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SOME ESTIMATORS FOR A FINITE POPULATION MEAN BY USING ESTIMATED MULTI-AUXILIARY INFORMATION

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Abstract

The objective of this paper is to derived difference, regression and predictive estimators for two stage sampling by using double sampling technique when multi-auxiliary information is unavailable. The mean square errors for these estimators have been derived. A numerical study is given for finding the relative comparison.

1. Introduction

The estimation of the population mean is a persistent issue in sampling practice and many efforts have been made to improve the precision of the estimates. The literature of survey sampling describes a great variety of techniques when auxiliary information is utilized the estimation stage. The ratio, product, difference and regression estimators are employed in many situations. Particularly, in the presence of multi-auxiliary variables, wide varieties of estimators have been proposed. Cochran (1940) firstly suggested

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the estimators by using known auxiliary information. Olkin (1958) was the first author to deal the problem of estimating the mean of a survey variable when auxiliary variable are made available. He suggested the use of information on more than one auxiliary variable analogously to Olkin; Singh (1967a) gave a multivariate expression of Murthyfs (1964) product estimator. Raj (1965) suggested a method for using multi-auxiliary variables through a linear combination of single difference estimators, while Shukla (1965) suggested a multiple regression estimator.

A variety of approaches are available to construct more efficient estimators for the population mean including design based and model based methods. In a predictive approach a model is specified for the population values and is used to predict the non sampled values. Prediction theory for sampling surveys (or model- based theory) can be considered as a general framework for statistical inferences on the character of finite population. Srivastava (1983) suggested that the predictive estimator for product estimator could utilize the prediction criterion given by Basu (1971) and it subsequently studied by Sampford (1978). On the other hand, Scott and Smith (1969) and Chaudhuri and Stenger (1992) proposed some prediction criteria for two stage sampling. Hossain and Ahmed (2001) suggested some predictive estimators by using known auxiliary variable. In view of the above facts here some estimators are proposed by extending the idea for multiple auxiliary variables when it is estimated by using double sampling technique. Difference, regression and predictive estimators for p-auxiliary variables for two stage sampling are proposed. Before suggesting the estimators we provide suggested procedure and some useful notations in the following section.

2. Suggested Procedures and Notations

Suppose a finite population U be partitioned into N first stage units (fsu) denoted by $(U_1, \cdots, U_i, \cdots, U_n)$ such that the number of second stage units (ssu) in U_i is M_i and $M = \sum_{i}^{N} M_i$. When information on auxiliary character is not available, a double sampling procedure is proposed as an alternative under such a situation for the estimation of population mean $\overline{Y}$. Assume that a sample s' of n' denote the size of first phase sample of fsu (usually large) is drawn from U by using SRSWOR for measuring information on auxiliary character x, let $\{x+o\}$ $(i = 1, 2, \cdots, n')$ denote the x observations and

$$\overline{x}^* = \frac{1}{n'} \sum_{i}^{n'} x_i$$

the sample means. A second phase sample s of size n $(< n')$ is drawn as a subsample from n' i.e. the first-phase sample by using SRSWOR and then a sample s_i of m_i ssu'fs from the i^{th} selected fsu U_i of M_i ssus according to the sampling scheme. We define similar notations as Cochran(1977).

$$\begin{aligned} \overline{y}_i &= \tfrac{1}{m_i} \sum_{j \in s_i} y_{ij} & \overline{y} &= \tfrac{1}{n} \sum_{i \in s} w_i \overline{y}_i \\ \overline{x} &= \tfrac{1}{n} \sum_{i \in s} w_i \overline{x}_i & \overline{y}^* &= \tfrac{1}{n'} \sum_{i \in s'} w_i \overline{Y}_i \\ \text{and} \quad \overline{x}^* &= \tfrac{1}{n'} \sum_{i \in s'} w_i \overline{X}_i \end{aligned}$$

where $w_i = \frac{M_i}{\overline{M}} = \frac{NM_i}{M}$, also $M = \sum_{i}^{N} M_i$ and $\overline{M} = \frac{1}{N} \sum_{i}^{N} M_i$.

3. Proposed Estimators and their Properties

Under the usual predictive set-up, it possible to express, for a given non empty set s, we can partition

$$\begin{aligned} \overline{Y} &= \frac{\sum_{i}^{N} \sum_{j}^{M_i} y_{ij}}{\sum_{i}^{N} M_i} = \frac{\sum_{i}^{N} M_i \overline{Y}_i}{\sum_{i}^{N} M_i} \\ &= \frac{1}{M} [\sum_{i}^{N} M_i \overline{Y}_i] \\ &= \frac{1}{M} [\sum_{i \in s'} M_i \overline{Y}_i + \sum_{i \in s'} M_i \overline{Y}_i] \\ &= \frac{1}{M} [\sum_{i \in s'} \{ \sum_{i \in s} M_i \overline{Y}_i + \sum_{i \in \overline{s}} M_i \overline{Y}_i \} + \sum_{i \in \overline{s}'} M_i \overline{Y}_i] \\ &= \frac{1}{M} [\sum_{i \in s'} \{ \sum_{i \in s_i} y_{ij} + \sum_{j \in \overline{s}_i} y_{ij} \} + \sum_{i \in \overline{s}} M_i \overline{Y}_i + \sum_{i \in \overline{s}'} M_i \overline{Y}_i] \qquad (3.1) \end{aligned}$$

where $\overline{s}'$ denotes the set of $(N - n')$ first phase of fsu's in U which are not included in s', $\overline{s}$ denoted the set of $(n' - n)$ second phase of fsu's in U which are not included in s

and $\overline{s}_i$ the set of $(M_i - m_i)$ ssus of U_i which are not included in $s_i, i = 1, 2, \cdots, n$.

$$\overline{Y} = \frac{1}{M}[\sum_{i\in s'}\{\sum_{i\in s}(m_i\overline{y}_i + (M_i - m_i)\overline{Y}_{ir})\}] + \frac{1}{M}\sum_{i\in\overline{s}} M_i\overline{Y}_i + \frac{1}{M}\sum_{i\in\overline{s}'} M_i\overline{Y}_i. \tag{3.2}$$

Let $\overline{Y}_{r^*} = \frac{1}{N-n'}\sum_{i\in\overline{s}'} w_i\overline{Y}_i, \overline{Y}_r = \frac{1}{n'-n}\sum_{i\in\overline{s}_i} w_i\overline{Y}_i$ and $\overline{Y}_{ir} = \frac{1}{M_i-m_i}\sum_{i\in\overline{s}_i}\sum_{i\in\overline{s}_i} y_{ij}$. We have

$$\overline{Y} = \frac{1}{M}[\sum_{i\in s'}\{\sum_{i\in s}(m_i\overline{y}_i + (M_i - m_i)\overline{Y}_{ir})\}] + \frac{(n'-n)}{N}\overline{Y}_r + \frac{N-n'}{N}\overline{Y}_r, \tag{3.3}$$

where $\overline{Y}_{r^*} = \frac{N\overline{Y}-n'\overline{y}'}{N-n'}$, $\overline{Y}_r = \frac{n'\overline{y}'-n\overline{y}}{n'-n}$ and $\overline{Y}_{ir} = \frac{M_i\overline{Y}_i - m_i\overline{y}_i}{M_i-m_i}$. To estimate $\overline{Y}$, we therefore have to predict the quantities $\overline{Y}_{ir}, \overline{Y}_r$ and $\overline{Y}_{r^*}$, from the sample data because the first component of the right hand side of (3.8) is already known. Using $Z_{r'}, Z_r$ and Z_{ir} as their respective predictors.

Then, the predictive estimator of the population mean $\overline{Y}$ is

$$(\hat{\overline{Y}})_{pre} = \frac{1}{M}[\sum_{i\in s'}\{\sum_{i\in s}(m_i\overline{y}_i + (M_i - m_i)Z_{ir})\}] + \frac{(n'-n)}{N}Z_r + \frac{N-n'}{N}Z_r^* \tag{3.4}$$

where Z_{r^*}, Z_r are the predictors of $\overline{Y}_{r^*}, \overline{Y}_r$ for first stage unit using double sampling and Z_{ir} is the predictor of $\overline{Y}_{ir}$ for second stage unit respectively.

In equation (3.4) we combine the last two terms i.e. Non sampled part of first phase and second phase in first stage units because in first stage using double sampling when we go from one phase to second sampling unit is not changed and non sampled units are $N - n' + n' - n = N - n$.

Equation (3.4) can be written as

$$(\hat{\overline{Y}})_{pre} = \frac{1}{M}[\sum_{i\in s} m_i\overline{y}_i + (M_i - m_i)Z_{ir})] + \frac{N-n}{N}Z_r^{**}.$$

Define, Z_r^{**} and Z_{ir} as the class of estimators using single auxiliary variable x for first stage unit using double sampling and second stage unit proposed as

$$\left.\begin{aligned} Z_r^{**} &= \overline{y} + t(\overline{x}_r^* - \overline{x}) \\ Z_{ir} &= \overline{y}_i + t_i(\overline{X}_{ir} - \overline{x}_i) \end{aligned}\right\} \tag{3.5}$$

and t and t' are suitably chosen statistics and assume $E(t) = E_1E_2(t) = T$ or $E(t) \approx T$ and $E(t_i) = T_i$ or $E(t_i) \approx T_i$. Define

$$f' = \frac{n'}{N}, \quad f = \frac{n}{n'} \quad f_i = \frac{m_i}{M_i}$$

$$S_{yx} = \frac{1}{N-1}\sum_{i}^{N}(w_i\overline{Y}_i - \overline{Y})(w_i\overline{X}_i - \overline{X}) \qquad S_{iy}^2 = \frac{1}{M_i-1}\sum_{j}^{M_i}(y_{ij} - \overline{Y}_i)^2$$

and S_y^2 and S_x^2 can be obtained from S_{yx} using $y = x$.

Now for first order approximation the MSE (3.5) are given respectively

$$M(Z_r^{**}) \cong \left(\frac{1-f'}{n'}\right)S_y^2 + \left(\frac{1-f}{n}\right)\{S_y^2 - 2TS_{yx} + T^2S_x^2\} \tag{3.6}$$

$$M(Z_{ir}) \cong \left(\frac{1-f_i}{m_i}\right)\{S_{iy}^2 - 2T_iS_{iyx} + T_i^2S_{iyx} + T_i^2S_{ix}^2\} \tag{3.7}$$

where equation (3.6) is the mean square error for first stage unit using double sampling and equation (3.7) is the mean square error for second stage unit respectively.

After some simplification, we have the first order of mean square error of $(\hat{\overline{Y}})_{pre}$ is as follows:

$$\begin{aligned} M(\hat{\overline{Y}}_{pre}) &= \left(\frac{1-f'}{n'}\right)S_y^2 + \left(\frac{1-f}{n}\right)\{S_y^2 + T^2S_x^2 - 2TS_{yx}\} \\ &\quad + \frac{1}{nN}\sum_{i}^{N}w_i^2\left(\frac{1-f_i}{m_i}\right)\{S_{iy}^2 + \alpha_i^2S_{ix}^2 - 2\alpha_iS_{iyx}\} \end{aligned} \tag{3.8}$$

where $\alpha_i = T - f(T - T_i)$.

The optimum values of T and T_i which minimize $M(\hat{\overline{Y}}_{pre})$ are given respectively as

$$T_{opt} = \frac{S_{yx}}{S_x^2} = B \qquad \text{and} \qquad T_{iopt} = \frac{S_{iyx}}{S_{ix}^2} = B_i.$$

Then the minimum mean square error of $\hat{\overline{Y}}$ is

$$\begin{aligned} M(\hat{\overline{Y}})_{opt} &= \left(\frac{1-f'}{n'}\right)S_y^2 + \left(\frac{1-f}{n}\right)\{S_y^2 + B^2S_x^2 - 2BS_{yx}\} \\ &\quad + \frac{1}{nN}\sum_{i}^{N}w_i^2\left(\frac{1-f_i}{m_i}\right)\{S_{iy}^2 + B_i^2S_{ix}^2 - 2B_iS_{iyx}\}. \end{aligned} \tag{3.9}$$

Suppose $x_{1ij}, x_{2ij}, \cdots, x_{pij}$ are the values of p-auxiliary variables $x_1, x_2, \cdots, x_p$ respectively for the j^{th} ssu of i^{th} fsu $(j = 1, 2, \cdots, M_i, i = 1, 2, \cdots, N)$.

Define,

$$\begin{array}{llllllll} \overline{x}_k & = & \frac{1}{n}\sum\limits_{i\in s} w_i\overline{x}_{ki}, & \overline{x}_k^* & = & \frac{1}{n'}\sum\limits_{i\in s'} w_i\overline{X}_i & & \\ \underline{\overline{x}^*} & = & (\overline{x}_k^*)_{p\times 1}, & \underline{\overline{x}} & = & (\overline{x}_k)_{p\times 1}, & \underline{\lambda} & = & (\lambda_k)_{p\times 1} \\ S_{\underline{yx}} & = & (S_{yx_k})_{p\times 1}, & S_{\underline{xx}} & = & (S_{x_k x_k'})_{p\times 1}, & S_{\underline{iyx}} & = & (S_{iyx_k})_{p\times 1}, \\ S_{\underline{ixx}} & = & (S_{ix_k x_k'})_{p\times 1} & \text{where} & & k,k' = 1,2,\cdots,p & & \end{array}$$

Now, the multivariate difference estimator for population mean is

$$\hat{\overline{Y}}_d = \overline{y} + (\underline{\overline{x}^*} - \underline{\overline{x}})'\underline{\lambda} \tag{3.10}$$

where $\underline{\lambda}$ is predetermined constant vector. For predetermined constant vector where $\underline{\lambda}, \hat{\overline{Y}}_d$ is unbiased estimator.
The variance of $\hat{\overline{Y}}_d$ is

$$\begin{aligned} V(\hat{\overline{Y}}_d) &= \left(\frac{1-f'}{n'}\right)S_y^2 + \left(\frac{1-f}{n}\right)\{S_y^2 - 2S'_{\underline{yx}}\underline{\lambda} + \underline{\lambda}'S_{\underline{xx}}\underline{\lambda}\} \\ &\quad + \frac{1}{nN}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\{S_{iy}^2 + \underline{\lambda}'S_{\underline{ixx}}\underline{\lambda} - 2\underline{S'_{iyx}}\underline{\lambda} - 2\underline{S'_{iyx}}\underline{\lambda}\}. \end{aligned} \tag{3.11}$$

By minimizing (3.11), we may get the optimum choice of $\underline{\lambda}$.

$$\begin{aligned} \frac{\partial V(\hat{\overline{Y}}_d)}{\partial \underline{\lambda}} &= \left(\frac{1-f}{n}\right)\left\{-2\underline{S'_{yx}} + 2\underline{S_{xx}}\underline{\lambda}\right\} \\ &\quad + \frac{1}{nN}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\left\{2\underline{S_{ixx}}\underline{\lambda} - 2\underline{S'_{iyx}}\right\} = 0 \end{aligned}$$

or

$$S'_{yx} + \frac{1}{N-n}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\underline{S'_{iyx}} = \left(\underline{S_{xx}} + \frac{1}{N-n}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\underline{S_{ixx}}\right)\underline{\lambda}. \tag{3.12}$$

Let

$$\begin{aligned} \underline{G} &= S'_{yx} + \frac{1}{N-n}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\underline{S'_{iyx}} \quad \text{and} \\ \underline{H} &= \underline{S_{xx}} + \frac{1}{N-n}\sum_i^N w_i^2\left(\frac{1-f_i}{m_i}\right)\underline{S_{ixx}}. \end{aligned}$$

Hence, $\underline{H\lambda} = \underline{G}$.

Multiply both sides of (3.12) by $\underline{H^{-1}}$, we get the optimum $\underline{\lambda}$

$$\underline{\lambda_{opt}} = \underline{H^{-1}G}. \tag{3.13}$$

Now, the multivariate regression estimator for population mean is

$$\hat{\overline{Y}}_{reg} = \overline{y} + (\underline{\overline{x}^*} - \underline{\overline{x}})'\underline{B} \tag{3.14}$$

where $\underline{B} = \underline{S_{xx}^{-1}}\,\underline{S_{yx}}$.

The estimator (3.14) is an unbiased estimator for population mean, $\overline{Y}$ and the variance of $\hat{\overline{Y}}_{reg}$ is

$$\begin{aligned} V(\hat{\overline{Y}}_{reg}) &= \left(\frac{1-f'}{n'}\right) S_y^2 + \left(\frac{1-f}{n}\right) \left\{ S_y^2 - 2\underline{S'_{yx}}\underline{B} + \underline{B'}\,\underline{S_{xx}}\,\underline{B} \right\} \\ &\quad + \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) \left\{ S_{iy}^2 + \underline{B'}\,\underline{S_{ixx}}\,\underline{B} - 2\underline{S'_{iyx}}\,\underline{B} \right\} \end{aligned} \tag{3.15}$$

Now the multivariate predictive estimator is

$$(\hat{\overline{Y}})_{pre} = \frac{1}{M}\left[\sum_{i \in s} m_i \overline{y}_i + (M_i - m_i) Z_{irm} \right] + \frac{N-n}{N} Z_{rm}^{**} \tag{3.16}$$

where $Z_{rm}^{**} = \overline{y} + t(\overline{x}_{\underline{r}}^* - \underline{\overline{x}})'\underline{B}$ and $Z_{irm} = \overline{y}_i + t_i(\overline{X}_{\underline{ir}} - \overline{x}_{\underline{i}})'B_{\underline{i}}$.

Since the mean square error of Z_{rm}^{**} and Z_{irm} are respectively,

$$\begin{aligned} M(Z_{rm}^{**}) &\cong \left(\frac{1-f'}{n'}\right) S_y^2 + \left(\frac{1-f}{n}\right) \left\{ S_y^2 - 2\underline{S'_{yx}}\,\underline{B} + \underline{B'}\,\underline{S_{xx}}\,\underline{B} \right\} \\ M(Z_{irm}) &\cong \left(\frac{1-f_i}{m_i}\right) \{ S_{iy}^2 - 2\underline{S'_{iyx}}\,\underline{B_i} + \underline{B'_i}\,\underline{S_{ixx}}\,\underline{B_i} \}. \end{aligned}$$

Then the mean square error of (3.16) is

$$\begin{aligned} M(\hat{\overline{Y}})_{pre} &= \left(\frac{1-f'}{n'}\right) S_y^2 + \left(\frac{1-f}{n}\right) \left\{ S_y^2 - 2\underline{S'_{yx}}\,\underline{B} + \underline{B'}\,\underline{S_{xx}}\,\underline{B} \right\} \\ &\quad + \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) \left\{ S_{iy}^2 + \underline{\alpha'_i}\,\underline{S_{ixx}\alpha_i} - 2\underline{S'_{iyx}}\,\underline{\alpha_i} \right\} \end{aligned} \tag{3.17}$$

where $\underline{\alpha_i} = \underline{B} - f(\underline{B} - \underline{B_i})$.

4. Efficiency Comparison

Now compare the proposed estimator with the other estimators considered here.

The usual mean per element for two stage sampling is

$$V(\overline{Y}) = \left(\frac{1-f}{n}\right) S_y^2 + \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) S_{iy}^2. \tag{4.1}$$

From (3.11) and (4.1) we have

$$\begin{aligned} V(\overline{Y}) - V(\hat{\overline{Y}}_d) &= \left(\frac{1-f}{n}\right) \{\underline{\lambda S'_{yx}} - \underline{\lambda}^2 \underline{S}_{xx}\} \\ &+ \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) \{\underline{\lambda S'_{iyx}} - \underline{\lambda}^2 \underline{S}_{ixx}\} - \left(\frac{1-f'}{n'}\right) S_y^2 \end{aligned}$$

which is greater than zero i.e. $V(\hat{\overline{Y}}_d) < V(\overline{Y})$ if

$$\underline{\delta}_{yx} > \frac{\underline{\lambda}}{2} \quad \text{and} \quad \underline{\beta}_{iyx} > \frac{\underline{\lambda}}{2} \tag{4.2}$$

where $\underline{\delta}_{yx} = \frac{\underline{S'_{yx}}}{\underline{S}_{ixx}}$ be the regression coefficient y on $\underline{x}$ in U and $\underline{\beta}_{iyx} = \frac{\underline{S'_{iyx}}}{\underline{S}_{ixx}}$ be the regression coefficient y on $\underline{x}$ in U_i.

From (3.15) and (4.1) we have

$$\begin{aligned} V(\overline{Y}) - V(\hat{\overline{Y}}_{reg}) &= \left(\frac{1-f}{n}\right) \{\underline{BS'_{yx}} - \underline{B}^2 \underline{S}_{xx}\} \\ &+ \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) \{\underline{BS'_{iyx}} - \underline{B}^2 \underline{S}_{ixx}\} - \left(\frac{1-f'}{n'}\right) S_y^2 \end{aligned}$$

which is greater than zero i.e. $V(\hat{\overline{Y}}_{reg}) < V(\overline{Y})$ if

$$\underline{\delta}_{yx} > \frac{\underline{B}}{2} \quad \text{and} \quad \underline{\beta}_{iyx} > \frac{\underline{B}}{2} \tag{4.3}$$

where $\underline{\delta}_{yx}$ and $\underline{\beta}_{iyx}$ be the regression coefficient of y on $\underline{x}$ in U and U_i.

From (4.1) and (3.17) we have

$$\begin{aligned} V(\overline{Y}) - M(\hat{\overline{Y}}_{pre}) &= \left(\frac{1-f}{n}\right) \{\underline{BS'_{yx}} - \underline{B}^2 \underline{S}_{xx}\} \\ &+ \frac{1}{nN} \sum_i^N w_i^2 \left(\frac{1-f_i}{m_i}\right) \{\underline{\alpha}_i \underline{S'_{iyx}} - \underline{\alpha}_i^2 \underline{S}_{ixx}\} - \left(\frac{1-f'}{n'}\right) S_y^2 \end{aligned}$$

which is greater than zero i.e. $M(\hat{\overline{Y}})_{pre} < V(\overline{Y})$ if

$$\underline{\delta}_{yx} > \frac{B}{2} \qquad \text{and} \qquad \underline{\beta}_{iyx} > \frac{\alpha_i}{2} \tag{4.4}$$

where $\underline{\delta}_{yx}$ and $\underline{\beta}_{iyx}$ be the regression coefficient of y on $\underline{x}$ in U and U_i.
Expression (4.2), (4.3) and (4.4) provide the conditions under which the estimators defined in (3.10), (3.14) and (3.16) will be more efficient than estimator defined in (4.1).

5. Numerical Study

To show the importance of suggested methodology presented in this paper is highlighted by a numerical study.

Population : We consider the 2001 census data which relates to the total number of agricultural laboures and the total no. of cultivators of 444 villages of Bhiwani district of Haryana. The whole population of Bhiwani district (444 villages) is divided into 9 blocks (fsus) where i^{th} $(i = 1, 2, \cdots, 9)$ block consists of M_i villages (ssus) $i = 1, 2, \cdots, 9$. Let y = the number of agricultural laborers, x_1 = the area of villages, x_2 = total population, we consider classical, difference, regression and predictive estimator for two stage sampling when auxiliary information is estimated. The percentage relative efficiencies of the different estimators using two auxiliary variables, x_1 and x_2 are given in Table-I.

Table I

The percentage relative efficiencies over mean per element

Estimator	Auxiliary Variables	MSE ($\times 10^7$)	PRE[a]
$\overline{y}$	None	970.30	100
Difference estimator	x_1, x_2	275.06	352.76
Regression estimator	x_1, x_2	262.5	369.64
Predictive estimator	x_1, x_2	254.45	381.33

[a]PRE $= \frac{v(\overline{y})}{M(\cdot)} \times 100\%$.

6. Conclusion

Section 4 provides the conditions under which the proposed estimators has less mean squared error as compared to mean per element. In Section 5, table III shows that the proposed estimators have greater percent relative efficiency w.r.t. mean per element. It is also observed from table III that the predictive estimator has highest percent relative efficiency as compare to difference, regression and traditional estimators. Thus the predictive estimator is recommended for used in practice.

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