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ON SOME RECENT $g\mu$-CLOSED FUZZY SETS, FUZZY $g\mu$-CONTINUOUS MAPS AND FUZZY $g\mu$-CLOSED MAPS IN FUZZY TOPOLOGICAL SPACES

SADANAND N. PATIL
Professor & HOD, Department of Mathematics,
College of Engineering & Technology, Srinivas University, Manglore 574146, India.
(Working as Professor & HOD, Department of Mathematics
at Jain Institute of Technology, Davangere 577003)
Email: patilsadu@gmail.com

Abstract

The aim of this paper is to introduce a new class of fuzzy sets, namely $g\mu$ -closed fuzzy sets for fuzzy topological spaces. This new class is properly lies between the class of closed fuzzy sets and the class of g-closed fuzzy sets. We also introduce and study some new spaces, namely fuzzy cT $g\mu$-spaces, fuzzy gT $g\mu$-spaces, as applications of $g\mu$-closed fuzzy sets, the concept of fuzzy $g\mu$ -continuous, fuzzy $g\mu$-irresolute mappings, fuzzy $g\mu$ -closed maps, fuzzy $g\mu$ -open maps and fuzzy $g\mu$ -homeomorphism in fuzzy topological spaces are also introduced, studied and some of there properties are obtained.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh

Key Words and Phrases : *$g\mu$ -closed fuzzy sets, $fg\mu$ -continuous, $fg\mu$ -irresolute, $fg\mu$ -open, $fg\mu$ -closed mappings and $fg\mu$ -homeomorphism.*

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in his classical paper [20] in the year 1965. Subsequently several researchers have worked on topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang [3]. N. Levine [4] introduced the concepts of generalized closed sets in general topology in the year 1970. G. Balasubramanian and P. Sundaram [2] introduced and studied generalized closed fuzzy sets in fuzzy topology. K. K. Azad [1] introduced semi-closed fuzzy sets in the year 1981. H. Maki, T. Fukutake, M. Kojima and H. Harada [5] introduced semi-generalized closed fuzzy sets (briefly fsg - closed) in fuzzy topological space in the year 1998.

In the year 2006, $g\mu$ - closed sets, $g\mu$ - continuous, $g\mu$ - irresolute, $g\mu$ -closed, $g\mu$ -open maps were introduced and studied by M. K. R. S. Veera Kumar [18] for general topology.

Recently author introduced and studied Ψ-closed fuzzy sets[7], pre-semi-closed fuzzy sets[8], g^*-semi-closed fuzzy sets[6], $g\#$- closed fuzzy sets[7], $g\#$-semi- closed fuzzy sets [7], $\#g$ closed fuzzy sets[9], $\#g$-semi-closed fuzzy sets[9], g^*-closed fuzzy sets [7], μ-closed fuzzy sets[10], $\hat{g}$-closed fuzzy sets[7], $^*g$- closed fuzzy sets[11], $^*g$-semi- closed fuzzy sets [12], $\alpha -^* g$-closed fuzzy sets[13], μ-semi-closed fuzzy sets [14], μ-pre-closed fuzzy sets [15], semi-μ-closed fuzzy sets[16] and $g^*\Psi$- closed fuzzy sets[17].

The class of Ψ-closed fuzzy sets is properly placed between the class of semi- closed fuzzy sets and the class of semi-pre-closed fuzzy sets. The class of pre-semi- closed fuzzy sets is placed properly between the class of semi-pre- closed fuzzy sets and the class of gsp-closed fuzzy sets. The class of $g\#$- closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of g*- closed fuzzy sets. The class of $g\#$-semi- closed fuzzy sets is properly placed between the class of semi- closed fuzzy sets and the class of gs- closed fuzzy sets. The class of g^*- closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of g-closed fuzzy sets. The class of $\#g$- closed fuzzy sets is placed properly between the class of g^*- closed fuzzy sets and the class of g-closed fuzzy sets. The class of $\#g$-semi- closed fuzzy sets is placed properly between the class of $\#g$- closed fuzzy sets and the class of gs-closed fuzzy sets. This class also lies between the class of semi- closed fuzzy sets and the class of gs-closed fuzzy sets. The class of μ- closed fuzzy sets is a super class of the classes of $g\#$- closed fuzzy sets, α- closed fuzzy sets and the class of closed fuzzy sets. The class of $\hat{g}$-closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of g- closed fuzzy sets. The class of $^*g$- closed fuzzy sets is placed properly between the class of g^*-closed fuzzy sets and the class of g- closed fuzzy sets. The class of $^*g$-semi- closed fuzzy sets is placed properly between the class of $g\#s$- closed fuzzy sets and the class of gs-closed fuzzy sets. The class of $\alpha -^* g$-closed fuzzy sets is properly placed between the class of α- closed fuzzy sets and the class of αg- closed fuzzy sets. The class of μs- closed fuzzy sets is a super class of the classes of semi- closed fuzzy sets, α- closed fuzzy sets, closed fuzzy sets, μ closed fuzzy sets, $g\#$- closed fuzzy sets and the class of $g\#$s- closed fuzzy sets. The class of μp- closed fuzzy sets is a super class of the classes of pre closed fuzzy sets, α- closed fuzzy sets, closed fuzzy sets, $g\alpha$- closed fuzzy sets, $\hat{g}$-closed fuzzy sets, μ-closed fuzzy sets and the class of $g\#$- closed fuzzy sets. The class of $s\mu$- closed fuzzy sets properly contains the classes of semi-closed fuzzy sets, α- closed fuzzy sets and the class of closed fuzzy sets. The class of $g^*\Psi$-closed fuzzy sets is properly placed between the class of Ψ-closed fuzzy sets and the class of gsp-closed fuzzy sets. The class of $g\mu$-closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of g-closed fuzzy sets.

The aim of this paper is to introduce a new class of fuzzy sets, namely $g\mu$ - closed fuzzy sets for fuzzy topological spaces. This new class is properly lies between the class of closed fuzzy sets and the class of g-closed fuzzy sets. We also introduce and study some new spaces, namely fuzzy cT $g\mu$-spaces, fuzzy gT $g\mu$-spaces, as applications of $g\mu$-closed fuzzy sets, the concept of fuzzy $g\mu$ -continuous, fuzzy μ -irresolute mappings, fuzzy $g\mu$ -closed maps, fuzzy $g\mu$ -open maps and fuzzy $g\mu$ -homeomorphism in fuzzy topological spaces are also introduced, studied and some of their properties are obtained.

2. Literature Survey

Let X, Y and Z be sets. Throughout the present paper $(X,T),(Y,\sigma)$ and (Z,η) and (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitly stated. Let A be a fuzzy set of X. We denote the closure, interior and complement of A by *cl* (A), *int* (A) and $C(A)$ respectively.

Before entering into our work we recall the following definitions, which are due to various authors.

Definition 2.01 : A fuzzy set A in a fts (X,T) is called:

(1) a semi - open fuzzy set, (2) a pre - open fuzzy set, (3) a α- open fuzzy set and 4) a

semi pre - open fuzzy set can be found in [4] and [7].

The semi - closure (resp. pre closure fuzzy, α - closure fuzzy and semipre closure fuzzy) of a fuzzy set A in a fts (X,T) is the intersection of all semi - closed (resp. pre closed fuzzy sets, *al* - closed fuzzy sets and *sp*-closed fuzzy sets) fuzzy sets containing A and is denoted by $scl(A)$ (resp. $pcl(A)$, α $cl(A)$ and $spcl(A)$).

The following definitions are useful in the sequel.

Definition 2.02: A fuzzy set A of a fts (X,T) is called:

(1) a generalized closed (g - closed fuzzy) fuzzy set, (2) a generalized - pre closed (gp -closed fuzzy) fuzzy set, (3) a α-generalized closed (αg-closed fuzzy) fuzzy set, (4) a generalized α-closed ($g\alpha$-closed fuzzy) fuzzy set, (5) a generalized semi - pre-closed (gsp-closed fuzzy) fuzzy set, (6) a generalized semi - closed (gs-closed fuzzy) fuzzy set, (7) a semi - generalized closed (sg-closed fuzzy) fuzzy set, 8) a $\hat{g}$ - closed fuzzy set and (9) a Ψ-closed fuzzy sets can be found in [7].

Definition 2.03 : LetX, Y be two fuzzy topological spaces. A function $f : X \to Y$ is called:

1. fuzzy continuous (f - continuous),
2. fuzzy α-continuous ($f\alpha$ - continuous),
3. fuzzy semi - continuous (fs - continuous) function,
4. fuzzy pre - continuous (fp - continuous) function,
5. fg - continuous function,
6. fgp - continuous function,
7. fgs - continuous function,
8. fsg - continuous function,
9. $fg\alpha$ - continuous function,
10. $f\alpha g$ - continuous function,
11. $fgsp$ - continuous functions,
12. $f\hat{g}$ - continuous function,

13. $f\Psi$-continuous function,

14. $f\Psi$-irresolute,

15. fgc-irresolute and

16. $f\hat{g}$ -irresolute can be found in [7].

Definition 2.04 : Let X, Y be two fuzzy topological spaces. A function $f : X \to Y$ is called:

1. fuzzy $T^{1/2}$ - space can be found in [7].

Definition 2.05 : Let X, Y be two fuzzy topological spaces. A function $f : X \to Y$ is called:

1. fuzzy-homeomorphisms,

2. fuzzy g^*s-homeomorphisms,

3. fuzzy $g\#$-homeomorphisms,

4 fuzzy $g\#s$-homeomorphisms and

5. fuzzy $g\#\alpha$-homeomorphisms can be found in [7].

2.1 . Objective of the Work

Finding new fuzzy closed sets, continuous, irresolute, homomorphism mapping are to be obtained. Also introducing stronger forms, compactness and separations axioms in fuzzy topological spaces.

3. Results and Discussion of $g\mu$-closed Fuzzy Sets in fts

Definition 3.01 : A fuzzy set A of a fuzzy topological space (X, T) is called generalized μ-closed (Briefly: $g\mu$-closed fuzzy set) fuzzy set if $cl(A) \leq U$ whenever $A \leq U$ and U is μ-open fuzzy set in (X, T).

Theorem 3.02 : Every closed fuzzy set is $g\mu$ -closed fuzzy set in any fts X.

Proof : Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 3.03 : Let $X = \{a, b, c\}$ and the fuzzy sets A and B be defined as follows: $A = \{(a, 0.4), (b, 0.4), (c, 0.7)\}$, $B = \{(a, 1), (b, 0.9), (c, 0.8)\}$. Consider the fts (X, T), where $T = \{0, 1, A\}$. Note that the fuzzy subset B is $g\mu$ -closed fuzzy set but not a closed fuzzy set in (X, T).

Theorem 3.04 : Every $g\mu$ -closed fuzzy set is pre-closed fuzzy set, semi-pre-closed fuzzy set, pre-semi-closed fuzzy set, g-closed fuzzy set, αg-closed fuzzy set, $g\alpha$-closed fuzzy set, gs-closed fuzzy set, sg-closed fuzzy set gp-closed fuzzy set and gsp-closed fuzzy set in fts in X.

Proof : Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 3.05 : Let X = $\{a, b, c\}$ and the fuzzy sets A and B defined as follows. $A = \{(a, 0.2), (b, 0.5), (c, 0.3)\}$, $B = \{(a, 0.5), (b, 0.3), (c, 0.3)\}$. Consider $T = \{0, 1, A\}$. Then (X, T) is fts. Here the fuzzy set B is pre-closed fuzzy set, semi-pre-closed fuzzy set, pre-semi-closed fuzzy set, g-closed fuzzy set,αg-closed fuzzy set, $g\alpha$-closed fuzzy set, gs-closed fuzzy set, sg-closed fuzzy set gp-closed fuzzy set and gsp-closed fuzzy set in fts in X but not $g\mu$ -closed fuzzy set in (X, T).

Theorem 3.06 : In a fts X, if a fuzzy set A is both g - open fuzzy set and $g\mu$ -closed fuzzy set, then A is closed fuzzy set.

Theorem 3.07 : If A is $g\mu$ -closed fuzzy set and $cl(A) \wedge (1.cl(A)) = 0$. Then there is no non - zero μ - closed fuzzy set F such that $F \leq cl(A) \wedge (1 - A)$.

Theorem 3.08 : If a fuzzy set A is $g\mu$ -closed fuzzy set in X such that $A \leq B \leq cl(A)$, then B is also a $g\mu$ -closed fuzzy set in X.

Definition 3.09 : A fuzzy set A of a $fts(X, T)$ is called $g\mu$ -open fuzzy ($g\mu$ -open fuzzy set) set if its complement $1 - A$ is $g\mu$ -closed fuzzy set.

Theorem 3.10 : A fuzzy set A of a fts is $g\mu$ -open iff $F \leq int(A)$, whenever F is g-closed fuzzy set and $F \leq A$.

Theorem 3.11 : Every open fuzzy set is a $g\mu$ -open fuzzy set but not conversely.

Proof : Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 3.12 : In the example 3.03, the fuzzy subset $-B = \{(a, 0), (b, 0.1), (c, 0.2)\}$ is $g\mu$ -open fuzzy set but not a open fuzzy set in (X, T).

Theorem 3.13 : In a fts X, Every $g\mu$-open fuzzy set is pre-open fuzzy set, semi-pre-open fuzzy set, pre-semi-open fuzzy set, g-open fuzzy set, αg-open fuzzy set, $g\alpha$-open fuzzy set, gs-open fuzzy set, sg-open fuzzy set, gp-open fuzzy set and gsp-open fuzzy set in fts in X but not conversely.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.14: In the example 3.05, the fuzzy subset $1-B = \{(a, 0.5), (b, 0.7), (c, 0.7)\}$ is pre- open fuzzy set, semi-pre- open fuzzy set, pre-semi- open fuzzy set, g-open fuzzy set, αg-open fuzzy set, $g\alpha$-open fuzzy set, gs-open fuzzy set, sg-open fuzzy set, gp-open fuzzy set and gsp-open fuzzy set but not $g\mu$ -open fuzzy set in (X, T).

Theorem 3.15 : If $int(A) \leq B \leq A$ and if A is $g\mu$ -open fuzzy set, then B is $g\mu$ -open fuzzy set in a fts (X, T).

Theorem 3.16 : If $A \leq B \leq X$ where A is $g\mu$ -open fuzzy relative to B and B is $g\mu$ -open fuzzy relative to X, then A is $g\mu$ -open fuzzy relative to fts X.

Theorem 3.17 : Finite intersection (Union) of $g\mu$-open fuzzy set is a $g\mu$ - open fuzzy set.

Remark 3.18 : The following diagram 1 shows the relationships of $g\mu$ -closed fuzzy sets with some other fuzzy sets.

4. Fuzzy $g\mu$-Closure and Fuzzy $g\mu$-Interior Fuzzy Sets in FTS

In this section we introduce the concepts of fuzzy $g\mu$ -closure $(fg\mu - cl)$ and fuzzy $g\mu$ -interior $(fg\mu - int)$, and investigate their properties.

Definition 4.01 : For any fuzzy set A in any fts is said to be fuzzy $g\mu$ -closure and is denoted by $fg\mu - cl(A)$, defined by $fg\mu - cl(A) = \wedge\{U : U$ is $g\mu$ -closed fuzzy set and $A \leq U\}$.

Definition 4.02 : For any fuzzy set A in any fts is said to be fuzzy $g\mu$ -interior and is denoted by $fg\mu - int(A)$, defined by $fg\mu - int(A) = \wedge\{V : V$ is $g\mu$ -interior fuzzy set and $A \geq V\}$.

Theorem 4.03 : Let A be any fuzzy set in a $fts(X, T)$.

Then $fg\mu - cl(A) = fg\mu - cl(1.A) = 1 - fg\mu - int(A). = fg\mu - int(1 - A) = 1 - fg\mu - cl(A)$.

Proof : Omitted.

Theorem 4.04 : In a fts (X, T), a fuzzy set A is $g\mu$ -closed iff $A = fg\mu - cl(A)$.

Proof : Omitted.

Theorem 4.05: In a fts X the following results hold for fuzzy $g\mu$ -closure.

1. $g\mu - cl(0) = 0$.
2. $g\mu - cl(A)$ is $g\mu$ -closed fuzzy set in X.
3. $g\mu - cl(A) \leq g\mu - cl(B)$ if $A \leq B$.
4. $g\mu - cl(g\mu - cl(A)) = g\mu - cl(A)$.
5. $g\mu - cl(A \vee B) \geq g\mu - cl(A) \vee g\mu - cl(B)$.
6. $g\mu - cl(A \wedge B) \leq g\mu - cl(A) \wedge g\mu - cl(B)$.

Proof : The easy verification is omitted.

Theorem 4.06 : In a fts X, a fuzzy set A is $g\mu$ -open iff $A = fg\mu - int(A)$.

Proof : Omitted.

Theorem 4.07 : In a fts X, the following results hold for $g\mu$ -interior.

1. $g\mu - int(0) = 0$.
2. $g\mu - int(A)$ is $g\mu$ -open fuzzy set in X.
3. $g\mu - int(A) \leq g\mu - int(B)$ if $A \leq B$.
4. $g\mu - int(g\mu - int(A)) = g\mu - int(A)$.
5. $g\mu - int(A \vee B) \geq g\mu - int(A) \vee g\mu - int(B)$.
6. $g\mu - int(A \wedge B) \leq g\mu - int(A) \wedge g\mu - int(B)$.

Proof: Omitted.

NOW WE INTRODUCE THE FOLLOWING.

Definition 4.08 : A fts (X, T) is called a fuzzy $-cTg\mu$ space if every $g\mu$ - closed fuzzy set is a closed fuzzy set.

Theorem 4.09 : A fts (X, T) is called a fuzzy $-cTg\mu$ space iff every $g\mu$ - open fuzzy set is a open fuzzy set in X.

Theorem 4.10 : Every fuzzy $-T^{1/2}$ space is fuzzy $-cTg\mu$- space.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.11 : Let $X = \{a, b, c\}$. The fuzzy sets A, B and C defined as follows: $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 0), (b, 1), (c, 1)\}$ and $C = \{(a, 0), (b, 1), (c, 0)\}$. Then (X, T) is a fts space with $T = \{0, 1, A\}$.

Then (X, T) is fuzzy-$cTg\mu$ space as $g\mu$ closed fuzzy set B is closed in X. But (X, T) is not fuzzy $-T^{1/2}$ space since g-closed fuzzy set C is not closed fuzzy set in X.

Definition 4.12 : A fts (X, T) is called a fuzzy $-gTg\mu$ space if every g - closed fuzzy set is a $g\mu$-closed fuzzy set.

Theorem 4.13 : Every fuzzy $-T^{1/2}$ space is fuzzy $-gTg\mu$ space.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.14 : Let $X = \{a, b, c\}$. The fuzzy sets A, B and C defined as follows: $A = \{(a, 1), (b, 0), (c, 0)\}, B = \{(a, 1), (b, 0), (c, 1)\}$ and $C = \{(a, 1), (b, 1), (c, 0)\}$. Let (X, T) be fts with $T = \{0, 1, A, B\}$. Then X is fuzzy $-gTg\mu$ space but not fuzzy $-T^{1/2}$. space as the fuzzy set C is g-closed fuzzy set and is $g\mu$- closed fuzzy set but not closed fuzzy set.

Theorem 4.15 : A fts X fuzzy $-T^{1/2}$ space iff it is fuzzy $-cTg\mu$ space and fuzzy $-gTg\mu$ space.

Theorem 4.16 : A fts X is called a fuzzy $-gTg\mu$ space iff every open fuzzy set in X is a $g\mu$- open fuzzy set in X.

5. Fuzzy $g\mu$-Continuous and Fuzzy $g\mu$-Irresolute Mappings in FTS

Definition 5.01 : A function $f : X \to Y$ is said to be fuzzy $g\mu$ -continuous ($fg\mu$ -continuous) if the inverse image of every open fuzzy set in Y is $g\mu$ -open fuzzy set in X.

Theorem 5.02 : A function $f : X \to Y$ is $fg\mu$ -continuous if the inverse image of every closed fuzzy set in Y is $g\mu$ -closed fuzzy set in X.

Theorem 5.03 : Every fuzzy continuous function is fuzzy $g\mu$ -continuous.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 5.04 : Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C defined as follows. $A = \{(a, 0), (b, 0.1), (c, 0.2)\}$, $B = \{(a, 0.4), (b, 0.4), (c, 0.7)\}$, $C = \{(a, 1), (b, 0.9), (c, 0.8)\}$. Consider $T = 0, 1, B$ and $\sigma = \{0, 1, A\}$. Then (X, T) and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $fg\mu$ -continuous but not f-continuous. As the fuzzy set C is closed fuzzy set in Y and $f^{-1}(C) = C$ is not closed fuzzy set in X but $g\mu$ -closed fuzzy set in X. Hence f is fuzzy $g\mu$ -continuous.

Remark 5.05 : The following diagram 2 shows the relationships of $fg\mu$-continuous maps with some other fuzzy maps.

Theorem 5.06 : Every fuzzy $g\mu$ -continuous function is fuzzy pre- continuous, fuzzy semi-pre-continuous, fuzzy pre-semi-continuous, fuzzy g-continuous, fuzzy αg-continuous, fuzzy $g\alpha$-continuous, fuzzy gs-continuous, fuzzy sg-continuous, fuzzy gp-continuous and fuzzy gsp-continuous function.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 5.07 : Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 0), (b, 1), (c, 1)\}$, $C = \{(a, 0), (b, 1), (c, 0)\}$. Consider $T = \{0, 1, A, B\}$. Then (X, T) is fts. Define $f : X \to Y$ by $f(a) = b, f(b) = a$ and $f(c) = c$. Then fis fuzzy pre-continuous, fuzzy semi-pre-continuous, fuzzy pre-semi-continuous, fuzzy g-continuous, fuzzy αg-continuous, fuzzy $g\alpha$-continuous, fuzzy gs-continuous, fuzzy sg-continuous, fuzzy gp-continuous and fuzzy gsp-continuous function. But not fuzzy $g\mu$ -continuous as the fuzzy set B is closed fuzzy set in Y and its inverse image $f - 1(B) = B$ which is not $g\mu$ -closed fuzzy set in X which is gsp-closed fuzzy set in X.

We introduce the following definitions.

Definition 5.08 : A function $f : X \to$Y is said to be fuzzy $g\mu$ -irresolute ($fg\mu$-irresolute) if the inverse image of every $g\mu$ -closed fuzzy set in Y is $g\mu$ -closed fuzzy set in X.

Theorem 5.09: A function $f : X \to Y$ is f $g\mu$-irresolute function iff the inverse image of every $g\mu$ -open fuzzy set in Y is $g\mu$ -open fuzzy set in X.

Theorem 5.10 : Every $fg\mu$ -irresolute function is $fg\mu$ -continuous.

Proof : Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 5.11 : Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B, C, D and E be defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 0), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 1), (c, 0)\}$, $D = \{(a, 1), (b, 0), (c, 1)\}$, $E = \{(a, 100), (b, 1), (c, 1)\}$. Consider $T = \{0, 1, A, B, C, D\}$ and $\sigma = \{0, 1, C\}$. Then (X, T) and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is $fg\mu$ -continuous but not $fg\mu$ -irresolute as the fuzzy set E is $g\mu$ -closed fuzzy set in Y but $f^{-1}(E) = C$ is not $g\mu$ -closed fuzzy set in X.

Theorem 5.12 : Let X, Y, Z be three fuzzy topological spaces. Let $f : X \to Y$ and $g : Y \to Z$ be any two fuzzy functions. Then

(1) gof is $g\mu$ -continuous if g is continuous and f is $g\mu$ -continuous.

(2) gof is $g\mu$ -irresolute if g is $g\mu$ - irresolute and f is $g\mu$ - irresolute.

(3) gof is $g\mu$ -continuous if g is $g\mu$ - continuous and f is $g\mu$ - irresolute.

(4) gof is $g\mu$ -continuous if g is $g\mu$ - continuous and f is gc - irresolute.

Proof : Omitted.

6. Fuzzy $g\mu$-Open Maps and Fuzzy $g\mu$-Closed Maps in FTS

This study was further carried out by Sadanand N. Patil [7]. We introduced the following concepts.

Definition 6.01 : A function $f : X \to Y$ is said to be fuzzy $g\mu$-open (briefly $fg\mu$-open) if the image of every open fuzzy set in X is $g\mu$-open fuzzy set in Y.

Definition 6.02 : A function $f : X \to Y$ is said to be fuzzy $g\mu$-closed (briefly $fg\mu$-closed) if the image of every closed fuzzy set in X is $g\mu$-closed fuzzy set in Y.

Theorem 6.03 : Every fuzzy-open map is fuzzy $g\mu$-open map.

Proof : The proof is follows from the definition 6.01.

The converse of the above theorem need not be true as seen from the following example.

Example 6.04 : Let $X = Y = \{a, b, c\}$. Fuzzy sets A, B and C be defined as follows. $A = \{(a, 0), (b, 0.1), (c, 0.2)\}$, $B = \{(a, 0.4), (b, 0.4), (c, 0.7)\}$ and $C = \{(a, 1), (b, 0.9), (c, 0.8)\}$. Consider $T = \{0, 1, A\}$ and $\sigma = \{0, 1, B\}$. Then (X, T) and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is $fg\mu$-open map but not an

f-open map as the fuzzy set A is fuzzy open in X, its image $f(A) = A$ is not open fuzzy set in Y which is $g\mu$-open fuzzy set in Y.

Theorem 6.05 : Every fuzzy $g\mu$-open map is fuzzy pre-open map, fuzzy semi-pre-open map, fuzzy pre-semi- open map, fuzzy g-open map, fuzzy αg-open map, fuzzy $g\alpha$-map, fuzzy gs-open map, fuzzy sg-open map, fuzzy gp-open map and fuzzy gsp-open map.

Proof : The proof is follows from the definition 6.01.

The converse of the above theorem need not be true as seen from the following example.

Example 6.06 : Let $X = Y = \{a, b, c\}$. Fuzzy sets A, B and C be defined as follows. $A = \{(a, 0.2), (b, 0.5), (c, 0.3)\}$, $B = \{(a, 0.8), (b, 0.5), (c, 0.7)\}$ and $C = \{(a, 0.5), (b, 0.3), (c, 0.3)\}$. Consider $T = \{0, 1, A\}$ and $\sigma = \{0, 1, A, B\}$. Then (X, T) and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = b, f(b) = aandf(c) = c$. Then the function f is fuzzy pre-open map, fuzzy semi-pre-open map, fuzzy pre-semi-open map, fuzzy g-open map, fuzzy αg-open map, fuzzy $g\alpha$-map, fuzzy gs-open map, fuzzy sg-open map, fuzzy gp-open map and fuzzy gsp-open map but not an fuzzy $g\mu$-open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $g\mu$-open fuzzy set in Y.

Theorem 6.07 : Every fuzzy-closed map is fuzzy $g\mu$-closed map.

Proof : The proof is follows from the definition 6.02.

The converse of the above theorem need not be true as seen from the following example.

Example 6.08 : In the example 6.04, the function f is fuzzy $g\mu$-closed map but not closed fuzzy map as the fuzzy set C is closed in X and its image $f(C) = C$ is $g\mu$-closed fuzzy set in Y but not closed in Y.

Theorem 6.09 : A function $f : X \to Y$ is $fg\mu$ -closed iff for each fuzzy set S of Y and for each open fuzzy set U such that $f^{-1}(S) \leq U$, there is a $g\mu$-open fuzzy set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Theorem 6.10 : If a map $f : X \to Y$ is fuzzy gc - irresolute and $fg\mu$ -closed and A is $g\mu$ -closed fuzzy set of X, then $f(A)$ is $g\mu$ -closed fuzzy set in Y.

Theorem 6.11 : Let $f : X \to Y$, $g : Y \to Z$ be two maps such that $gof : X \to Z$ is f $g\mu$ -closed map.

(i) If f is f - continuous and surjective, then g is $fg\mu$ -closed map.

(ii) If g is $fg\mu$ -irresolute and injective, then f is $fg\mu$ -closed map.

Definition 6.12 [17] : Let X and Y be two fts. A bijective map $f : X \to Y$ is called fuzzy - homeomorphism (briefly f - homeomorphism) if f and f^{-1} are fuzzy - continuous.

We introduce the following.

Definition 6.13 : A function $f : X \to Y$ is called fuzzy $fg\mu$ -homeomorphism (briefly $fg\mu$ -homeomorphism) if f and f^{-1} are $fg\mu$ -continuous.

Theorem 6.14 : Every f- homeomorphism is fg-homeomorphism, $fg\#$ -homeomorphism, $fg\#s$-homeomorphism, $fg\#\alpha$-homeomorphism, fg^*s-homeomorphism, $f\hat{g}$-homeomorphism and $fg\mu$ -homeomorphism.

Proof : The proof is follows from the definition 6.23.

The converse of the above theorem need not be true as seen from the following example.

Example 6.15 : Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 1), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 0), (c, 1)\}$. Consider $T = \{0, 1, A\}$ and $\sigma = \{0, 1, B\}$. Then (X, T) and (Y, σ) are fts. Define $f : X \to Y$ by $f(a) = a, f(b) = c$ and $f(c) = b$. Then f is fg-homeomorphism, $fg\#$-homeomorphism, $fg\#s$-homeomorphism, $fg\#\alpha$-homeomorphism, fg^*s-homeomorphism, $f\hat{g}$-homeomorphism and $fg\mu$ -homeomorphism but not f - homeomorphism as A is open in $Xf(A) = A$ is not open in $Y.f^{-1} : Y \to X$ is not f-continuous.

Theorem 6.16 : Let $f : X \to Y$ be a bijective function. Then the following are equivalent:

(a) f is $fg\mu$ -homeomorphism.

(b) f is $fg\mu$ -continuous and $fg\mu$ -open maps.

(c) f is $fg\mu$ -continuous and $fg\mu$ -closed maps.

Definition 6.17 : Let X and Y be two fts. A bijective map $f : X \to Y$ is called fuzzy $g\mu - c$ - homeomorphism (briefly $fg\mu - c$ - homeomorphism) if f and f^{-1} are fuzzy $g\mu$ -irresolute.

Theorem 6.18 : Every $fg\mu - c$- homeomorphism is $fg\mu$ -homeomorphism.

Remark 6.19 : The following diagram 3 shows the relationships of $fg\mu$- homeomorphism maps with some other fuzzy homeomorphisms.

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