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A COMMON FIXED POINT THEOREM FOR FOUR SELP-MAPS

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Abstract

A common fixed point theorem of Bouhadjera (2010) has been extended to four self-maps satisfying any two of three inequalities under contractive modulus.

1. Introduction

Let X be a nonempty set. A mapping $d : X \times X \to [0, \infty)$ such that $d(x, y) = 0$ if and only if $x = y$ and $d(x, y) = d(y, x)$ for x, y in X is called a symmetric on X. Self-maps f and g on X are said to be occasionally weakly compatible (shortly (owc)), if there is a point $t \in X$ which is coincidence point of f and g, at which f and g commute. Several fixed point theorems in metric space setting have been proved through contraction type conditions involving different types of auxiliary functions. One such auxiliary function is a mapping $\phi : [0, \infty) \to [0, \infty)$, known as a contractive modulus [3], with the choice

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$$\phi(0) = 0 \quad \text{and} \quad (t) < t \quad \text{for} \quad t > 0. \tag{1.1}$$

For instance, $\phi(t) = t/(t+1)$ and $\phi(t) = t^2/(t+1)$ are contractive moduli. Bouhadjera [1] proved the following

Theorem 1.1 : Let X be a set with symmetric d and f, g and r be three self-maps on (X, d) satisfying:

$$\begin{aligned} d(fx, gy) \quad \leq \quad & \max\{\phi(d(rx, ry))\phi(d(rx, fx)), \phi(d(rx, hy))\phi(d(ry, fx)), \\ & \phi(d(rx, ry))\phi(d(ry, gy)), \phi(d(rx, fx))\phi(d(ry, gy)), \\ & \phi(d(rx, gy))\phi(d(ry, fx))\} \quad \text{for all} \quad x, y \in X, \end{aligned} \tag{1.2}$$

where ϕ is a contractive modulus. If the pair (f, r) or (g, r) is owc, then f, g and r have a unique common fixed point.

In this paper we obtain a common fixed point for four self-maps satisfying any two of three inequalities under a contractive modulus.

2. Main Result

We prove the following main result:

Theorem 2.1 : Let X be a nonempty set with symmetric d and f, g, h and r be self-maps on (X, d) satisfying any two of the following three inequalities:

$$\begin{aligned} d^2(fx, gy) \quad \leq \quad & \max\{\phi(d(rx, ry))\phi(d(rx, fx)), \phi(d(rx, ry))\phi(d(ry, fx)), \\ & \phi(d(rx, ry))\phi(d(ry, gy)), \phi(d(rx, fx))\phi(d(ry, gy)), \\ & \phi(d(rx, gy))\phi(d(ry, fx))\} \end{aligned} \tag{2.1}$$

$$\begin{aligned} d^2(gx, hy) \quad \leq \quad & \max\{\phi(d(rx, ry))\phi(d(rx, gx)), \phi(d(rx, ry))\phi(d(ry, gx)), \\ & \phi(d(rx, ry))\phi(d(ry, hy)), \phi(d(rx, gx))\phi(d(ry, hy)), \\ & \phi(d(rx, hy))\phi(d(ry, gx))\} \end{aligned} \tag{2.2}$$

$$\begin{aligned} d^2(hx, fy) \quad \leq \quad & \max\{\phi(d(rx, ry))\phi(d(rx, hx)), \phi(d(rx, ry))\phi(d(ry, hx)), \\ & \phi(d(rx, ry))\phi(d(ry, fy)), \phi(d(rx, hx))\phi(d(ry, fy)), \\ & \phi(d(rx, fy))\phi(d(ry, hx))\} \end{aligned} \tag{2.3}$$

for all $x, y \in X$, where ϕ is a contractive modulus. If any of the pairs $(f, r), (g, r)$ or (h, r) is owc, then f, g, h and r have a unique common fixed point.

Proof : Suppose that (f, r) is owc, then there exists $u \in X$ such that

$$fu = ru \quad \text{and} \quad fru = rfu. \tag{(2.4}$$

Writing $x = y = u$ in (2.1) and using (2.4), we get

$$\begin{aligned} d^2(fu, gu) \quad \leq \quad & \max\{\phi(d(ru, ru))\phi(d(ru, fu)), \phi(d(ru, ru))\phi(d(rufu)), \\ & \phi(d(ru, ru)\phi(d(ru, gu)), \phi(d(ru, fu))\phi(d(ru, gu)), \phi(d(ru, gu))\phi(ru, fu))\} \end{aligned}$$

so that $d^2(fu, gu) \leq 0$ or $fu = gu$. Therefore,

$$fu = gu = ru. \tag{2.5}$$

Now, if $gfu \neq ffu$, then again by (2.1) with $x = y = fu$, we get

$$\begin{aligned} d^2(ffu, gfu) \quad \leq \quad & \max\{\phi(d(rfu, rfu))\phi(d(rfu, ffu)), \phi(d(rfu, rfu))\phi(d(rfu, ffu)), \\ & \phi(d(rfu, rfu))\phi(d(rfu, gfu)), \phi(d(rfu, ffu))\phi(d(rfu, gfu)), \\ & \phi(rfu, gfu))\phi(d(rfu, ffu))\} \end{aligned}$$

or $d^2(ffu, gfu) \leq 0$, in view of (2.4) and (2.5) so that $ffu = gfu$. Therefore,

$$gfu = ffu = fru = rfu. \tag{2.6}$$

Again (2.1) with $x = fu$, $y = u$ and (2.4), (2.5) and (2.6), imply that

$$\begin{aligned} d^2(ffu, gu) \quad \leq \quad & \max\{\phi(d(rfu, ru))\phi(d(rfu, ffu)), \phi(d(rfu, ru))\phi(d(ru, ffu)), \\ & \phi(d(rfu, ru))\phi(d(ru, gu)), \phi(d(rfu, ffu))\phi(d(ru, gu)), \\ & \phi(rfu, gu))\phi(d(ru, ffu))\} \end{aligned}$$

$$d^2(ffu, fu) \leq \max\{\phi(d(ffu, fu))\phi(d(fu, ffu)), \phi(d(ffu, fu))\phi(d(fu, ffu))\}$$

so that $d^2(ffu, fu) \leq (\phi(d(ffu, fu)))^2$ or $ffu = fu)$. Thus $ffu = fru = rfu = gfu = fu$. fu = gu = ru = p is a common fixed point for f, g, and r

That is

$$fp = rp = gp = p. \tag{2.7}$$

On one hand using (2.7) in (2.2) with $x = y = p$, we have

$$\begin{aligned} d^2(gp, hp) \leq\ & \max\{\phi(d(rp, rp))\phi(d(rp, gp)), \phi(d(rp, rp))\phi(d(rp, gp)), \\ & \phi(d(rp, rp))\phi(d(rp, hp)), \phi(d(rp, gp))\phi(d(rp, hp)), \\ & \phi(d(rp, hp))\phi(d(rp, gp))\} \leq 0 \end{aligned}$$

or $gp = hp$. Hence

$$fp = gp = hp = rp = p. \tag{2.8}$$

Thus p is a common fixed point of f, g, h and r.
On the otherhand, taking $x = y = p$ in (2.3) and using (2.7),

$$\begin{aligned} d^2(hp, fp) \leq\ & \max\{\phi(d(rp, rp))\phi(d(rp, hp)), \phi(d(rp, rp))\phi(d(rp, hp)), \\ & \phi(d(rp, rp))\phi(d(rp, fp)), \phi(d(rp, hp))\phi(d(rp, fp)), \\ & \phi(d(rp, fp))\phi(d(rp, hp))\} \leq 0 \end{aligned}$$

so that $hp = fp$. From this (2.8) follows.
Suppose that (2.2) and (2.3) hold good. Now writing $x = y = u$ in (2.3) and using (2.4), we get

$$\begin{aligned} d^2(hu, fu) \leq\ & \max\{\phi(d(ru, ru))\phi(d(ru, hu)), \phi(d(ru, ru))\phi(d(ru, hu)), \\ & \phi(d(ru, ru))\phi(d(ru, fu)), \phi(d(ru, hu))\phi(d(ru, fu)), \\ & \phi(d(ru, fu))\phi(d(ru, hu))\} \leq 0 \end{aligned}$$

or $hu = fu$. Therefore,

$$fu = hu = ru = q. \tag{2.9}$$

If $hfu \neq ffu$, then again by (2.3) with $x = y = fu$, we get

$$\begin{aligned} d^2(hfu, ffu) \leq\ & \max\{\phi(d(rfu, rfu))\phi(d(rfu, hfu)), \phi(d(rfu, rfu))\phi(d(rfu, hfu)), \\ & \phi(d(rfu, rfu))\phi(d(rfu, ffu)), \phi(d(rfu, hfu))\phi(d(rfu, ffu)), \\ & \phi(d(rfu, ffu))\phi(d(rfu, hfu))\} \leq 0, \end{aligned}$$

in view of (2.9) so that $hfu = ffu$. Therefore, $hfu = ffu = fru = rfu$ or

$$hq = fq = rq. \tag{2.10}$$

Now, by (2.2) with $x = y = q$ and using (2.10), we get

$$\begin{aligned} d^2(gq, hq) \leq\ & \max\{\phi(d(rq, rq))\phi(d(rq, gq)), \phi(d(rq, rq))\phi(d(rq, gq)), \\ & \phi(d(rq, rq))\phi(d(rq, hq)), \phi(d(rq, gq))\phi(d(rq, hq)), \\ & \phi(d(rq, hq))\phi(d(rq, gq))\} \leq 0 \end{aligned}$$

or $gq = hq$. Therefore,

$$fq = hq = gq = rq. \tag{2.11}$$

Now, by (2.3) with $x = hu = q, y = u$ and using (2.9), (2.10) and (2.11), we get

$$\begin{aligned} d^2(hhu, fu) =\ & d^2(hq, q) \leq \max\{\phi(d(rhu, ru))\phi(d(rhu, hhu)), \phi(d(rhu, ru))\phi(d(ru, hhu)), \\ & \phi(d(rhu, ru))\phi(d(ru, fu)), \phi(d(rhu, hhu))\phi(d(ru, fhu)), \\ & \phi(d(rhu, fu))\phi(d(ru, hhu))\} \end{aligned}$$

or

$$\begin{aligned} d^2(hq, q) \leq\ & \max\{\phi(d(rq, q))\phi(d(rq, hq)), \phi(d(rq, q))\phi(d(q, hq)), \\ & \phi(d(rq, q))\phi(d(q, q)), \phi(d(rq, hq))\phi(d(q, fq)), \\ & \phi(d(rq, q))\phi(d(q, q))\} \end{aligned}$$

so that

$$d^2(hq, q) \leq (\phi(d(hq, q)))^2$$

giving $hq = q$. Thus $fq = hq = gq = rq = q$. In other words, if (f, r) is owc, and any two of (2.1), (2.2) and (2.3) hold good, then f, g, h and r have a common fixed point. Suppose that (g, r) is owc. Then there exists $u \in X$ such that

$$gu = ruandgru = rgu. \tag{2.12}$$

Writing $x = y = u$ in (2.1) and using (2.12), we get

$$\begin{aligned} d^2(fu, gu) \leq\ & \max\{\phi(d(ru, ru))\phi(d(ru, fu)), \phi(d(ru, ru))\phi(d(ru, fu)), \\ & \phi(d(ru, ru))\phi(d(ru, gu)), \phi(d(ru, fu))\phi(d(ru, gu)), \\ & \phi(d(ru, gu))\phi(d(ru, fu))\} \leq 0 \end{aligned}$$

or $fu = gu$. Therefore $fu = gu = ru$. Now, if $gfu \neq ffu$, then again by (2.1) with $x = y = fu$, we get

$$\begin{aligned} d^2(ffu, gfu) &\leq \max\{\phi(d(rfu, rfu))\phi(d(rfu, ffu)), \phi(d(rfu, rfu))\phi(d(rfu, ffu)), \\ &\quad \phi(d(rfu, rfu))\phi(d(rfu, gfu)), \phi(d(rfu, ffu))\phi(d(rfu, gfu)), \\ &\quad \phi(d(rfu, gfu))\phi(d(rfu, ffu))\} \leq 0, \end{aligned}$$

or $ffu = gfu$ so that

$$ffu = gfu = rgu = rfu. \tag{2.13}$$

Again by (2.1) with $x = fu$, $y = u$ and using (2.13) we get

$$\begin{aligned} d^2(ffu, gu) &\leq \max\{\phi(d(rfu, ru))\phi(d(rfu, ffu)), \phi(d(rfu, ru))\phi(d(ru, ffu)), \\ &\quad \phi(d(rfu, ru))\phi(d(rf, gu)), \phi(d(rfu, ffu))\phi(d(ru, gu))\phi(d(ru, ffu))\}, \\ &\leq (\phi(d(ffu, fu)))^2 \end{aligned}$$

or $ffu = fu$. Therefore,

$$ffu = gfu = rgu = rfu = fu. \tag{2.14}$$

Thus $fu = gu = ru = p$ is a common fixed point for f, g and r. Then by (2.14) we get

$$fp = gp = rp = p. \tag{2.15}$$

On one hand, writing $x = y = p$ in (2.2) and using (2.15), we get

$$\begin{aligned} d^2(gp, hp) &\leq \max\{\phi(d(rp, rp))\phi(d(rp, gp)), \phi(d(rp, rp))\phi(d(rp, gp)), \\ &\quad \phi(d(rp, rp))\phi(d(rp, hp)), \phi(d(rp, gp))\phi(d(rp, hp)), \\ &\quad \phi(d(rp, hp))\phi(d(rp, gp))\} \leq 0 \end{aligned}$$

or $gp = hp$. Therefore,

$$fp = gp = hp = rp = p. \tag{2.16}$$

Thus p is a common fixed point of f, g, h and r.
Now, taking $x = y = p$ in (2.3) and using (2.15), we get

$$\begin{aligned} d^2(hp, fp) &\leq \max\{\phi(d(rp, rp))\phi(d(rp, hp)), \phi(d(rp, rp))\phi(d(rp, hp)), \\ &\quad \phi(d(rp, rp))\phi(d(rp, fp)), \phi(d(rp, hp))\phi(d(rp, fp)), \\ &\quad \phi(d(rp, fp))\phi(d(rp, hp))\} \leq 0 \end{aligned}$$

or $hp = fp$. Hence (2.16) follows.

Suppose that (2.2) and (2.3) hold good. Now writing $x = y = u$ in (2.2) and using (2.12), we get

$$\begin{aligned} d^2(gu, hu) \quad \leq \quad & \max\{\phi(d(ru, ru))\phi(d(ru, gu)), \phi(d(ru, ru))\phi(d(ru, gu)), \\ & \phi(d(ru, ru))\phi(d(ru, hu)), \phi(d(ru, gu))\phi(d(ru, hu)), \\ & \phi(d(ru, hu))\phi(d(ru, gu))\} \leq 0 \end{aligned}$$

or $gu = hu$. Therefore,

$$gu = hu = ru = q. \tag{2.17}$$

Now, if $ggu \neq hgu$, then again by (2.2) with $x = y = gu$, we get

$$\begin{aligned} d^2(ggu, hgu) \quad \leq \quad & \max\{\phi(d(rgu, rgu))\phi(d(rgu, ggu)), \phi(d(rgu, rgu))\phi(d(rgu, ggu)), \\ & \phi(d(rgu, rgu))\phi(d(rgu, hgu)), \phi(d(rgu, ggu))\phi(d(rgu, hgu))\phi(d(rgu, hgu))\}, \\ \leq \quad & \max\{\phi(d(gru, ggu))\phi(d(rgu, hgu)), \phi(d(rgu, hgu))\phi(d(gru, ggu)), \\ & \phi(d(gq, gq))\phi(d(rq, hq)), \phi(d(rq, hq))\phi(d(gq, gq))\} \end{aligned}$$

or $ggu = hgu$. Therefore,

$$hgu = ggu = gru = rgu \tag{2.18}$$

or

$$hq = gq = rq. \qquad ((2.19)$$

By (2.3) with $x = y = q$ and (2.19), we get

$$\begin{aligned} d^2(hq, fq) \quad \leq \quad & \max\{\phi(d(rq, rq))\phi(d(rq, hq)), \phi(d(rq, rq))\phi(d(rq, hq)), \\ & \phi(d(rq, rq))\phi(d(rq, fq)), \phi(d(rq, hq))\phi(d(rq, fq)), \\ & \phi(d(rq, fq))\phi(d(rq, hq))\} \leq 0 \end{aligned}$$

or $hq = fq$ so that

$$gq = rq = hq = fq. \tag{2.20}$$

Now, by (2.2) with $x = u$ and $y = hu$, we have

$$\begin{aligned} d^2(gu, hhu) \quad \leq \quad & \max\{\phi(d(ru, rhu))\phi(d(ru, gu)), \phi(d(ru, rhu))\phi(d(rhu, gu)), \\ & \phi(d(ru, rhu))\phi(d(rhu, hhu)), \phi(d(ru, gu))\phi(d(rhu, hhu)), \\ & \phi(d(ru, hhu))\phi(d(rhu, gu))\}. \end{aligned}$$

On simplification, this gives $d(q, hq) \leq \phi(d(hq, q))$ or $hq = q$. Therefore, $gq = rq = fq = hq = q$.

Thus if (g, r) is owc, and any two of (2.1), (2.2), (2.3) hold good, then f, g, h and r have a common fixed point.

Finally, suppose that (h, r) is owc. Then there exists some $u \in X$ such that

$$hu = ru \quad \text{and} \quad hru = rhu. \tag{2.21}$$

Writing $x = y = u$ in (2.2) and then using (2.21), we get

$$\begin{aligned} d^2(gu, hu) \quad \leq \quad & \max\{\phi(d(ru, ru))\phi(d(ru, gu)), \phi(d(ru, ru))\phi(d(ru, gu)), \\ & \phi(d(ru, ru))\phi(d(ru, hu)), \phi(d(ru, gu))\phi(d(ru, hu)), \\ & \phi(d(ru, hu))\phi(d(ru, gu))\} \leq 0 \end{aligned}$$

or $gu = hu$. Therefore,

$$gu = hu = ru. \tag{2.22}$$

If $ghu \neq hhu$, then again by (2.2) with $x = y = hu$, we get

$$\begin{aligned} d^2(ghu, hhu) \quad \leq \quad & \max\{\phi(d(rhu, rhu))\phi(d(rhu, ghu)), \phi(d(rhu, rhu))\phi(d(rhu, ghu)), \\ & \phi(d(rhu, rhu))\phi(d(rhu, hhu)), \phi(d(rhu, ghu))\phi(d(rhu, hhu)), \\ & \phi(d(rhu, hhu))\phi(d(rhu, ghu))\} \leq 0, \end{aligned}$$

in view of (2.21) and (2.22) or $ghu = hhu$. Therefore,

$$ghu = hhu = hru = rhu. \tag{2.23}$$

Again by (2.2) with $x = hu, y = u$ and using (2.21), (2.22) and (2.23), we get

$$\begin{aligned} d^2(gu, hu) \quad \leq \quad & \max\{\phi(d(rhu, ru))\phi(d(rhu, ghu)), \phi(d(rhu, ru))\phi(d(ru, ghu)), \\ & \phi(d(rhu, ru))\phi(d(ru, hu)), \phi(d(rhu, ghu))\phi(d(ru, hu)), \\ & \phi(d(rhu, gu))\phi(d(ru, ghu))\} \end{aligned}$$

or $ghu = hu$. Thus by (2.23) we get $ghu = hhu = rhu = hu$. In other words, $hu = gu = ru = p$ is a common fixed point for g, h and r. That is

$$hp = gp = rp = p. \tag{2.24}$$

On one hand, using (2.24) in (2.1) wrth $x = y = p$, we get

$$\begin{aligned} d^2(fp, gp) \leq\ & \max\{\phi(d(rp, rp))\phi(d(rp, fp)), \phi(d(rp, rp))\phi(d(rp, fp)), \\ & \phi(d(rp, rp))\phi(d(rp, gp)), \phi(d(rp, fp))\phi(d(rp, gp)), \\ & \phi(d(rp, gp))\phi(d(rp, fp))\} \leq 0 \end{aligned}$$

or $fp = gp$. Hence

$$hp = rp = gp = fp = p. \tag{2.25}$$

Thus p is a common fixed point of h, r, g and f.
On the other hand, taking $x = y = p$ in (2.3) and using (2.24), we get

$$\begin{aligned} d^2(hp, fp) \leq\ & \max\{\phi(d(rp, rp))\phi(d(rp, hp)), \phi(d(rp, rp))\phi(d(rp, hp)), \\ & \phi(d(rp, rp))\phi(d(rp, fp)), \phi(d(rp, hp))\phi(d(rp, fp)), \\ & \phi(d(rp, fp))\phi(d(rp, hp))\} \leq 0 \end{aligned}$$

or $hp = fp$ and (2.25) follows.
Suppose that (2.1) and (2.3) hold good. Then writing $x = y = u$ in (2.3) and using (2.21), we get

$$\begin{aligned} d^2(hu, fu) \leq\ & \max\{\phi(d(ru, ru))\phi(d(ru, hu)), \phi(d(ru, ru))\phi(d(ru, hu)), \\ & \phi(d(ru, ru))\phi(d(ru, fu)), \phi(d(ru, hu))\phi(d(ru, fu)), \\ & \phi(d(ru, fu))\phi(d(ru, hu))\} \leq 0 \end{aligned}$$

or $hu = fu$. Therefore,

$$hu = fu = ru = q. \tag{2.26}$$

If $hhu \neq fhu$, then again by (2.3) with $x = y = hu$, we get

$$\begin{aligned} d^2(hhu, fhu) \leq\ & \max\{\phi(d(rhu, rhu))\phi(d(rhu, hhu)), \phi(d(rhu, rhu))\phi(d(rhu, hhu)), \\ & \phi(d(rhu, rhu))\phi(d(rhu, fhu)), \phi(d(rhu, hhu))\phi(d(rhu, fhu)), \\ & \phi(d(rhu, fhu))\phi(d(rhu, hhu))\} \leq 0, \end{aligned}$$

in view of (2.21) and (2.26), so that $hhu = fhu$. Therefore, $fhu = hhu = hru = rhu$ or

$$fq = hq = rq. \tag{2.27}$$

By (2.1) with $x = y = q$ and (2.27), we get

$$\begin{aligned} d^2(fq, gq) &\leq \max\{\phi(d(rq, rq))\phi(d(rq, fq)), \phi(d(rq, rq))\phi(d(rq, fq)), \\ &\quad \phi(d(rq, rq))\phi(d(rq, gq)), \phi(d(rq, fq))\phi(d(rq, gq)), \\ &\quad \phi(d(rq, gq))\phi(d(rq, fq))\} \leq 0 \end{aligned}$$

or $fq = gq$. Therefore,

$$hq = rq = fq = gq. \tag{2.28}$$

By (2.3) with $x = hu = q$ and $y = u$ and (2.26), (2.27) and (2.28),

$$\begin{aligned} d^2(hhu, fu) &\leq \max\{\phi(d(rhu, ru))\phi(d(rhu, hu)), \phi(d(rhu, ru))\phi(d(ru, hhu)), \\ &\quad \phi(d(rhu, ru))\phi(d(ru, fu)), \phi(d(rhu, hhu))\phi(d(ru, fu)), \\ &\quad \phi(d(rhu, fu))\phi(d(ru, hhu))\} \leq (\phi(d(hq, q)))^2, \end{aligned}$$

or $hq = q$ so that $hq = rq = fq = gq = q$. Thus if (h, r) is owc, and any two of (2.1), (2.2), (2.3) hold good, then f, g, h and r have a common fixed point.

Uniqueness : Suppose that z and w are two common fixed points of f, g, h and r so that $fz = gz = hz = rz = z$ and $fw = gw = hw = rw = w$. Then, by (2.1) with $x = z$ and $y = w$, we obtain

$$\begin{aligned} d^2(fz, gw) &\leq \max\{\phi(d(rz, rw))\phi(d(rz, fz)), \phi(d(rz, rw))\phi(d(rw, fz)), \\ &\quad \phi(d(rz, rw))\phi(d(rw, gw)), \phi(d(rz, fz))\phi(d(rw, gw)), \\ &\quad \phi(d(rz, gw))\phi(d(rw, fz))\} \leq (\phi(d(z, w)))^2 \end{aligned}$$

or $z = w$, proving that the common fixed point is unique. □

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