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ON DIFFERENCE CORDIAL GRAPHS

J. DEVARAJ[1] AND M. TEFFILIA[2]
[1] Associate Professor (Rtd),
Research Dept. of Mathematics,
NMC College, Marthandam, India
[2] Assistant Professor, Dept. of Mathematics,
WCC College, Nagercoil, India

Abstract

Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1, 2, \cdots, p\}$. For each edge uv, assign the label $|f(u)-f(v)|$. f is called difference cordial labeling if f is 1-1 and $|e_f(0)-e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper we prove that vertex switching of cycle C_n, One point union of t copies of path P_n, P_n^2, shipping graph, $H_n \odot S_3$ are difference cordial graphs.

1. Introduction

For all terminology and notations in Graph theory we follow Harary [4]. Unless mentioned or otherwise a graph in this paper shall mean a simple finite graph without isolated vertices.

Key Words : *Vertex Switching graph, Square graph, Shipping graph.*

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In [7] Ponraj and others defined the notion of difference cordial labeling. Consider the injective function $f : V(G) \rightarrow \{1, 2, \cdots, |V(G)|\}$. This induces the map f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$. This is equal to 1 if the difference is 1 and equal to 0 otherwise. If $|e_f(0) - e_f(1)| \leq 1$ we say f is a difference cordial labeling, where $e_f(i)$ equal to number of edges labeled with i, here $i = 0, 1$. A graph which admits difference cordial labeling is called a difference cordial graph.

Definition 1.2 : A vertex switching of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Theorem 1.3 : A vertex switching of cycle $C_n(VSC_n)$ is difference cordial, for all $n \geq 4$.

Proof : Let G be a (p, q) graph.

VSC_n means vertex switching of cycle C_n. It is obtained by taking a vertex a_1 of C_n removing all the edges incident with a_1 and adding edges joining a_1 to every vertex which are not adjacent to a_1 in C_n.

$$\begin{aligned} V(VSC_n) &= \{a_i/1 \leq i \leq n\} \quad \text{and} \\ E(VSC_n) &= \{(a_i a_{i+1})/2 \leq i \leq n-1\} \cup \{(a_1 a_j)/3 \leq j \leq n-1\}. \end{aligned}$$

Then the graph VSC_n has $2n - 5$ edges and n vertices.

Define $f : V(G) \rightarrow \{1, 2, \cdots, p\}$ as follows

$$\begin{aligned} f(a_1) &= n \\ f(a_i) &= i - 1, 2 \leq i \leq n. \end{aligned}$$

Then the function f induces the function f^* on $E(VSC_n)$ as follows.

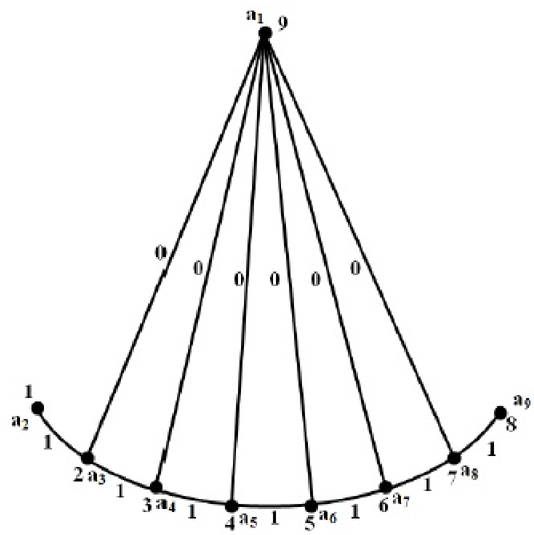
$$\begin{aligned} f^*(a_i a_{i+1}) &= 1, 2 \leq i \leq n-1 \\ f^*(a_1 a_{i+2}) &= 0, 1 \leq i \leq n-3. \end{aligned}$$

Now $e_f(0) = n - 3$ and $e_f(1) = n - 2$

$$\therefore |e_f(0) - e_f(1)| \leq 1.$$

Hence VSC_n is Difference cordial, for all $n \geq 4$.

Illustration 1.4 : Vertex switching of cycle C_9 is shown below.



Definition 1.5 : The square G^2 of a graph G has $V(G^2) = V(G)$ with uv adjacent in G^2, whenever $d(u, v) \leq 2$ in G.

Theorem 1.6 : P_n^2 is difference cordial, for all $n \geq 4$.

Proof : Let G be a (p, q) graph.

$$V(P_n^2) = \{u_i/1 \leq i \leq n\}.$$

$$E(P_n^2) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{u_iu_{i+2}/1 \leq i \leq n-2\}.$$

P_n^2 has n vertices and $2n - 3$ edges.

Define $f : V(P_n^2) \to \{1, 2, \cdots, p\}$ as follows

$$f(u_i) = i, 1 \leq i \leq n.$$

Then the function f induces the function f^* on $E(P_n^2)$ as follows.

$$f^*(u_iu_{i+1}) = 1, 1 \leq i \leq n-1$$

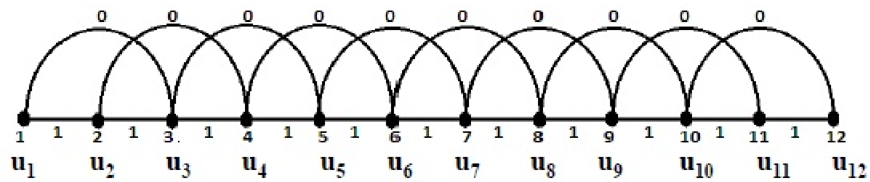
$$f^*(u_iu_{i+2}) = 0, 1 \leq i \leq n-2.$$

Now $e_f(0) = n - 2$, $e_f(1) = n - 1$

$$\therefore \quad |e_f(0) - e_f(1)| \leq 1.$$

Hence P_n^2 is difference cordial, for all $n \geq 4$.

Illustration 1.7 : Difference cordial labeling of P_{12}^2 is shown below.



Theorem 1.8 : $P_n^{(t)}$, one point union of t copies of path P_n, where n is odd and even t is difference cordial.

Proof : Let G be a (p, q) graph.

$$\begin{aligned} E(P_n^{(t)}) &= \{(vv_{i1})/1 \leq i \leq t\} \cup \{v_{ij}v_{i(j+1)}/1 \leq i \leq t, 1 \leq j \leq n-1\} \\ V(P_n^{(t)}) &= \{v_{ij}/1 \leq i \leq t, 1 \leq j \leq n-1\} \cup \{v\}. \end{aligned}$$

Then the graph $P_n^{(t)}$ has $(n-1)t+1$ vertices and $(n-1)t$ edges.
Define $f : V(P_n^{(t)}) \to \{1, 2, \cdots, p\}$ as follows.

$$\begin{aligned} f(v) &= 1 \\ f(v_{1(2i)}) &= 2i, 1 \leq i \leq \frac{n-1}{2} \\ f(v_{1(2i-1)}) &= 2i+1, 1 \leq i \leq \frac{n-1}{2} \\ f(v_{(i+1)(2j)}) &= f(v_{(i)(2j)}) + (n-1), 1 \leq i \leq t-1, 1 \leq j \leq \frac{n-1}{2} \\ f(v_{(i+1)(2j-1)}) &= f(v_{(i)(2j-1)}) + (n-1), 1 \leq i \leq t-1, 1 \leq j \leq \frac{n-1}{2}. \end{aligned}$$

Then the function f induces the function f^* on $E(P_n^{(t)})$ as follows.

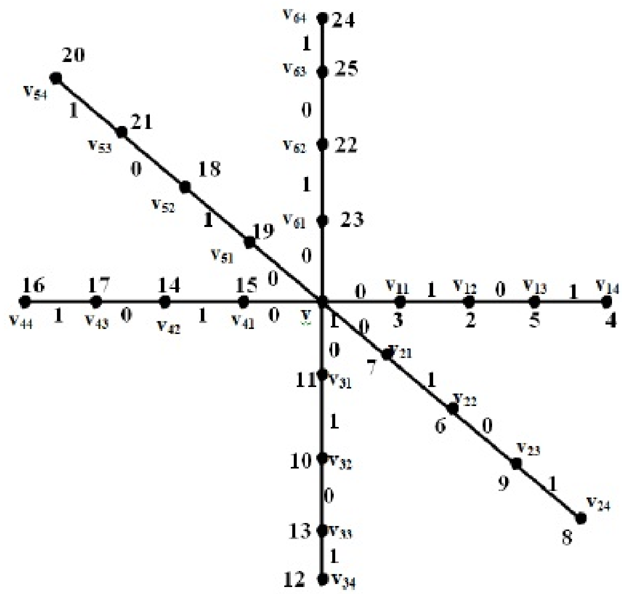
$$\begin{aligned} f^*(vv_{t1}) &= 0, \ \forall \ \ t \\ f^*(v_{ti}v_{t(i+1)}) &= 1, \ \forall \ t, \quad \text{if } i \text{ is odd.} \\ f^*(v_{ti}v_{t(i+1)}) &= 0, \ \ \forall \ \ t, \quad \text{if } i \text{ is even.} \end{aligned}$$

Now $e_f(0) = \left[\frac{n}{2}\right] t, e_f(1) = \left[\frac{n}{2}\right] t$.

$$\therefore \quad |e_f(0) - e_f(1)| \leq 1.$$

Hence $P_n^{(t)}$ is Difference cordial, where n is odd and even t.

Illustration 1.9 : Difference cordial labeling of $P_n^{(6)}$ is shown below.



Definition 1.10 : Let P_n $(n \geq 6)$ and two new vertices u and v on either side of P_n. Join the vertex v to first two vertices from the left and last two vertices of P_n from the right. Join the vertex u to the remaining vertices of P_n in the middle. The resulting graph is called shipping graph and is denoted by SP_n.

Theorem 1.11 : The shipping graph SP_n is Difference cordial.

Proof : Let G be a (p, q) graph.

The vertex set of G is $V(G) = \{u, v, v_1, v_2, \cdots, v_n\}$ and the edge set of G is

$$E(G) = \{vv_1, vv_2, vv_{n-1}, vv_n\} \cup \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{uv_i/3 \leq i \leq n-2\}.$$

Then G has $n+2$ vertices and $2n-1$ edges.

Define $f : V(G) \rightarrow \{1, 2, \cdots, p\}$ as follows.

$$\begin{aligned} f(v_i) &= i, 1 \leq i \leq n \\ f(u) &= n+1 \\ f(v) &= n+2. \end{aligned}$$

Then the function f induces the function f^* on $E(G)$ as follows.

$$f^*(v_i v_{i+1}) = 1, 1 \leq i \leq n-1$$

$$f^*(uv_i) = 0, 3 \leq i \leq n-2$$

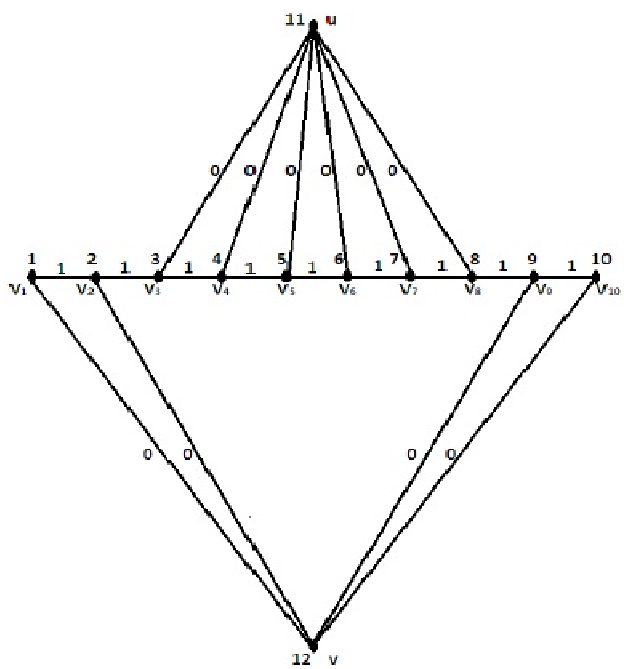
$$\begin{aligned} f^*(vv_1) &= 0 \\ f^*(vv_2) &= 0 \\ f^*(vv_{n-1}) &= 0 \\ f^*(vv_n) &= 0. \end{aligned}$$

Now $e_f(0) = n, e_f(1) = n-1$.

$$\therefore \quad |e_f(0) - e_f(1)| \leq 1.$$

Hence shipping graph is Difference cordial.

Illustration 1.12 : Difference cordial SP_{10} is shown below.



Definition 1.13 : Let H_n - graph of a path P_n is the graph obtained from the two copies of P_n with vertices $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_n$ by joining the vertices $\mathbf{v}_{\frac{n+1}{2}}$ and $\mathbf{u}_{\frac{n+1}{2}}$ by means of an edge if n is odd and the vertices $\mathbf{v}_{\frac{n}{2}+1}$ and $\mathbf{u}_{\frac{n}{2}}$ if n is even.

Definition 1.14 : The graph $H_n \odot S_m$ is obtained from H_n by identifying the centre vertex of the star S_m at each vertex of H_n.

Theorem 1.15 : The graph $H_n \odot S_3$ is difference cordial graph.

Proof : Let G be a (p,q) graph.

The vertex set of G is

$$V(G) = \{u_i/1 \leq i \leq n, v_i/1 \leq i \leq n\} \cup = \{u_{ij}, v_{ij}/1 \leq i \leq n, 1 \leq j \leq 3\}.$$

The edge set of G is

$$E(G) = \{u_i u_{i+1}, v_i v_{i+1}/1 \leq i \leq n-1\} \cup \left\{\mathbf{u}_{\frac{n}{2}+1}\mathbf{v}_{\frac{n}{2}+1} \text{ if } n \text{ is even}\right\}$$

or

$$\left\{\mathbf{u}_{\frac{n+1}{2}}\mathbf{v}_{\frac{n+1}{2}} \text{ if } n \text{ is odd}\right\} \cup \{u_i u_{ij}, v_i v_{ij}/1 \leq i \leq n, 1 \leq j \leq 3\}.$$

Then G has $8n$ vertices and $8n-1$ edges.

Define $f : V(G) \to \{1, 2, \cdots, p\}$ as follows.

$$\begin{aligned}
f(u_i) &= 4i-2, \quad 1 \leq i \leq n \\
f(u_{i1}) &= f(u_i - 1, \quad 1 \leq i \leq n \\
f(u_{i2}) &= f(u_i) + 2, \quad 1 \leq i \leq n \\
f(u_{i3}) &= f(u_i) + 1, \quad 1 \leq i \leq n \\
f(v_1) &= f(u_{n2}) + 2 \\
f(v_i) &= f(v_{i-1}) + 4, \quad 2 \leq i \leq n \\
f(v_{i1}) &= f(v_i) - 1, \quad 1 \leq i \leq n \\
f(v_{i2}) &= f(v_i) + 2, \quad 1 \leq i \leq n \\
f(v_{i3}) &= f(v_i) + 1, \quad 1 \leq i \leq n.
\end{aligned}$$

Then the function f induces the function f^* on $E(G)$ as follows.

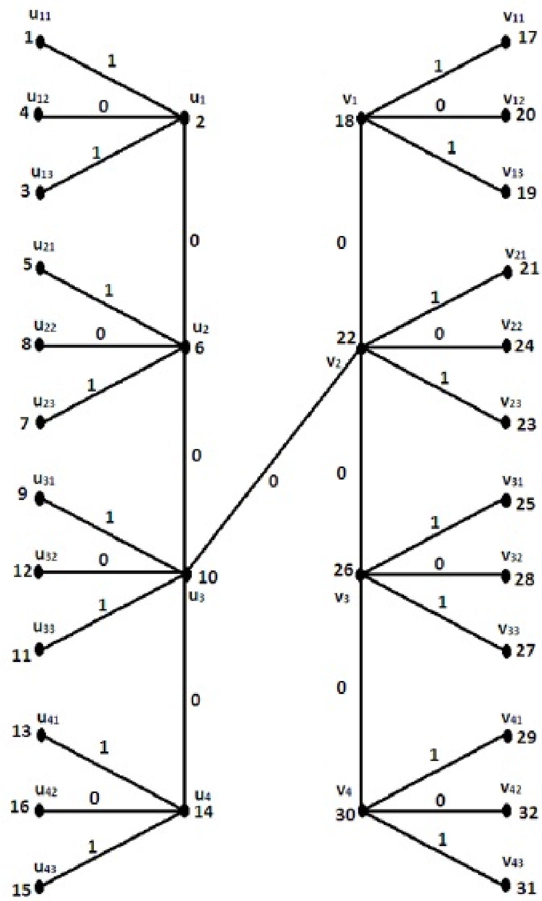
$$\begin{aligned}
f^*(u_iu_{i1}) &= 1, \quad 1 \leq i \leq n \\
f^*(u_iu_{i2}) &= 0, \quad 1 \leq i \leq n \\
f^*(u_iu_{i3}) &= 1, \quad 1 \leq i \leq n \\
f^*(v_iv_{i1}) &= 1, \quad 1 \leq i \leq n \\
f^*(v_iv_{i2}) &= 0, \quad 1 \leq i \leq n \\
f^*(v_iv_{i3}) &= 1, \quad 1 \leq i \leq n \\
f^*(u_iu_{i+1}) &= 0, 1 \leq i \leq n-1 \\
f^*(v_iv_{i+1}) &= 0, \quad 1 \leq i \leq n-1 \\
f^*\left(\mathbf{u}_{\frac{n}{2}+1}\mathbf{v}_{\frac{n}{2}}\right) &= 0, \quad \text{if } n \text{ is even} \\
f^*\left(\mathbf{u}_{\frac{n+1}{2}}\mathbf{v}_{\frac{n+1}{2}}\right) &= 0, \quad \text{if } n \text{ is odd.}
\end{aligned}$$

Now $e_f(0) = 4n-1, e_f(1) = 4n$.

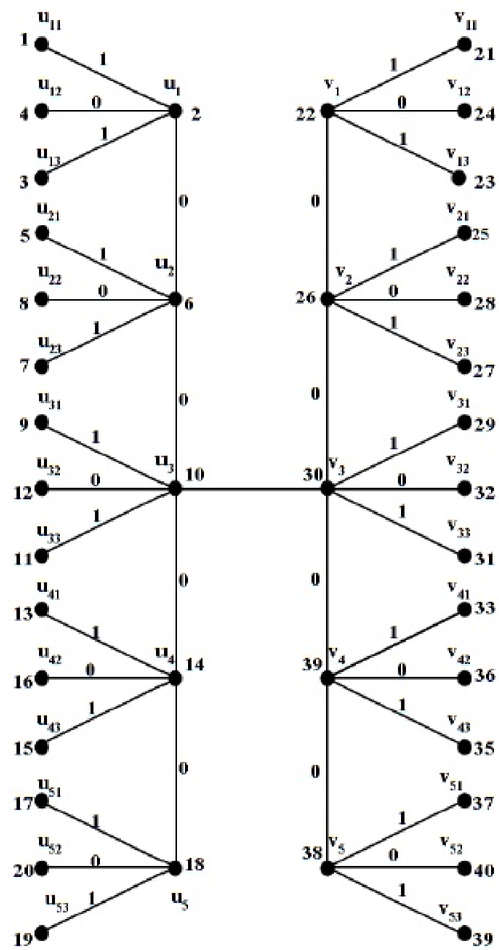
$$\therefore \quad |e_f(0) - e_f(1)| \leq 1.$$

Hence $H_n \odot S_3$ is Difference cordial graph.

Illustration 1.16 : Difference cordial labeling of $H_4 \odot S_3$ is shown below.



Difference cordial labeling of $H_5 \odot S_3$ is shown below.



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