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FUZZY GENERALISED LATTICES

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Abstract

This paper, deals with the concept of fuzzy subgeneralised lattice of a generalised lattice which can also be call as fuzzy generalised lattice. The concepts of fuzzy ideal (filter), fuzzy prime ideal (filter) and fuzzy convex subgeneralised lattice of a generalised lattice are also introduced and obtained their characterizations. Discussed about the products of fuzzy subgeneralised lattices (ideals, filters, convex subgeneralised lattices) and obtain a necessary and sufficient condition for a fuzzy ideal on the direct product of generalised lattices to be representable as a direct product of fuzzy ideals on each generalised lattice.

1. Introduction

Murty and Swamy [5] introduced the concept of generalised lattice. In [2] the author developed the theory of generalised lattices that can play an intermediate role between the theories of lattices and posets. The concept of fuzzy lattice is well known by [1] and [7]. In this paper section 2 contains preliminaries which are taken from the references. In section 3 introduced the concepts of fuzzy subgeneralised lattice, fuzzy ideal (filter) and fuzzy prime ideal (filter) of a generalised lattice with proper examples. Proved that these are characterized by their level subsets. In section 4 the concept of fuzzy convex subgeneralised lattice introduced and discussed. In section 5 discussed about the products of fuzzy subgeneralised lattices (ideals, filters, convex subgeneralised lattices) and obtain a necessary and sufficient condition for a fuzzy ideal on the direct product of generalised lattices to be representable as a direct product of fuzzy ideals on each generalised lattice.

2. Preliminaries

Definition 2.1 [Murty [5]] : Let $(P, \leq)$ be a poset. P is said to be a generalised meet semilattice if for every non empty finite subset A of P, there exist a non empty finite subset B of P such that, $x \in L(A)$ if and only if $x \leq b$ for some $b \in B$. P is said to be a generalised join semilattice if for every non empty finite subset A of P, there exist a non empty finite subset B of P such that, $x \in U(A)$ if and only if $b \leq x$ for some $b \in B$. P is said to be a generalised lattice if it is both generalised meet and join semilattice.

It is observed that if P is a generalised meet (join) semilattice, then for any $L(A) \in \mathcal{L}(P)$ $(U(A) \in \mathcal{U}(P))$ there exists a unique finite subset B of P such that $L(A) = \bigcup_{b\in B} L(b)$ $(U(A) = \bigcup_{b\in B} U(b))$ and the elements of B are mutually incomparable and the set is denoted by $ML(A)$ $(mu(A))$.

Definition 2.2 [Kishore [2]] : Let P be a generalised lattice. A non empty subset S of P is said to be a subgeneralised lattice if for any finite subset A of S, $ML(A) \subseteq S$ and $mu(A) \subseteq S$. An initial segment I of a P is said to be a strong ideal if $mu(A) \subseteq I$, for any finite subset A of I. Dually a final segment F of P is said to be a strong filter if $ML(A) \subseteq F$, for any finite subset A of F.

It is observed that in a generalised lattice, every strong ideal (filter) is a subgeneralised lattice and an ideal (filter)(see [5]).

Definition 2.3 [Kishore [4]] : An ideal (filter) I(F) of a poset P is said to be prime if for any $a, b \in P - I$ $(a, b \in P - F)$ there exist $c \in P - I$ $(c \in P - F)$ such that $c \leq a, b$ $(c \geq a, b)$.

It is observed that in a generalized lattice, an ideal (filter) $I(F)$ is prime if and only if for any $a, b \in P - I$, $(a, b \in P - F)$ there exist $c \in ML(\{a, b\})$ $(c \in mu(\{a, b\}))$ such that $c \in P - I$ $(c \in P - F)$.

Definition 2.4 [Kishore [3]] : A subset C of a poset P is said to be convex if for any $a,\ b \in C,\ c \in P$ and $a \leq c \leq b$, we have $c \in C$. If a subgeneralised lattice of a generalised lattice is convex then it is said to be a convex subgeneralised lattice.

Definition 2.5 : Let μ be a fuzzy set in P. For any $t \in [0,\ 1]$, the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called level subset of μ. The fuzzy set μ' defined by $\mu'(x) = 1 - \mu(x)$ for all $x \in P$ is called complement of μ.

Definition 2.6 : Let $\mu,\ \theta$ be a fuzzy sets in P. Then their intersection $\mu \cap \theta$ defined by $(\mu \cap \theta)(x) = min\{\mu(x), \theta(x)\}$.

3. Fuzzy Subgeneralised Lattices and Fuzzy Ideals

Throughout this section, we shall denote by P a generalised lattice. In a generalised lattice, first we define the concepts: fuzzy subgeneralised lattice, fuzzy ideal, fuzzy filter, fuzzy prime ideal and fuzzy prime filter with proper examples.

Definition 3.1 : A fuzzy set μ in P is said to be a fuzzy subgeneralised lattice if for any finite subset A of P, (i) $\mu(s) \geq \min_{a \in A}\{\mu(a)\}$ for all $s \in mu(A)$ and (ii) $\mu(t) \geq \min_{a \in A}\{\mu(a)\}$ for all $t \in ML(A)$.

Definition 3.2 : Let μ be a fuzzy subgeneralised lattice of P. μ is called a fuzzy ideal if $x \leq y$ in P implies $\mu(x) \geq \mu(y)$. μ is called a fuzzy filter if $x \leq y$ in P implies $\mu(x) \leq \mu(y)$.

Definition 3.3 : A fuzzy ideal μ of P is said to be a fuzzy prime ideal if for any finite subset A of P, (iii) $\mu(t) \leq \mathrm{Max}_{a \in A}\{\mu(a)\}$ for all $t \in ML(A)$.

Definition 3.4 : A fuzzy filter μ of P is said to be a fuzzy prime filter if for any finite subset A of P, (iv) $\mu(s) \leq \mathrm{Max}_{a \in A}\{\mu(a)\}$ for all $s \in mu(A)$.

For a fuzzy set μ in P and $x, y \in P$, consider the following conditions: (i') $\mu(s) \geq \min\{\mu(x), \mu(y)\}$ for all $s \in mu(\{x, y\})$, (ii') $\mu(t) \geq \min\{\mu(x), \mu(y)\}$ for all $t \in ML(\{x, y\})$, (iii') $\mu(t) \leq \mathrm{Max}\{\mu(x), \mu(y)\}$ for all $t \in ML(\{x, y\})$ and (iv') $\mu(s) \leq \mathrm{Max}\{\mu(x), \mu(y)\}$ for all $s \in mu(\{x, y\})$. Then we have $(i) \Leftrightarrow (i')$, $(ii) \Leftrightarrow (ii')$, $(iii) \Leftrightarrow (iii')$ and $(iv) \Leftrightarrow (iv')$.

Example 3.5 : Consider the generalised lattice $P = \{0, a, b, c, d, 1\}$ under the order $0 < x < 1$ for all $x \in P$ and $a < c,\ a < d,\ b < c,\ b < d$. Define the fuzzy sets $\mu_1,\ \mu_2 : P \to [0,\ 1]$ by $\mu_1(0) = 0.3,\ \mu_1(a) = 0.2,\ \mu_1(b) = 0.5,\ \mu_1(c) = 0.6,\ \mu_1(d) = 0.2,\ \mu_1(1) = 0.4$ and $\mu_2(0) = 0.6,\ \mu_2(a) = 0.2,\ \mu_2(b) = 0.4,\ \mu_2(c) = \mu_2(d) = \mu_2(1) = 0.2$. Then μ_1 is a fuzzy subgeneralised lattice but not a fuzzy ideal of P and μ_2 is a fuzzy ideal but not a fuzzy prime ideal of P. More over every fuzzy prime ideal of P is a constant fuzzy set.

Example 3.6 : Consider the generalised lattice $P' = \{0, a, b, c, d, e, 1\}$ under the order $0 < x < e < 1$ for all $x \in P$ and $a < c,\ a < d,\ b < c,\ b < d$. Define the fuzzy set $\mu_3 : P' \to [0,\ 1]$ by $\mu_3(0) = \mu_3(a) = \mu_3(b) = \mu_3(c) = \mu_3(d) = \mu_3(e) = 0.4,\ \mu_3(1) = 0.2$. Then μ_3 is a fuzzy prime ideal of P' which is also a non-const map.

In the following Propositions we obtain equivalent conditions for a fuzzy set to be a fuzzy ideal (filter, prime ideal, prime filter).

Proposition 3.7 : Let μ be a fuzzy set in P. Then for any $x,\ y \in P$ and for any finite subset A of P, the following conditions are equivalent: (i) $\mu(x) \geq \mu(y)$ whenever $x \leq y$, (ii) $\mu(t) \geq \text{Max}_{a\in A}\{\mu(a)\}$ for all $t \in ML(A)$ and (iii) $\mu(s) \leq \min_{a\in A}\{\mu(a)\}$ for all $s \in mu(A)$.

Proof : (i) $\Leftrightarrow$ (ii): For any $t \in ML(A)$, by (i) we have $\mu(t) \geq \mu(a)$ for all $a \in A$ and then $\mu(t) \geq \text{Max}_{a\in A}\{\mu(a)\}$. Conversely suppose $x \leq y$, then by (ii) we have $\mu(x) \geq \text{Max}\{\mu(x),\ \mu(y)\}$ and then $\mu(x) \geq \mu(y)$. Similarly we can prove (i) $\Leftrightarrow$ (iii).

Observe that a fuzzy subgeneralised lattice μ of P is a fuzzy ideal if and only if μ satisfies any one of the three conditions of above theorem. Moreover a fuzzy subset μ of P is a fuzzy ideal if and only if for any finite subset A of P, $\mu(s) = \min_{a\in A}\{\mu(a)\}$ for all $s \in mu(A)$.

Proposition 3.8 : Let μ be a fuzzy set in P. Then for any $x,\ y \in P$ and for any finite subset A of P, the following conditions are equivalent: (i) $\mu(x) \leq \mu(y)$ whenever $x \leq y$, (ii) $\mu(s) \geq \text{Max}_{a\in A}\{\mu(a)\}$ for all $s \in mu(A)$ and (iii) $\mu(t) \leq \min_{a\in A}\{\mu(a)\}$ for all $t \in ML(A)$.

Observe that a fuzzy subgeneralised lattice μ of P is a fuzzy filter if and only if μ satisfies any one of the three conditions of above theorem. Moreover a fuzzy subset μ of P is a fuzzy filter if and only if for any finite subset A of P, $\mu(t) = \min_{a\in A}\{\mu(a)\}$ for all $t \in ML(A)$.

Proposition 3.9 : Let μ be a fuzzy ideal of P. Then for any finite subset A of P, the following conditions are equivalent: (i) μ is a fuzzy prime ideal, (ii) $\mu(t) = \text{Max}_{a\in A}\{\mu(a)\}$ for all $t \in ML(A)$ and (iii) $\mu(t) = \mu(a)$ for some $a \in A$, for all $t \in ML(A)$.

Proof : (i) $\Leftrightarrow$ (ii): If μ is a fuzzy prime ideal then by the Definition 3.3 and the Proposition 3.7, we can get (ii). Clearly the converse is true. (iii) $\Leftrightarrow$ (ii): Let $t \in ML(A)$ and suppose $\mu(t) = \mu(a)$ for some $a \in A$. Then $\mu(t) = \mu(a) \geq \mu(b)$ for all $b \in A$.

Therefore $\text{Max}_{b\in A}\{\mu(b)\} = \mu(a) = \mu(t)$. The converse is clear.

Proposition 3.10 : Let μ be a fuzzy filter of P. Then for any finite subset A of P, the following conditions are equivalent: (i) μ is a fuzzy prime filter, (ii) $\mu(s) = \text{Max}_{a\in A}\{\mu(a)\}$ for all $s \in mu(A)$ and (iii) $\mu(s) = \mu(a)$ for some $a \in A$, for all $s \in mu(A)$.

Recall the Definition 2.5 for the complement of a fuzzy set.

Corollary 3.11 : If μ is a fuzzy ideal of P, such that the complement μ' is a fuzzy filter then μ and μ' are fuzzy prime.

Corollary 3.12 : Let μ be a fuzzy set in P. Then μ is a fuzzy prime ideal if and only if μ' is a fuzzy prime filter.

In the following Theorems we characterize fuzzy subgeneralised lattices (ideals, filters, prime ideals, prime filters) by their level subsets.

Theorem 3.13 : A fuzzy set μ in P is a fuzzy subgeneralised lattice if and only if each non-empty level subset μ_t is a subgeneralised lattice of P if and only if for any $t \in \text{Im}\mu$, μ_t is a subgeneralised lattice of P.

Proof : Suppose μ is a fuzzy subgeneralised lattice of P and μ_t is a non-empty level subset of P. Let A be a finite subset of μ_t. Then for any $r \in mu(A)$ and $s \in ML(A)$, we have $\mu(r), \mu(s) \geq \min_{a\in A}\{\mu(a)\} \geq t$. So that $mu(A),\ ML(A) \subseteq \mu_t$. Conversely suppose each non-empty level subset μ_t is a subgeneralised lattice of P. Let $A = \{a_1, a_2, \cdots a_n\}$ be a finite subset of P. For $1 \leq i \leq n$, suppose $\mu(a_i) = t_i$ and $t_i \leq t_{i-1}$. Then we have $\mu_{t_{i-1}} \subseteq \mu_{t_i}$ and $a_i \in \mu_{t_i}$. Now $mu(A),\ ML(A) \subseteq \mu_{t_n}$, since $A \subseteq \mu_{t_n}$. Therefore for any $r \in mu(A)$ and $s \in ML(A)$, $\min_{a\in A}\{\mu(a)\} = t_n \leq \mu(r), \mu(s)$.

Definition 3.14 : If μ is a fuzzy subgeneralised lattice of P, then for any $t \in \text{Im}\mu$, the subgeneralised lattice μ_t is called a level subgeneralised lattice of μ.

Observe that the set of level subgeneralised lattices of a fuzzy subgeneralised lattice of a generalised lattice forms a chain.

Theorem 3.15 : A fuzzy set μ in P is a fuzzy ideal (filter) if and only if each non-empty level subset μ_t is a strong ideal (filter) of P if and only if for any $t \in \text{Im}\mu$, μ_t is a strong ideal (filter) of P.

Proof : Suppose μ is a fuzzy ideal of P and μ_t is a non-empty level subset of P. Let $x \in \mu_t,\ y \in P$ and suppose $y \leq x$. Since $\mu(y) \geq \mu(x) \geq t,\ y \in \mu_t$. By Theorem 3.13, μ_t is a strong ideal of P. Conversely suppose each non-empty level subset μ_t is a strong ideal of P. By theorem 3.13, μ is a subgeneralised lattice of P. Let $x,\ y \in P,\ \mu(x) = t_1,\ \mu(y) = t_2$

and suppose $x \leq y$. If $\mu(x) \ngeq \mu(y)$ then since μ_{t_2} is an initial segment, x not in μ_{t_2} but $y \in \mu_{t_2}$ leads to a contradiction. Therefore μ is a fuzzy ideal of P.

Definition 3.16 : If μ is a fuzzy ideal (filter) of P, then for any $t \in \text{Im}\mu$, the strong ideal (filter) μ_t is called a level strong ideal (filter) of μ.

Theorem 3.17 : A fuzzy set μ in P is a fuzzy prime ideal (filter) if and only if each non-empty level subset μ_t is a prime strong ideal (filter) of P if and only if for any $t \in \text{Im}\mu$, μ_t is a prime strong ideal (filter) of P.

Proof : Suppose μ is a fuzzy prime ideal of P and μ_t is a non-empty level subset of P. Then by theorem 3.15, μ_t is a strong ideal of P. Now for any $a, b \in P - \mu_t$ and $c \in ML(\{a,\ b\})$, by Theorem 3.9 we have $\mu(c) \ngeq t$, i.e., $c \in P - \mu_t$. Conversely suppose each non-empty level subset μ_t is a prime strong ideal of P. Assume that the fuzzy ideal μ is not prime. Then by theorem 3.9 there exists $a,\ b \in P$ and $c \in ML(\{a,\ b\})$ such that $\mu(c) \neq \mu(a)$ and $\mu(c) \neq \mu(b)$. Since μ is a fuzzy ideal, $\mu(c) > \mu(a)$ and $\mu(c) > \mu(b)$. Now for $t = \mu(c)$, we have $a \in P - \mu_t,\ b \in P - \mu_t$ but $c \in \mu_t$ which leads to a contradiction. Therefore μ is a fuzzy prime ideal.

Theorem 3.18 : Let μ_i be any family of fuzzy subgeneralised lattices (ideals, filters) of P. Then $\bigcap \mu_i$ is a fuzzy subgeneralised lattice (ideal, filter) of P.

Definition 3.19 : Let μ be a fuzzy set in P. Then the least fuzzy subgeneralised lattice (ideal, filter) of P containing μ is called the fuzzy subgeneralised lattice (ideal, filter) generated by μ, denoted by $[\mu]$ $((\mu],\ [\mu))$.

4. Fuzzy Convex Subgeneralised Lattices

Throughout this section, we shall denote by P a generalised lattice.

Definition 4.1 : A fuzzy subgeneralised lattice μ of P is fuzzy convex if for every interval $[a, b] \subseteq P$, $\mu(x) \geq min\{\mu(a), \mu(b)\}$ for all $x \in [a, b]$.

Theorem 4.2 : A fuzzy set μ in P is a fuzzy convex subgeneralised lattice if and only if each non-empty level subset μ_t is a convex subgeneralised lattice of P if and only if for any $t \in \text{Im}\mu$, μ_t is a convex subgeneralised lattice of P.

Proof : Suppose μ is a fuzzy convex subgeneralised lattice of P and μ_t is a non-empty level subset of P. By Theorem 3.13 μ_t is a subgeneralised lattice of P. Let $a,\ b \in \mu_t$ and $a < b$. Since $\mu(x) \geq min\{\mu(a), \mu(b)\} \geq t$ for any $x \in [a, b]$, we have $[a,\ b] \subseteq \mu_t$ and then μ_t is convex. Conversely suppose each non-empty level subset μ_t is a convex subgeneralised lattice of P. Again by Theorem 3.13, μ is a fuzzy subgeneralised lattice of P. Let $[a,\ b] \subseteq P,\ x \in [a,\ b]$ and take $t = min\{\mu(a), \mu(b)\}$. Since $a,\ b \in \mu_t$, we have $x \in \mu_t$ i.e., $\mu(x) \geq t$. Therefore μ is a fuzzy convex subgeneralised lattice of P.

Proposition 4.3 : In a generalised lattice, every fuzzy ideal (filter) is fuzzy convex.

Example 4.4 : Consider the generalised lattice P as in the Example 3.5. Define the fuzzy set $\mu_4 : P \to [0,\ 1]$ by $\mu_4(0) = \mu_4(a) = 0.2,\ \mu_4(b) = 0.4,\ \mu_4(c) = 0.6,\ \mu_4(d) = \mu_4(1) = 0.2$. Then μ_4 is a fuzzy convex subgeneralised lattice but not a fuzzy ideal.

Observe that the fuzzy subgeneralised lattice μ_1 (defined in the Example 3.5) is not fuzzy convex.

Theorem 4.5 : Let μ_i be any family of fuzzy convex subgeneralised lattices P. Then $\bigcap \mu_i$ is a fuzzy convex subgeneralised lattice of P.

Observe that in a generalised lattice, the intersection of a fuzzy ideal and a fuzzy filter is a fuzzy convex subgeneralised lattice provided the intersection is non-empty.

5. Products of Fuzzy Subgeneralised Lattices

Through out this section, we denote $P_1,\ P_2$ are generalised lattices.

Definition 5.1 : Let $\mu_1,\ \mu_2$ be fuzzy subsets of $P_1,\ P_2$ respectively. Define the product $\mu_1 \times \mu_2$, a fuzzy subset of $P_1 \times P_2$, by $(\mu_1 \times \mu_2)(x,\ y) = \min\{\mu_1(x), \mu_2(y)\}$ for all $(x,\ y) \in P_1 \times P_2$.

Definition 5.2 : Let μ be a fuzzy subgeneralised lattice of $P_1 \times P_2$. Then the fuzzy subsets $\pi_1(\mu),\ \pi_2(\mu)$ defined by $\pi_1(\mu)(x) = \text{Sup}\{\mu(x,\ y) \mid y \in P_2\},\ \pi_2(\mu)(y) = \text{Sup}\{\mu(x,\ y) \mid x \in P_1\}$ are called projections of μ on $P_1,\ P_2$ respectively.

Proposition 5.3 : (i) If $\mu_1,\ \mu_2$ are fuzzy subgeneralised lattices (ideals, filters, fuzzy convex) of $P_1,\ P_2$ respectively, then $\mu_1 \times \mu_2$ is a fuzzy subgeneralised lattice (ideal, filter, fuzzy convex) of $P_1 \times P_2$. (ii) If μ is a fuzzy subgeneralised lattice (ideal, filter, fuzzy convex) of $P_1 \times P_2$, then $\pi_1(\mu),\ \pi_2(\mu)$ are fuzzy subgeneralised lattices (ideals, filters, fuzzy convex) of $P_1,\ P_2$ respectively.

Proof : (i) Suppose $\mu_1,\ \mu_2$ are fuzzy subgeneralised lattices of $P_1,\ P_2$ respectively. Let C be a finite subset of $P_1 \times P_2$. Then there exists finite subsets $A,\ B$ of $P_1,\ P_2$ respectively such that $L(C) = L(A \times B)$. Now for any $(s,\ t) \in ML(C) = ML(A) \times ML(B)$, we have $(\mu_1 \times \mu_2)(s,\ t) = \min\{\mu_1(s),\ \mu_2(t)\} \geq \min\{\min_{a \in A}\mu_1(a),\ \min_{b \in B}\mu_2(b)\} = \min_{(a,\ b) \in C}(\mu_1 \times \mu_2)(a,\ b)$. Therefore $\mu_1 \times \mu_2$ is a fuzzy subgeneralised lattice of $P_1 \times P_2$.

(ii) Suppose μ is a fuzzy subgeneralised lattice of $P_1 \times P_2$. Let A be a finite subset of P_1 and $s \in mu(A)$. Then for any $z \in P_2$, we have $\mu(s,\ z) \geq min_{a\in A}\mu(a,\ z)$, since $(s,\ z) \in mu(A) \times mu(z) = mu(A \times \{z\})$. So that $\pi_1(\mu)(s) \geq \text{Sup}\{\min_{a\in A}\mu(a,\ z) \mid z \in P_2\} = \min_{a\in A}\pi_1(\mu)(a)$. Therefore $\pi_1(\mu)$ is a fuzzy subgeneralised lattice of P_1.

Definition 5.4 : Let μ be a fuzzy subset of $P_1 \times P_2$ and $a \in P_2,\ b \in P_1$. Then the fuzzy subsets $m_1^a(\mu),\ m_2^b(\mu)$ defined by $m_1^a(\mu)(x) = \mu(x,\ a)$ and $m_2^b(\mu)(y) = \mu(b,\ y)$ are called marginal fuzzy subsets of $P_1,\ P_2$ respectively.

Proposition 5.5 : If μ is a fuzzy subgeneralised lattice (ideal, filter, fuzzy convex) of $P_1 \times P_2$, then for all $a \in P_2,\ b \in P_1;\ m_1^a(\mu),\ m_2^b(\mu)$ are fuzzy subgeneralised lattices (ideals, filters, fuzzy convex) of $P_1,\ P_2$ respectively.

Lemma 5.6 : If μ is a fuzzy ideal of $P_1 \times P_2$, then for all $a \in P_2,\ b \in P_1;\ m_1^a(\mu) \times m_2^b(\mu) \subseteq \mu \subseteq \pi_1(\mu) \times \pi_2(\mu)$.

Proof : For any $(x,\ y) \in P_1 \times P_2$, we have $(m_1^a(\mu) \times m_2^b(\mu))(x,\ y) = \min\{m_1^a(\mu)(x), m_2^b(\mu)(y)\} = mu(\{\mu(x,\ a),\ \mu(b,\ y)\}) = mu(\{\mu(x,\ y),\ \mu(b,\ a)\}) \leq \mu(x,\ y)$ and clearly $\mu(x,\ y) \leq \min\{\pi_1(\mu)(x), \pi_2(\mu)(y)\} = \pi_1(\mu) \times \pi_2(\mu)(x,\ y)$.

Theorem 5.7 : Let $P_1,\ P_2$ be generalised lattices with 0 and μ is a fuzzy ideal of $P_1 \times P_2$. Then μ is the product of a fuzzy ideal of P_1 and a fuzzy ideal of P_2 if and only if $m_1^0(\mu) \times m_2^0(\mu) = \pi_1(\mu) \times \pi_2(\mu)$.

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