Reprint ISSN 0973-9424

INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES AND ENGINEERING APPLICATIONS

(IJMSEA)

ASCENT PUBLICATION @ ASCENT PUNE, INDIA

www.ascent-journals.com

International J. of Math. Sci. Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 16 No. I June, 2022, pp. 15-22

NEW STRONGER FORMS OF FUZZY $g\mu$-CONTINUOUS AND ITS RELATED FUZZY FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

SADANAND N. PATIL[1], B. M. PRAVEEN[2] AND AMBANNA S. MADABHAVI[3]
[1] Professor & HOD, Department of Mathematics,
College of Engineering & Technology, Srinivas University, Manglore 574146, India.
(Working as Professor & HOD, Department of Mathematics
at Jain Institute of Technology, Davangere 577003)
[2] Professor & HOD, Department of Chemistry,
College of Engineering & Technology, Srinivas University, Mangalore 574146, India.
[3] Assistant Professor, Department of Computer Applications, ISBR College,
Bengaluru-560100, India
Emails: patilsadu@gmail.com, praveen@yahoo.co.in & asmadabhavi@gmail.com

Abstract

The goal of this Paper is to develop a new class of fuzzy sets for fuzzy topological spaces, called $g\mu$-closed fuzzy sets. This new class is positioned between the closed fuzzy set and the $g\mu$-closed fuzzy set classes. We also introduce and study some new spaces, such as fuzzy $cTg\mu$-spaces and fuzzy $g\mu T$ $g\mu$-spaces, as applications of g-closed fuzzy sets, as well as the concept of fuzzy $g\mu$ -continuous, fuzzy $g\mu$ -irresolute mappings, fuzzy $g\mu$-closed maps, fuzzy $g\mu$ -open maps, and fuzzy $g\mu$ -homeomorphism in fuzzy topological spaces. Stronger variants of fuzzy $g\mu$-

Keywords and phrases : *$g\mu$ -closed fuzzy sets, $fg\mu$ -continuous, $fg\mu$ -irresolute, $fg\mu$ -open, $fg\mu$ -closed mappings and $fg\mu$ -homeomorphism, fuzzy $g\mu$-continuous functions, strongly fuzzy $g\mu$-continuous, perfectly fuzzy $g\mu$-continuous, and Absolutely fuzzy $g\mu$-continuous.*
2000 AMS Subject Classification : 54A40.

continuous functions have now been introduced and researched, including strongly fuzzy $g\mu$-continuous, perfectly fuzzy $g\mu$-continuous, and absolutely fuzzy $g\mu$-continuous.

1. Introduction

L. A. Zadeh [10] initially proposed the concept of fuzzy sets and fuzzy set operations in his classic paper [10] in 1965. Following that, other academics worked on fuzzy set topology and established the notion of fuzzy topological spaces. The concept of fuzzy subsets is naturally important in the study of fuzzy topology pioneered by C. L. Chang [3]. In the year 1970, N. Levine [4] presented the concepts of generalised closed sets in general topology. In fuzzy topology, G. Balasubramanian and P. Sundaram [2] proposed and investigated generalised closed fuzzy sets. In 1981, K. K .Azad [1] presented semi-closed fuzzy sets. In the year 1998, H. Maki, T. Fukutake, M. Kojima, and H. Harada [5] proposed semi-generalized closed fuzzy sets (briefly fsg - closed) in fuzzy topological space. M. K. R. S. Veera Kumar [8] introduced and explored g - closed sets, g - continuous, g - irresolute, g -closed, g -open maps for general topology in 2006. Stronger variants of fuzzy g-continuous functions have now been introduced and researched, including strongly fuzzy g-continuous, perfectly fuzzy g-continuous, and absolutely fuzzy g-continuous.

2 Stronger Forms of Fuzzy $g\mu$-Continuous Functions

Definition 2.1 : If the inverse image of every $g\mu$ - open fuzzy set in Y is open fuzzy set in X, a function $f : X \to Y$ is called strongly fuzzy $g\mu$ - continuous (briefly strongly $fg\mu$-continuous).

Theorem 2.2 : If the inverse image of every $g\mu$-closed fuzzy set in Y is a closed fuzzy set in X, a function $f : X \to Y$ is strongly $fg\mu$-continuous.

Proof : Assume f is a substantially $fg\mu$-continuous function. Let F in Y be a $g\mu$-closed fuzzy set. 1F is then a $g\mu$-open fuzzy set. $f - 11(1F)$ is an open fuzzy set in X because f is strongly $fg\mu$- continuous. However, because $f - 1(1F) = 1f - 1(F)$, $f - 1(F)$ is a closed fuzzy set in X. Assume that the closed fuzzy set in X is the inverse image of every $g\mu$-closed fuzzy set in Y. If V in Y is a $g\mu$-open fuzzy set, then $1V$ is a $g\mu$-closed fuzzy set in Y. $f - 1(1F)$ is a closed fuzzy set in X, according to hypothesis. As a result, $f - 1(1F) = 1f - 1(F)$, and $f - 1(F)$ is an open fuzzy set in X. As a result, f is $fg\mu$-continuous.

Theorem 2.3 : Every strongly $fg\mu$-continuous function is a f-continuous function.

Proof : Let $f : X \to Y$ a highly $fg\mu$- continuous function. If V is an open fuzzy set in Y, then V is also a $g\mu$-open fuzzy set in Y. Then in $X, f - 1(F)$ is an open fuzzy set. As a result, f is an f-continuous function.

As the following example shows, the converse of the preceding theorem does not have to be true.

Example 2.4 : f is f-continuous in Example 2.4.4 because B is an open fuzzy set in Y and $f-1(B)=B$ is an open fuzzy set in X. However, because the fuzzy set C is a g-closed fuzzy set in Y and $f-1(C)=C$ is not a closed fuzzy set in X, f is not strongly fg-continuous.

Theorem 2.5 : A strongly $fg\mu$-continuous function is an f-strongly continuous function.

Proof : Let f : Be an f-strongly continuous function for X and Y. Assume V is a $g\mu$-open fuzzy set in Y. Then, because f is an strongly $fg\mu$-continuous function, $f-1(V)$ is both an open and a closed fuzzy set in X. As a result, f is a $fg\mu$-continuous function.

As the following example shows, the converse of the preceding theorem does not have to be true.

Example 2.6 : As the fuzzy set $A1$ in Y is such that $f-1(A1)=A2$ is open but not closed fuzzy set in X, f is strongly $fg\mu$-continuous but not f-strongly continuous in Example 1.4.6.

Theorem 2.7 : Let $f : X \to Y$ should be f-continuous and fuzzy - $TS\#$ space. Then f is strongly $fg\mu$-continuous.

Proof : Assume V is a $g\mu$-open fuzzy set in Y. Because Y is fuzzy - $TS\#$ space, V is an open fuzzy set in Y. As f is f-continuous, $f-1(V)$ is an open fuzzy set in X. As a result, f is $fg\mu$-continuous.

Theorem 2.8 : If $f : X \to Y$ and $g : Y \to Z$ are both strongly $fg\mu$-continuous, then the composition map $gof : XZ$ then becomes a substantially $fg\mu$-continuous function.

Proof : Assume V is a $g\mu$-open fuzzy set in Z. Because g is strongly $fg\mu$-continuous, $g-1(V)$ is an open fuzzy set in Y. As a result, $g-1(V)$ in Y is a $g\mu$-open fuzzy set. $f-1(g-1(V) = (gof)-1(V)$ is also an open fuzzy set in X since f is strongly $fg\mu$-continuous. As a result, gof is a $fg\mu$-continuous function.

Theorem 2.9 : Let $f : X \to Y, g : Y \to Z$ be maps such that f is strongly $fg\mu$-continuous and g is $fg\mu$-continuous then $gof : X \to Z$ is f-continuous.

Proof : Let F be a closed fuzzy set in Z. Then $g-1(F)$ is $g\mu$-closed fuzzy set in Y. Since g is $fg\mu$-continuous. And since f is strongly $fg\mu$-continuous, $f-1(g-1(F) = (gof)-1(F)$ is closed fuzzy set in X. Hence gof is f-continuous.

Theorem 2.10 : If $f : X \to Y$ be strongly $fg\mu$-continuous and $g : Y \to Z$ is $fg\mu$-irresolute. Then the composition map $gof : X \to Z$ is strongly $fg\mu$-continuous.

Proof : Let V be $g\mu$-open fuzzy set in Z. Then $g-1(V)$ is $g\mu$-open fuzzy set in Y since g is $fg\mu$-irresolute. And then $f-1(g-1(V)) = (gof)-1(V)$ is open fuzzy set in X. Also since f is strongly $fg\mu$-continuous. Hence gof is strongly $fg\mu$-continuous.

3. Perfectly Fuzzy $g\mu$ - Continuous Functions

Definition 3.1 : A functions $f : X \to Y$ is called perfectly fuzzy $g\mu$ - continuous (briefly perfectly $fg\mu$-continuous) if the inverse image of every $g\mu$ - open fuzzy set in Y is both open and closed fuzzy set in X.

Theorem 3.2 : A map $f : X \to Y$ is perfectly $fg\mu$-continuous iff the inverse image of every $g\mu$-closed fuzzy set in Y is both open and closed fuzzy set in X.

Proof : Assume that f is perfectly $fg\mu$-continuous. Let F be $g\mu$-closed fuzzy set in Y. Then $1 - F$ is $g\mu$-open fuzzy set in Y. And therefore $f - 1(1 - F)$ is both open and closed fuzzy set in X. But $f - 1(1 - F) = 1 - f - 1(F)$ and so $f - 1(F)$ is both open and closed fuzzy set in X. Assume, on the other hand, that the inverse image of any $g\mu$-closed fuzzy set in Y is both an open and a closed fuzzy set in X. Assume V is a $g\mu$-open fuzzy set in Y. Then in Y, $1V$ is a $g\mu$-closed fuzzy set. In $X, f - 1(1V)$ is both an open and a closed fuzzy set, according to hypothesis. However, $f - 1(1V) = 1\ f - 1(V)$. As a result, in $X, f - 1(V)$ is both an open and a closed fuzzy set. As a result, f is $fg\mu$-continuous.

Theorem 3.3 : Every perfectly $fg\mu$-continuous function is a f-continuous function.

Proof : Let $f : X \to Y$ be perfectly $fg\mu$-continuous. Let V be open fuzzy set in Y, and so V is $g\mu$-open fuzzy set in Y. Since f is perfectly $fg\mu$-continuous, then $f - 1(V)$ is both open and closed fuzzy set in X. That is $f - 1(V)$ is open fuzzy set in X. Hence f is f-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.4 : In the Example 2.4.4, the function f is f-continuous but not perfectly $fg\mu$-continuous as the fuzzy set $1 - C = \{(a, .3), (b, .5), (c, .2)\}$ is $g\mu$-open fuzzy set in Y and $f - 1(1 - C) = 1 - C$ which is not both open and closed fuzzy set in X.

Theorem 3.5 : Every perfectly $fg\mu$-continuous function is a f-perfectly continuous function.

Proof : Let $f : X \to Y$be perfectly $fg\mu$-continuous. Let V be open fuzzy set in Y then V be $g\mu$-open fuzzy set in Y. Since f is perfectly $fg\mu$-continuous. Then $f - 1(V)$ is both open and closed fuzzy set in X. And hence f is f-perfectly continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6 : In the Example 2.4.4, the function f is f-perfectly continuous as the fuzzy set B is open fuzzy set in Y, and its inverse image $f - 1(B) = B$ is both open and closed fuzzy set in X. But f is not perfectly $fg\mu$-continuous as the fuzzy set $1 - C = \{(a, .3), (b, .5), (c, .2)\}$ is

$g\mu$-open fuzzy set in Y and $f-1(1-C) = 1-C$ which is not both open and closed fuzzy set in X.

Theorem 3.7 : If $f : X \to Y$ is f-perfectly continuous and Y is fuzzy- $TS\#$ space. Then f is a perfectly $fg\mu$-continuous function.

Proof : Let V be $g\mu$-open fuzzy set in Y. Then V is open fuzzy set in Y as Y is fuzzy- $TS\#$ space. Since f is f-perfectly continuous, $f-1(V)$ is both open and closed fuzzy set in X. And therefore, f is perfectly $fg\mu$-continuous function.

Theorem 3.8 : Every perfectly $fg\mu$-continuous function is strongly $fg\mu$- continuous function.

Proof : Let $f : X \to Y$ be perfectly $fg\mu$-continuous. Let V be $g\mu$-open fuzzy set in Y. And then $f-1(V)$ is both open and closed fuzzy set in X. Therefore $f-1(V)$ is open fuzzy set in X. Hence f is strongly $fg\mu$-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9 : In the Example 2.4.6, the function f is strongly $fg\mu$- continuous but not perfectly $fg\mu$-continuous as the fuzzy set $A3$ is $g\mu$-closed fuzzy set in Y and $f-1(A3) = A3$ is not both open and closed fuzzy set in X.

Theorem 3.10 : Let $f : X \to Y, g : Y \to Z$ be two perfectly $fg\mu$-continuous functions then $gof : X \to Z$ is perfectly $fg\mu$-continuous.

Proof : Let V be $g\mu$-open fuzzy set in Z. And then $g-1(V)$ is both open and closed fuzzy set in Y. Since g is perfectly $fg\mu$-continuous, and therefore $g-1(V)$ is $g\mu$-open fuzzy set in Y. Also since f is perfectly $fg\mu$-continuous $f-1(g-1(V)) = (gof)-1(V)$ is both open and closed fuzzy set in X. Hence gof is perfectly $fg\mu$-continuous.

Theorem 3.11 : Let $f : X \to Y$ is perfectly $fg\mu$-continuous and $g : Y \to Z$ is $fg\mu$-irresolute functions then $gof : X \to Z$ is perfectly $fg\mu$-continuous functions.

Proof : Let V be $g\mu$-open fuzzy set in Z. And then $g-1(V)$ is $g\mu$-open fuzzy set in Y. Since g is $fg\mu$-irresolute functions. Also since f is perfectly $fg\mu$ continuous. $f-1(g-1(V)) = (gof)-1(V)$ is both open and closed fuzzy set in X. Hence gof is perfectly $fg\mu$-continuous functions.

4. Completely Fuzzy $g\mu$ - Continuous Functions

Definition 4.1 : A map $f : X \to Y$ is completely fuzzy $g\mu$ - continuous (briefly completely $fg\mu$-continuous) if the inverse image of every $g\mu$ - open fuzzy set in Y is regular-open fuzzy set in X.

Theorem 4.2 : A map $f : X \to Y$ is completely $fg\mu$-continuous iff the inverse image of every

$g\mu$-closed fuzzy set in Y is regular-closed fuzzy set in X.

Proof : Suppose f is completely $fg\mu$-continuous. Let F be $g\mu$-closed fuzzy set in Y. Then $1-F$ is $g\mu$-open fuzzy set in Y. And therefore $f-1(1-F)$ is regular-open fuzzy set in X. Now $f-1(1-F) = 1-f-1(F)$. And therefore $f-1(F)$ is regular-closed fuzzy set in X.

Assume that the regular-closed fuzzy set in X is the inverse image of every $g\mu$-closed fuzzy set in Y. Assume V is a $g\mu$-open fuzzy set in Y. Then in $Y, 1V$ is a $g\mu$-closed fuzzy set. $f-1(1V)$ is a regular-closed fuzzy set in X, according to the hypothesis. Now $f-1(1V) = 1f-1f-1f-1f1f-1f-1f1f(V)$. As a result, in X, $f-1(V)$ is a regular-open fuzzy set. As a result, f is a $fg\mu$-continuous function.

Theorem 4.3 : Every completely $fg\mu$-continuous function is a f-continuous function.

Proof : Let $f : X \to Y$ be completely $fg\mu$-continuous function. Let V be open fuzzy set in Y. Then V is $g\mu$-open fuzzy set in Y. And then $f-1(V)$ is both regular-open fuzzy set in X, and therefore $f-1(V)$ is open fuzzy set in X. Hence f is f-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.4 : In the Example 1.4.4, the function f is f- continuous but not completely $fg\mu$-continuous as the fuzzy set $1-C = \{(a, .3), (b, .5), (c, .2)\}$ is $g\mu$-open fuzzy set in Y and $f-1(1-C) = 1-C$ which is not regular-open fuzzy set in X.

Theorem 4.5 : Every completely $fg\mu$-continuous function is a f-completely continuous function.

Proof : Let $f : X \to Y$ be completely $fg\mu$-continuous. Let V be open fuzzy set in Y. Then V be $g\mu$-open fuzzy set in Y. And then $f-1(V)$ is regular-open fuzzy set in X. Hence f is f-completely continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.6 : In the Example 1.4.4, the function f is f-completely continuous as the fuzzy set B is open fuzzy set in Y, and its inverse image $f-1(B) = B$ is regular-open fuzzy set in X. But f is not completely $fg\mu$-continuous as the fuzzy set $1-C = \{(a, .3), (b, .5), (c, .2)\}$ is $g\mu$-open fuzzy set in Y and $f-1(1-C) = 1-C$ which is not regular-open fuzzy set in X.

Theorem 4.7 : Every completely $fg\mu$-continuous function is strongly $fg\mu$-continuous.

Proof : Let $f : X \to Y$ be completely $fg\mu$-continuous. Let V be $g\mu$-open fuzzy set in Y. And then $f-1(V)$ is regular-open fuzzy set in X. And therefore $f-1(V)$ is open fuzzy set in X. Hence f is strongly $fg\mu$-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.8 : In the Example 1.4.6, the function f is strongly $fg\mu$- continuous but not completely $fg\mu$-continuous as the fuzzy set $A3$ is $g\mu$-closed fuzzy set in Y and its inverse image $f-1(A3) = A3$ is not regular-closed fuzzy set in X.

Theorem 4.9 : If $f : X \to Y$ is f-completely continuous and Y is fuzzy- $TS\#$ space. Then f is a completely $fg\mu$-continuous function.

Proof : Let V be $g\mu$-open fuzzy set in Y. Then V is open fuzzy set in Y since Y is fuzzy- $TS\#$ space. Also since f is f-completely continuous, $f-1(V)$ is regular-open fuzzy set in X. And hence f is completely $fg\mu$-continuous function.

Theorem 4.10 : Let $f : X \to Y$ is completely $fg\mu$-continuous and $g : Y \to Z$ is $fg\mu$-irresolute functions then $gof : X \to Z$ is completely $fg\mu$-continuous functions.

Proof : Let $Vg\mu$-open fuzzy set in Z. And then $g-1(V)$ is $g\mu$-open fuzzy set in Y. Since g is $fg\mu$-irresolute functions. Also since f is completely $fg\mu$- continuous. And then $f-1(g-1(V)) = (gof)-1(V)$ is regular-open fuzzy set in X. Hence gof is completely $fg\mu$-continuous functions.

Theorem 4.11 : If $f : X \to Y$ and $g : Y \to Z$ be two completely $fg\mu$-continuous functions then $gof : X \to Z$ is completely $fg\mu$-continuous functions.

Proof : Assume V is a $g\mu$-open fuzzy set in Z. The regular-open fuzzy set in Y is then $g-1(V)$. Because g is fully $fg\mu$-continuous. As a result, $g-1(V)$ is an open fuzzy set, and $g\mu$-open fuzzy set is an open fuzzy set in Y. $f-1(g-1(V)) = (gof)-1(V)$ is also a regular-open fuzzy set in X because f is a perfectly $fg\mu$-continuous function. As a result, gof is a fully $f\mu g$-continuous function.

References

[1] Azad K. K., On fuzzy continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981), 14-32.

[2] Balasubramanian G. and Sundaram P., On some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems, 86 (1997), 93-100.

[3] Chang C. L., Fuzzy topological spaces, J. Math. Anal. Appl. (1968), 182-190.

[4] Levine N., Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2) (1970), 89-96.

[5] Maki H., Fukutake T., Kojima M. and Harada H., Generalised closed sets in fts I Report for Meeting on Topological spaces, Theory and its Applications, (1998), 23-26.

[6] Patil S. N., Thesis On Some Recent Advances In Topology Awarded the Doctorate Degree in Karnataka University Dharwad 2009.

[7] Patil S. N., on pre-semi-closed fuzzy sets in fuzzy topological space. ...KSubmitted.

[8] Veerakumar M. K. R. S., Generalized-μ-closed sets, Antarctica J. Math., 3(1) (2006), 67-83.

[9] Wong C. K., Fuzzy points and Local properties of Fuzzy Topology, Jl. Math. Anal. Appl. 46(1974), 316-328.

[10] Zadeh L. A., Fuzzy sets, Inform. and Control, 8 (1965), 338-353.