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MIXED CONNECTEDNESS VIA SHADOW SPACES

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Abstract

Shadow spaces are quotient spaces of $A-$spaces. They provide a better understanding to the concept of $A-$spaces using their corresponding posets. In this paper, we introduce and investigate new types of $A-$space called upper bounded and lower bounded $A-$spaces. Then we study connectedness and types of mixed connectedness. we prove that an $A-$space X is $i-j-$connected iff its shadow space $[X]$ is $i-j-$connected, for $i, j \in \{\alpha,\ P,\ S,\ SP,\ \gamma\}$.

1. Introduction

An *Alexandroff space* (briefly $A-$space) (or *minimal neighborhood space*) X is a topological space in which the arbitrary intersection of open sets is open. In these spaces,

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Key Words : *Alexandroff spaces, Shadow spaces, Generalized open sets, $\alpha-$open sets, Preopen sets, Connectedness, Mixed connectedness, Preconnectedness, Semi-connectedness, $\alpha-$connectedness, $\beta-$connectedness.*

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each element x has a minimal open neighborhood set $V(x)$ which is the intersection of all open sets containing x. For every T_o $A-$space (X,τ), there is a corresponding poset $(X,\leq_\tau)$ in one to one and onto way, where each one of them is completely determined by the other. If $(X,\leq)$ is a poset, then $\mathcal{B}=\{\uparrow x : x\in X\}$ forms a base for a T_0 Alexandroff topology on X denoted by $\tau_{\leq}$. Moreover, if (X,τ) is an A-space, we define the pre-order $\leq_\tau$, called (*Alexandroff*) *specialization pre-order* as follows: $a\leq_\tau b$ if and only if $a\in\overline{\{b\}}$. This specialization pre-order is a partial order if and only if (X,τ) is T_o. So, we consider $(X,\tau(\leq))$ to be a T_o $A-$space (X,τ) together with its specialization order $\leq$. We see that $\forall x\in X$, $V(x)$ equals $\uparrow x$; the up set of x in the corresponding poset and $\overline{x}=\downarrow x$; the down set of x. A poset $(X,\leq)$ satisfies the *ascending chain condition* (ACC) if for any increasing sequence $x_1\leq x_2\leq\cdots\leq x_n\leq\cdots$ in X, there exists $k\in\mathbb{N}$ such that $x_k=x_{k+1}=\cdots$. The dual of (ACC) is the *descending chain condition* (DCC). If a poset satisfies both ACC and DCC, we say that X is of *finite chain condition* (FCC). Given a poset $(X,\leq)$, the set of all maximal elements is denoted by $M(X)$ (or simply M) and the set of all minimal elements is denoted by $m(X)$ (or simply m). Moreover, for each $x\in X$, we define $\hat{x}$ to be the set of all maximal elements grater than or equal to x and $\check{x}$ to be the set of all minimal elements less than or equal to x. That is, $\hat{x}=\uparrow x\cap M$ and $\check{x}=\downarrow x\cap m$.

A T_o $A-$space whose corresponding poset satisfies the ACC is called *Artinian T_o $A-$space*, and whose corresponding poset satisfies the DCC is called *Noetherian T_o $A-$space* [11]. An upper bounded T_0 $A-$space is introduced as a generalization of Artinian T_0 $A-$spaces. A T_0 $A-$space is an *upper bounded* T_0 $A-$space (briefly a UB T_0 $A-$space) [5]S if every chain of points in the corresponding posets is bounded above. There is a detail study of these spaces in [12]. A T_0 $A-$space is an *lower bounded* T_0 $A-$space (briefly an LB T_0 $A-$space) [5] if every chain of points in the corresponding poset is bounded below. If X is a topological space and $\mathfrak{D}$ a partition of X, then $\mathfrak{D}$ can be topologized as follows: $\mathfrak{F}\subseteq\mathfrak{D}$ is open in $\mathfrak{D}$ iff $\bigcup_{F\in\mathfrak{F}}F$ is open in X. The topology $\tau_{\mathfrak{D}}$ on $\mathfrak{D}$ is called the *quotient topology* of X induced by $\mathfrak{D}$, and the open sets U in X where $U=\bigcup\{F\in\mathfrak{F}:\mathfrak{F}\in\tau_{\mathfrak{D}}\}$ are called *saturated*. It should be noted that not all open sets in X are saturated. Nevertheless, each saturated open set has a corresponding open set in $\mathfrak{D}$. So there is a one to one correspondence between $\tau_{\mathfrak{D}}$ and the collection of all saturated open sets in X. For a topological space X, an equivalence relation $\sim$ can be defined on X as follows: $x \sim y$ iff x and y cannot be separated by open sets. The set of all equivalence classes $[X]$ forms a partition on X with quotient topology satisfies the separation axiom T_0. Again, not all open sets in X are saturated. In [**?**], it was proved that if the topology on X is Alexandroff, then each open set in X is saturated with respect to the equivalence relation $\sim$. Hence there is a one to one correspondence between τ on X and the quotient topology on $[X]$. So, the quotient topology on $[X]$ is called the *shadow topology* of τ and denoted by τ_s. The shadow space $([X], \tau_s)$ is a T_0 $A-$space and has a corresponding poset $([X], \leq_s)$. For two classes $[a], [b]$ in $[X]$, we have $[a] \leq [b]$ iff $b \in V(a)$ iff $V(b) \subseteq V(a)$ iff $a \in \bar{b}$. For a subset $A \subseteq X$, we define $[A] = \{[a] : \ a \in A\}$. It may happen that $A \neq B$ but $[A] = [B]$. In Some of our previous studies see [10] and [13], we observed that the relation between $A-$spaces and their shadow spaces is very interesting and used this relation in introducing new definitions and concepts defined on $A-$shadow spaces and carry over to any $A-$space. We gave a study of preopen, semi-open, and $\alpha-$ open sets on $A-$spaces. Then we proved that an $A-$space is connected (compact) iff its shadow space $[X]$ is connected (compact). From now on, and for any $A-$space (X, τ), we will consider its A-shadow space $([X], \tau_s)$.

2. Preliminaries

A subfamily m_x of a power set $\mathcal{P}(X)$ of a nonempty set X is said to be a *minimal structure* on X, if $\emptyset, X \in m_x$ and $\bigcup A_\alpha \in m_x$ whenever $A_\alpha \in m_x$. The sets in m_x are caled m_x-open sets, and the sets where their complements in m_x are called m_x-closed sets. It is clear that an arbitrary intersection of m_x-closed sets is m_x-closed set. For $A \subseteq X$, we define m_x interior and m_x closure of A as follows:

$$m_x - Int(A) = \bigcup \{U : U \subseteq A \text{ and } U \in m_x\}, \text{ and}$$

$$m_x - Cl(A) = \bigcap \{F : A \subseteq F \text{ and } X \backslash F \in m_x\}.$$

It is obvious that $m_x - Int(A)$ is the largest m_x-open set inside A and $m_x - Cl(A)$ is the smallest m_x-closed set containing A. Let (X, τ) be a topological space. A subset A of X is said to be $\alpha-$*open* [18] (resp. *preopen* [2], *semi-open* [17], *b-open* [3] (equiv. $\gamma-$*open* [1]), $\beta-$*open* [16] (equiv. *semi-preopen* [4])) if $A \subseteq \overline{A^o}^o$ (resp. $A \subseteq \overline{A}^o$, $A \subseteq \overline{A^o}$, $A \subseteq \overline{A}^o \cup \overline{A^o}$, $A \subseteq \overline{\overline{A}^o}$). A set F is called $j-$*closed* for $j \in \{\alpha, \text{ semi}, \text{ pre}, b, \beta\}$ if $X \backslash F$ is $j-$open. A set F is called $j-$*clopen* for $j \in \{\alpha, \text{ semi}, \text{ pre}, b, \beta\}$ if F is both $j-$open and

$j-$closed. The family of all $\alpha-$open (resp. preopen, preclosed, semi-open, semi-closed, b-open, b-closed, $\beta-$open, $\beta-$closed) is denoted by τ_α (resp. $PO(X)$, $PC(X)$, $SO(X)$, $SC(X)$, $BO(X)$, $BC(X)$, $SPO(X)$, $SPC(X)$). We have the following facts: The collection τ_α forms a topology on X [18]. $\tau_\alpha = PO(X) \cap SO(X)$ [21]. $PO(X) \cup SO(X) \subseteq BO(X) \subseteq SPO(X)$ [3]. $\tau \subseteq \tau_\alpha$ (resp. $\tau \subseteq JO(X)$ for $J \in \{S,\ P,\ B,\ SP\}$). For $j \in \{\text{semi},\ \text{pre},\ b,\ \beta\}$, the union (intersection) of any family of $j-$open ($j-$closed) sets is $j-$open ($j-$closed). Thus for $J \in \{S,\ P,\ B,\ SP\}$, $JO(X)$ is a minimal structure on X. If a minimal structure $m_x = \tau_\alpha$ (resp. $PO(X)$, $SO(X)$, $BO(X)$, $SPO(X)$), then $m_x - Int(A)$ is denoted by $Int_\alpha(A)$ (resp. $pInt(A)$, $sInt(A)$, $bInt(A)$, $\beta Int(A)$). Similarly $m_x - Cl(A)$ is denoted by $Cl_\alpha(A)$ (resp. $pCl(A)$, $sCl(A)$, $bCl(A)$, $\beta Cl(A)$). If X is a $UB\ T_0\ A-$space, then $PO(X) = \tau_\alpha$ and $PO(X) \subseteq SO(X)$. [12]

Proposition 2.1 [12] : Let X be a UB T_0 $A-$space, and A is a preopen subset of X. Then A is a UB T_0 $A-$space.

Proposition 2.2 [12] : Let X be a T_0 UB $A-$space. A set $A \subseteq X$ is preopen iff $\hat{x} \subseteq A$ for every $x \in A$.

Proposition 2.3 [12] : let X be a UB $T_\circ$ $A-$space. Then A is semi-open set if and only if $\hat{x} \cap A \neq \emptyset\ \forall x \in A$.

Proposition 2.4 [13] : Let X be an $A-$space and $A \subseteq X$. Then $[A]$ as a subspace of the shadow space $[X]$ is the same as the shadow space of the subspace (A, τ_A) of X.

Proposition 2.5 [13] : Let X be an $A-$space and $A \subseteq X$. Then A is preopen iff $[A]$ is preopen.

3. Upper Bounded $A-$spaces

In order to generalize some of basic concepts and ideas to be known in any $A-$space, we first start with a chain. A chain is a concept that related to the category of posets, and hence related in some sence to $A-$spaces that satisfy the separation axiom T_0. In the following definitions, we generalize this concept and others to any $A-$space.

Definition 3.1 : Let X be an $A-$space. A subset $C \subseteq X$ is said to be *a chain* if for every $x, y \in C$ either $V(x) \subseteq V(y)$ or $V(y) \subseteq V(x)$. A subset $B \subseteq X$ is said to be *bounded above* (*resp. below*) if there exist $z \in X$ such that $V(z) \subseteq V(x)$ (resp. $V(x) \subseteq V(z)$) for every $x \in B$. B is bounded if it is bounded above and bounded below.

Definition 3.2 : An $A-$space X is called *an upper (resp. a lower) bounded* (briefly,

UB, (resp. LB)) if every chain in X is bounded above (resp. bounded below). X is *bibounded* (briefly BB) if every chain in X is bounded, and hence X is both UB and LB.

One can easily prove the following:

Proposition 3.3 : If C is a chain in an $A-$space X, then the set $[C]$ is a chain in the corresponding poset $([X], \leq_s)$ of the shadow space $[X]$.

Propoition 3.4 : Let X be an $A-$space and C a chain in X. Then C is bounded above iff the chain $[C]$ is bounded above in the poset $([X], \leq_s)$.

Corollary 3.5 : An $A-$space X is UB iff its shadow space $[X]$ is UB.

Remark 3.6 : Let X be a UB T_0 $A-$space and Y an $A-$space such that $X = [Y]$. Then Y is a UB $A-$space. In other words, all $A-$spaces, where X is their shadow space are UB $A-$spaces.

Proposition 3.7 : Let X be a UB $A-$space and A a preopen subset of X. Then the subspace A is a UB $A-$space.

Proof : $[A]$ is a preoen subset of the shadow space $[X]$. So by Proposition 2.1 $[A]$ is a UB T_0 $A-$space. Hence by Corollary 3.5, A is a UB $A-$space. □

Definition 3.8 [13] : Let X be an $A-$space. An element $x \in X$ is said to be *maximal* (resp. *minimal*) if $V(x) = V(z)$ whenever $V(z) \subseteq V(x)$ (resp. whenever $V(z) \supseteq V(x)$). We denote the set of maximal elements of X by $M(X)$ (or simply M), and the set of minimal elements of X by $m(X)$ (or simply m). We define $\widehat{x} = V(x) \cap M$ and $\check{x} = \overline{x} \cap m$.

The following theorems, in various forms and with simple modifications, are proved for Artinian (Notherian) $A-$spaces in [13]. They still true in UB $A-$space without essential change in the mechanics of the proofs. The heart of the proofs of the theorems lie in the observation that; for both Artinian and UB $A-$spaces, $M \neq \emptyset$ and $\hat{x} \neq \emptyset$ for all $x \in X$. Similarly, $m \neq \emptyset$ and $\check{x} \neq \emptyset$ for all $x \in X$ for both Notherian and LB $A-$spaces.

Proposition 3.9 : Let X be a UB $A-$space, then $M \neq \emptyset$.

Proposition 3.10 : Let X be a UB (resp. LB) $A-$space, then M is open (resp. m is closed) set in X .

Proposition 3.11 : If X is a UB (resp. a LB) $A-$space, then $x \in M$ iff $[x] \in [M]$ (resp. $x \in m$ iff $[x] \in [m]$.)

Proposition 3.12 : Let X be a UB $A-$space. Then $V(x) \subseteq M$ iff $[V(x)] \subseteq [M]$.

Proposition 3.13 : Let X be a UB $A-$space and A a preopen subset of X. Then

(1) $[M(A)] = M[A]$.

(2) $Cl(A) = Cl(M(A))$.

(3) $M(A) \subseteq M$.

Proposition 3.14 : Let X be a UB $A-$space, and let $A \subseteq X$. If A is open (resp. closed), then $\widehat{x} \subseteq A$ (resp. $\check{x} \subseteq A$) for all $x \in A$.

Proposition 3.15 : If X is a UB (resp. a LB) $A-$space, then $[M] = M_s$ (resp. $[m] = m_s$).

Proposition 3.16 : If X is a UB (resp. LB)$A-$space, then $\hat{x} \neq \emptyset$ (resp. $\check{x} \neq \emptyset$).

Proposition 3.17 : Let X be a UB $A-$space, $A \subset X$. If $\hat{x} \subseteq A$ for every $x \in A$, then A is preopen.

Proof : If $\hat{x} \subseteq A$, then $\widehat{[x]} \subseteq [A]$. By Proposition 2.2 $[A]$ is preopen. Hence by Proposition 2.5 we get that A is preopen. $\square$

Proposition 3.18 : Let X be a UB $A-$space. If a set $A \subseteq X$ is semi-open, then $\hat{x} \cap A \neq \emptyset$ for all $x \in A$.

Proof : If $x \in A$, then $x \in Cl(Int(A))$. So $V(r) \subseteq V(x)$ for some $r \in Int(A)$, which implies that $V(r) \subseteq A$. Take $y \in \hat{r} \subseteq M$. Then $V(y) \subseteq V(r) \subseteq V(x)$. Therefore $y \in \hat{x} \cap A$. $\square$

The converse of the above two propositions need not be true as the following example shows:

Example 3.19 : Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{\emptyset, X, \{3\}, \{4, 5\}, \{3, 4, 5\}\}$. So $M = \{3, 4, 5\}$. If $A = \{1, 2, 4, 5\}$, then A is preopen. Moreover $\hat{1} = V(1) \cap M = \{3, 4, 5\} \nsubseteq A$. If $B = \{1, 2, 5\}$, then B is not semi-open. Moreover $\hat{1} = \hat{2} = \{3, 4, 5\}$, and $\hat{5} = \{4, 5\}$. So $\hat{x} \cap B \neq \emptyset$ for every $x \in B$.

4. Connectedness

A topological space (X, τ) is said to be *connected* (resp. *preconnected*) if X cannot be expressed as a union of two non-empty and disjoint open (resp. preopen) subsets of X. X is said to be *disconnected* (resp. *predisconnected*) if it is not connected (resp. preconnected). When X is connected, X contains a nontrivial clopen set. X is *hyperconnected* if every non-empty open subset of X is dense. If X is not hyperconnected, then it is *hyperdisconnected*.

Proposition 4.1 [13] : An $A-$space X is connected iff $[X]$ is connected.

Propostion 4.2 [10] : Let X be an $A-$space, $A \subseteq X$. Then A is dense iff $[A]$ is dense in $[X]$.

Proposition 4.3 [11] : Let X be a T_0 $A-$space. If X has a dense subset consisting of a single point, then X is connected.

Proposition 4.4 : Let X be an $A-$space. If $[x]$ is dense in X, for some $x \in X$, then X is connected.

Proof : By Proposition 4.2, $\{[x]\}$ is dense in $[X]$, so by Proposition 4.3, $[X]$ is connected and hence X is connected. $\square$

Proposition 4.5 [12] : Let X be a UB T_0 $A-$space and $|M| > 1$. If X is connected, then $\overline{x}$ is not open for all $x \in M$.

Proposition 4.6 : Let X be a connected UB $A-$space. If $|M| > 1$ and there is $x, y \in M$ such that $V(x) \neq V(y)$, then $Cl(x)$ is not open for all $x \in M$.

Proof : $[X]$ is connected, $[x], [y] \in [M]$ and $[x] \neq [y]$. So $|[M]| > 1$. By Proposition 4.5, $Cl_s([x])$ is not open and so $[Cl(x)]$ is not open in $[X]$. Therefore $Cl(x)$ is not open in X. $\square$

Recall that in any topological space, $Int(A \cup B) = Int(A) \cup Int(B)$ whenever $\overline{A} \cap \overline{B} = \emptyset$.

Proposition 4.7 : Let X be a UB $A-$space with a set of maximal elements M. Then X is connected iff for any proper subset C of M, $\overline{C} \cap \overline{M \setminus C} \neq \emptyset$.

Proof : If $\overline{C} \cap \overline{M \setminus C} = \emptyset$, then $\overline{C} \subseteq X \setminus (\overline{M \setminus C}) = ((M \setminus C)^{c})^{o} = (M^{c} \cup C)^{o} = (M^{c})^{o} \cup C^{o} = (\overline{M})^{c} \cup C^{o}$. Since $\overline{C} \cap \overline{M}^{c} = \emptyset$, then $\overline{C} \subseteq C^{o}$ and so C is clopen nonempty set. Conversely, let $X = P \cup Q$ where P and Q are two nonempty disjoint open subsets of X. Let $C = M \cap P$. By Proposition 3.13 (3), $C = M(P)$. This implies that $\overline{C} = \overline{M(P)} = \overline{P} = P$. Similarly $\overline{M \setminus C} = Q$. Therefore $\overline{C} \cap \overline{M \setminus C} = \emptyset$. $\square$

Proposition 4.8 [10] : An $A-$space X is hyperconnected iff its shadow space $[X]$ is hyperconnected.

Proposition 4.9 [16] : Let X be a T_0 $A-$space. If X contains a maximum element $\top$, then X is hyperconnected.

Proposition 4.10 : Let X be a UB $A-$space. If $V(x) = V(y)$ for all $x, y \in M$, then X is hyperconnected.

Proof : If $V(x) = V(y)$ for all $x, y \in M$, then $[x] = [y]$. Hence $|[M]| = 1$. Equivalently $[X]$ contains a top element. Therefore by Proposition 4.5, $[X]$ is hyperconnected. By

Proposition 4.8, X is hyperconnected. □

Proposition 4.11 : Let X be an $A-$space. If X is preconnected, then $[X]$ is preconnected.

Proof : If $[X]$ is predisconnected, then there is disjoint preopen sets $[U], [V]$ such that $[X] = [U] \cup [V]$. By Proposition 2.5, U and V are preopen sets. Let $U^* = \bigcup\{[x] : x \in U\}$ and $V^* = \bigcup\{[x] : x \in V\}$. Then, we have $X = U^* \cup V^*$, $U^* \cap V^* = \emptyset$ and U^*, V^* are preopen sets. Thus X is predisconnected. □

Directly from the definition, if a topological space X is preconnected, then it is connected. In [**?**], it was proved that if X is a UB T_0 $A-$space, then X is preconnected iff X is connected. In the following example, We first show that the converse of Proposition 4.11 need not be true. Then we show that if a connected UB $A-$space doesn't satisfy the separation axiom T_0, then X need not be preconnected.

Example 4.12 : Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{X, \emptyset, \{1\}, \{4, 5\}, \{3, 4, 5\}, \{1, 2, 4, 5\}, \{1, 4, 5\}, \{1, 3, 4, 5\}\}$. Then X is a UB $A-$space and doesn't satisfy the separation axiom T_0. Let $U = \{1, 2, 3, 4\}$ and $V = \{5\}$. Then U, V are preopen sets, so X is predisconnected. Now $[X] = \{[1], [2], [3], [4]\}$ and $\tau_s = \{[X], \emptyset, \{[1]\}, \{[4]\}, \{[3], [4]\}, \{[1], [2], [4]\}, \{[1], [4]\}, \{[1], [3], [4]\}$. So $[X]$ is connected and hence preconnected. Moreover X is connected.

5. Mixed Generalized Connectedness

Let X be a non-empty set and m_1, m_2 two minimal structures on X. Then X is said to be $m_1 - m_2-$*connected* [16], if X cannot be expressed as a union of two non-empty disjoint subsets $S_1, S_2 \in X$ such that $S_1 \in m_1$ and $S_2 \in m_2$. It is obvious that the notion $m_1 - m_2-$connectedness is equivalent to the notion $m_2 - m_1-$connectedness. If $m_1 = m_2$, then X is said to be m_1-*connected* [16]. For special cases, if (X, τ) is a topological space and if $m_1 = \tau$ (resp. τ_α, $PO(X)$, $SO(X)$, $BO(X)$, $SPO(X)$), then m_1-connected is denoted as connected (resp. $\alpha-$connected [**?**], $\mathcal{P}-$connected [24], $\mathcal{S}-$connected [22], $\gamma-$connected [6], $SP-$connected [23]). The mixed generalized connectedness is introduced in [25]. IF $m_1 = \tau$ and $m_2 = \tau_\alpha$ (resp. $m_2 = PO(X)$, $m_2 = SO(X)$, $m_2 = BO(X)$, $m_2 = SPO(X)$), then $m_1 - m_2-$connected is denoted as $\tau - \tau_\alpha-$connected (resp. $\tau - P-$connected, $\tau - S-$connected, $\tau - SP-$connected). [25] Similarly, we define $\alpha - P-$connectedness, $\alpha - S-$connectedness, $\alpha - SP-$connectedness,

$S-P-$connectedness, $S-SP-$connectedness, and $P-SP-$connectedness, (for more information, see [25]).

Proposition 5.1 [25] : In any topological space X the following statements are equivalent:

(1) (X,τ) is S-connected.

(2) (X,τ) is τ-SP-connected.

(3) (X,τ) is τ-B-connected.

(4) (X,τ) is τ-S-connected.

(5) (X,τ) is α-S-connected.

(6) (X,τ) is α-SP-connected.

(7) (X,τ) is α-B-connected.

(8) (X,τ) is S-B-connected.

(9) (X,τ) is S-SP-connected.

(10) (X,τ) is S-P-connected.

(11) (X,τ) is hyperconnected.

Corollary 5.2 : Let X be an $A-$space. Then for $J \in \{S,\ \tau-SP,\ \tau-B,\ \tau-S,\ \alpha-S,\ \alpha-SP,\ \alpha-B,\ S-P,\ S-B,\ S-SP\ \}$ X is J-connected $A-$space iff its shadow space $[X]$ is J-connected.

Proof : From Proposition 4.8, X is hyperconnected iff $[X]$ is hyperconnected. □

Proposition 5.3 : Let X be a UB $A-$space. If $V(x) = V(y)$ for all $x, y \in M$, then X is $J-$connected for $J \in \{S,\ \tau-SP,\ \tau-B,\ \tau-S,\ \alpha-S,\ \alpha-SP,\ \alpha-B,\ S-P,\ S-B,\ S-SP\}$.

Proof : Direct from Propositions 4.10 and Proposition 5.1. □

Proposition 5.4 [25] : Let (X,τ) be a topological space. Then the following statements are equivalent:

(1) X is SP-connected.

(2) X is S-connected and P-connected.

(3) X is B-SP-connected.

(4) X is P-SP-connected.

(5) X is γ-connected.

Proposition 5.5 : If an $A-$space X is J-connected, then $[X]$ is J-connected for $J \in \{SP,\ B-SP,\ P-SP,\ \gamma\}$

Proof : From Proposition 5.4 X is J-connected iff X is S-connected and P-connected. Then by Proposition 4.11 and Corollary 5.2, $[X]$ is S-connected and P-connected and hence J-connected. $\Box$

The converse of the above proposition need not be true as shown in the following example:

Example 5.6 : Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, X, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Then $U = \{1, 2, 3\}$ and $V = \{4\}$ are two nonempty disjoint preopen subsets of X. So X is not $P-$connected and hence is not $J-$connected for any $J \in \{SP,\ B-SP,\ P-SP,\ \gamma\}$. The shadow space $[X] = \{[1], [2], [3]\}$ with $\tau_s = \{\emptyset, [X], \{\{3\}\}, \{\{1\}, \{3\}\}, \{\{1\}, \{3\}\}\}$. It is obvious that $[X]$ is both S-connected and P-connected and hence J-connected.

Proposition 5.7 [25] : The following statements are equivalent for any topological space X :

(1) X is connected.

(2) X is $\tau - \tau^{\alpha}$-connected.

(3) X is $\tau - P$-connected.

Proposition 5.8 : Let X be an $A-$space. Then for $J \in \{\tau-\tau^{\alpha}, \tau-P\}$ X is J-connected iff $[X]$ is J-connected.

Proof : Direct from Propositions 4.1 and Proposition 5.7. $\Box$

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