

ON PENTAGONAL NUMBERS IN THE GENERALIZED PELL SEQUENCE

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Abstract

A positive integer N is called a Pentagonal Number if it is of the form $N = \frac{m(3m-1)}{2}$, where m any integer is greater than zero and a positive integer N is called a Generalized Pentagonal Number if $N = \frac{m(3m-1)}{2}$ for any integer m . The Pell Sequence $\{P_n\}$ defined by $P_0 = 0, P_1 = 1$ and $P_{n+2} = 2P_{n+1} + P_n$ for $n \geq 0$.

Now, we defined for a fixed integer $\alpha > 0$, a new sequence called Generalized Pell Sequence $\{P_n^{(\alpha)}\}$ defined by $P_0^{(\alpha)} = 0, P_1^{(\alpha)} = 1$ and

$$P_{n+2}^{(\alpha)} = (\alpha + 1)P_{n+1}^{(\alpha)} + \frac{\alpha(3\alpha - 1)}{2}P_n^{(\alpha)} \text{ for } n \geq 0.$$

We proved that there exists Pentagonal Numbers and Generalized Pentagonal Numbers in the sequence $\{P_n^{(\alpha)}\}$ and some other results relative to Pentagonal Numbers.

Key Words and Phrases: Pentagonal Number, Generalized Pentagonal Number, Perfect Square, Generalized Pell Sequence.

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