayley graph [17]. The properties of Circulant Graph have been studied extensively and surveyed by Bermond et al. [3]. The Circulant Graphs have been studied for the past two decades. Circulant graph have been used in the design of computer and telecommnucation networks due to their optimal fault-tolerence and routing capabilities [4]. It is also used in VLSI designs and distributed computation [3].

## 2. Literature Survey

Selvakumar et al. [18] have investigated the inverse domination semi-total block graph. Cockayne et al. [5] gave the first domination algorithm for trees in 1975 and about the same time, David Johnson constructed the first proof that, the domination problem for arbitrary graphs is NP complete.

Jasintha Quadras et al. [9] have found efficient algorithm for the inverse domination Number of t-layer cycles and have determined exact values of the domination number and inverse domination of this class. Indra Rajasingh et al. [14] have found a minimum connected dominating set for certain circulant graph.

S-(a,d) antimagic labeling of a class of circulant graph has been investigated by Cynthia et al. [11]. Shobana et al. [15] have found an efficient 2- domination number for circulant graph. Indra Rajasingh et al. [8] have investigated the embeddings of Circulant Networks. Cynthia et al. [10] have investigated the local metric dimension of Circulant Graphs.

## 3. Domination Number of Circulant Graph

Theorem 1: The domination number of Circulant Graph $G(n, \pm\{1,2\})$ is $\lceil n / 5\rceil$.


Figure 1: Domination number of $G(19,2)$

Proof Let $G$ be the undirected circulant graph $G(n, \pm 1,2)$. Let $\left\{v_{1}, v_{2}, \ldots v_{n}\right\}$ be the vertices of $G$. Any vertex $v_{i}$ of $G$ is adjacent to a set of four vertices $\left\{v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2}\right\}$ and hence it is clear that a vertex of $G$ can dominate atmost four vertices. We find
the dominating set of $G(n, \pm\{1,2\})$ by considering the following cases of $n$ namely $n \equiv 0,1,2,3,4(\bmod 5)$.

Case 1: $n \equiv 0(\bmod 5)$,
Consider the vertex $v_{3+j}$. It is adjacent to the set of vertices $\left\{v_{1+j}, v_{2+j}, v_{4+j}, v_{5+j}\right\}$ for $j=0,5,10, \ldots, n-5$. Hence we obtain a dominating set $D=\left\{v_{3+j} / j=0,5,10, \ldots, n-\right.$ $5\}$. Therefore $|D|=\lceil n / 5\rceil$.
Case 2: $n \equiv 1(\bmod 5), n \geq 6$
The vertex $v_{3+j}$ is adjacent to the set of vertices $\left\{v_{1+j}, v_{2+j}, v_{4+j}, v_{5+j}\right\}$ for $j=0,5,10, \ldots, n-$
6. Since the vertex $v_{n}$ is dominated by $v_{3+j}$ for any $j$. Hence we choose $v_{3+j}(j \in$ $\{0,5, \ldots n-6\}$ ) to be member of $D$. Since $v_{n}$ is not adjacent to $v_{3+j}$ for all j , we choose the vertex $v_{n}$ for the dominating set. Hence we obtain a dominating set $D=\left\{v_{3+j} / j=\right.$ $0,5,10, \ldots, n-6\} \bigcup\left\{v_{n}\right\}$. Thus $|D|=\lceil n / 5\rceil$.

Case 3: $n \equiv 2(\bmod 5), n \geq 7$
Consider the vertex $v_{3+j}$. It is adjacent to the set of vertices $\left\{v_{1+j}, v_{2+j}, v_{4+j}, v_{5+j}\right\}$ for $j=0,5,10, \ldots, n-7$. Then for the set of remaining vertices $\left\{v_{n-1}, v_{n}\right\}$, we choose the vertex $v_{n-1}$ to be a member of dominating set. Hence we obtain a dominating set $D=\left\{v_{3+j} / j=0,5,10, \ldots, n-7\right\} \bigcup\left\{v_{n-1}\right\}$. Thus $|D|=\lceil n / 5\rceil$.
Case 4: $n \equiv 3(\bmod 5), n \geq 8$
Consider the vertex $v_{3+j}$. It is adjacent to the set of vertices $\left\{v_{1+j}, v_{2+j}, v_{4+j}, v_{5+j}\right\}$ for $j=0,5,10, \ldots, n-8$ and the remaining set of three vertices $\left\{v_{n-2}, v_{n-1}, v_{n}\right\}$ is dominated by the vertex $v_{n-1}$. Hence we obtain a dominating set $D=\left\{v_{3+j} / j=\right.$ $0,5,10, \ldots, n-8\} \bigcup\left\{v_{n-1}\right\}$. Thus $|D|=\lceil n / 5\rceil$.
Case 5: $n \equiv 4(\bmod 5), n \geq 9$
The vertex $v_{3+j}$ is adjacent to the set of vertices $\left\{v_{1+j}, v_{2+j}, v_{4+j}, v_{5+j}\right\}$ for $j=0,5,10$, $\ldots, n-9$ and the remaining set of four vertices $\left\{v_{n-3}, v_{n-2}, v_{n-1}, v_{n}\right\}$ is dominated by the vertex $v_{n-1}$. Hence we obtain a dominating set $D=\left\{v_{3+j} / j=0,5,10, \ldots, n-\right.$ $9\} \bigcup\left\{v_{n-1}\right\}$. Thus $|D|=\lceil n / 5\rceil$.

## 4. Inverse Domination Number of Circulant Graph

Theorem 2: The Inverse domination number of Circulant Graph $G(n, \pm\{1,2\})$ is $\lceil n / 5\rceil$.

Proof: Let $G$ be the undirected circulant graph. Let $\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$ be the vertices of $G$. By the above theorem $\gamma(G)=\lceil n / 5\rceil$, and let the dominating set of $G$ be $D$ as in
theorem 1. We consider the following cases of $n$ to determine the inverse dominating set of $D^{\prime}$.


Figure 2: Inverse domination number of $\mathrm{G}(19,2)$

Case 1: $n \equiv 0(\bmod 5), n \geq 5$
Consider the vertex $v_{4+j}$. It is adjacent to the set of vertices $\left\{v_{2+j}, v_{3+j}, v_{5+j}, v_{6+j}\right\}$ for $j=0,5,10, \ldots, n-5$. Hence we obtain an inverse dominating set $D^{\prime}=\left\{v_{4+j} / j=\right.$ $0,5,10, \ldots, n-5\}$. Therfore $\left|D^{\prime}\right|=\lceil n / 5\rceil$.
Case 2: $n \equiv 1(\bmod 5), n \geq 6$
The vertex $v_{4+j}$ is adjacent to the set of vertices $\left\{v_{2+j}, v_{3+j}, v_{5+j}, v_{6+j}\right\}$ for $j=0,5,10$, $\ldots, n-6$. Hence we choose $v_{4+j}(j \in\{0,5, \ldots n-6\})$ to be member of $D^{\prime}$. Since $v_{1}$ is not adjacent to any $v_{4+j}$ for all j , we choose the vertex $v_{1}$ for the inverse dominating set. Hence we obtain an inverse dominating set $D^{\prime}=\left\{v_{4+j} / j=0,5,10, \ldots, n-6\right\} \bigcup\left\{v_{1}\right\}$. Thus $\left|D^{\prime}\right|=\lceil n / 5\rceil$.
Case 3: $n \equiv 2(\bmod 5), n \geq 7$
Consider the vertex $v_{4+j}$ is adjacent to the set of vertices $\left\{v_{2+j}, v_{3+j}, v_{5+j}, v_{6+j}\right\}$ for $j=0,5,10, \ldots, n-7$. Then for the set of remaining vertices $\left\{v_{n}, v_{1}\right\}$, we choose the
vertex $v_{n}$ to be a member of an inverse dominating set $D^{\prime}$. Hence we obtain an inverse dominating set $D^{\prime}=\left\{v_{4+j} / j=0,5,10, \ldots, n-7\right\} \bigcup\left\{v_{n}\right\}$. Thus $\left|D^{\prime}\right|=\lceil n / 5\rceil$.

Case 4: $n \equiv 3(\bmod 5), n \geq 8$
Consider the vertex $v_{4+j}$. It is adjacent to the set of vertices $\left\{v_{2+j}, v_{3+j}, v_{5+j}, v_{6+j}\right\}$ for $j=0,5,10, \ldots, n-8$ and the remaining set of three vertices $\left\{v_{n-1}, v_{n}, v_{1}\right\}$ is dominated by the vertex $v_{n}$. Hence we obtain an inverse dominating set $D^{\prime}=\left\{v_{4+j} / j=\right.$ $0,5,10, \ldots, n-8\} \bigcup\left\{v_{n}\right\}$. Thus $\left|D^{\prime}\right|=\lceil n / 5\rceil$.
Case 5: $n \equiv 4(\bmod 5), n \geq 9$
Consider the vertex $v_{4+j}$. It is adjacent to the set of vertices $\left\{v_{2+j}, v_{3+j}, v_{5+j}, v_{6+j}\right\}$ for $j=0,5,10, \ldots, n-9$ and the remaining set of four vertices $\left\{v_{n-2}, v_{n-1}, v_{n}, v_{1}\right\}$ is dominated by the vertex $v_{n}$. Hence we obtain an inverse dominating set $D^{\prime}=\left\{v_{4+j} / j=\right.$ $0,5,10, \ldots, n-9\} \bigcup\left\{v_{n}\right\}$. Thus $\left|D^{\prime}\right|=\lceil n / 5\rceil$.

## 5. Remark

The domination number and inverse domination number of a Circulant Graph $G(n, \pm\{1,2\})$ are equal of cardinality $\lceil n / 5\rceil$.

## 6. Conclusion

In this paper we have investigated the inverse domination number of Circulant graph $G(n, \pm\{1,2\})$. Futher we intend to study the inverse domination of Hypercube Networks.

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