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ON ARTINCOKERNEL OF THE GROUP $(Q_{2m} \times C_5)$ WHERE $m = p^2, p > 2$, p IS PRIME NUMBER

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Abstract

The main purpose of this paper is to find Artin's character table $Ar(Q_{2m} \times C_5)$ when $m = p^2, p > 2$, p is prime number; where Q_{2m} is denoted to Quaternion group of order $4m$, time is said to have only one dimension and space to have three dimension, the mathematical quaternion partakes of both these elements; in technical language it may be said to be "time plus space", or "space plus time" and in this sense it has, or at least involves a reference to four dimensions and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols., 1882, 1885, 1889))" and C_5 is Cyclic group of order 5. In 1962, C. W. Curits and I. Reiner studied Representation Theory of finite groups. In 1976, I. M. Isaacs studied Charactrs Theory of finite groups. In 1982, M. S. Kirdar studied the Factor Group of the Z-Valued class function modulo the group of the Generalized Characters. In 1995, N. R. Mahmood studies the Cyclic Decomposition of the factor Group cf$(Q_{2m}, Z)/R(G)(Q_{2m})$. In 2002, K-Sekiguchi studies Extensions and the Irreducibilies of the Induced Characters of Cyclic P-Group. In 2008, A. H. Abdul-Munem studied Artin Cokernel of The Quaternion group Q_{2m} when m is an Odd number.

Key Words : *Even number, Prime number, Quaternion group and Cyclic group.*

1. Introduction

Representation Theory is a branch of mathematics that studies abstract algebra structures by representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication in which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication. Moreover, representation and character theory provide applications ,no only in other branches of mathematics but also in physics and chemistry.

For a finite group G, the factor group $\overline{R}(G)/T(G)$ is called the Artincokernel of G denoted by $AC(G)$, $\overline{R}(G)$ denoted the a belian group generated by Z-valued characters of G under the operation of pointwise addition. $T(G)$ is a subgroup of $\overline{R}(G)$ which is generated by Artin's characters.

2. Preliminars :(3.1):[1]

The Generalized Quaternion Group Q_{2m} : For each positive integer $m \geq 2$, the generalized Quaternion Group Q_{2m} of order $4m$ with two generators x and y satisfies $Q_{2m} = \{x^h y^k, 0 \leq h \leq 2m-1, k = 0, 1\}$ which has the following properties $\{x^{2m} = y^4 = I, yx^m y^{-1} = x^{-m}\}$.

Let G be a finite group, all the characters of group G induced from a principal character of cyclic subgroup of G are called Artin characters of G. Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by $Ar(G)$. The first row is Γ-conjugate classes. The second row is the number of elements in each conjugate class. The third row is the size of the centralized $|C_G(CL_\alpha)|$ and other rows contains the values of Artin characters.

Theorem 3.2 [2] : The general form of Artin characters table of Cp^s when p is a prime number and s is a positive integer number is given by :-

$Ar(Cp^s) =$

Γ-classes						
$\lvert CL_a \rvert$	1	1	1	1		1
$\lvert C_{p^s}(CL_\alpha) \rvert$						
φ'_1		0	0	0		0
φ'_2			0	0		0
φ'_3				0		0
φ'_s						
φ'_{s+1}	1	1	1	1		1

Table(3,1)

Example 3.3 : We can write Artin characters table of the group C_5

$Ar(C_5) =$

Γ- classes	[1]	[x]
$\lvert CL_\alpha \rvert$	1	1
$\lvert C_{C_5}(CL\alpha) \rvert$	5	5
φ'_1	5	0
φ'_2	1	1

Table 3.2

Corollary 3.4 [2] : Let $m = p_1^{\alpha_1} \cdot p_2^{\alpha_2}, \cdots, p_n^{\alpha_n}$ where $g.c.d(p_i, p_j) = 1$, if $i \neq j$ and p_i's are primes numbers and α_n any positive integers, then;

$$Ar(C_m) = Ar(C_{p_1}\alpha_1) \otimes Ar(C_{p_2}\alpha_2) \otimes \cdots \otimes Ar(C_{p_n}\alpha_n).$$

Example 3.5 : Consider the cyclic group C_{18}. To find Artin characterstable for it, we use corollary(3,4)as the following $Ar(C_{18}) = Ar(C_2) \otimes Ar(C_3^2)$ by using theorem (3.2) to find $Ar(C_2)$ and $Ar(C_3^2)$ are as follows :

$Ar(C_2)=Ar(C_{3^2})=$

Γ- classes	[1]	$[x]$
$\lvert CL_\alpha \rvert$	1	1
$\lvert C_{C_2}(CL_\alpha)\rvert$	2	2
φ'_1	2	0
φ'_2	1	1

Table(3,4)

Γ- classes	[1]	$[x^3]$	$[x]$
$\lvert CL_\alpha \rvert$	1	1	1
$\lvert C_{C_{3^2}}(CL_\alpha)\rvert$	3^2	3^2	3^2
φ'_1	3^2	0	0
φ'_2	3	3	0
φ'_3	1	1	1

Table(3,5)

$Ar(C_{18})=$

Γ- classes	[1]	$[x^6]$	$[x^2]$	$[x^9]$	$[x^3]$	$[x]$
$\lvert CL_\alpha \rvert$	1	2	2	1	2	2
$\lvert C_{C_{18}}(CL_\alpha)\rvert$	18	9	9	18	9	9
φ'_1	18	0	0	0	0	0
φ'_2	6	6	0	0	0	0
φ'_3	2	2	2	0	0	0
φ'_4	9	0	0	9	0	0
φ'_5	3	3	0	3	3	0
φ'_6	1	1	1	1	1	1

Table(3,6)

Theorem 3.6 [1] : The Artin characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

$Ar(Q_{2m})=$

	Γ-Classes of C_{2m}								
Γ-Classes	x^{2r}				x^{2r+1}				$[y]$
$\lvert CL_\alpha\rvert$	1	2	…	2	1	2	…	2	2m
$\lvert C_{Q_{2m}}(CL_\alpha)\rvert$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	$2.Ar(C_{2m})$								0
Φ_2									0
:									:
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

Table (3.7)

where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m}, for all $1 \le j \le l+1$.

Example 3.7 : To construct $Ar(Q_{18})$ by using theorem (3.5)

$Ar(Q_{18}) =$

Γ-Classes	Γ-Classes of C_{18}						[y]
$\|CL_\alpha\|$	1	2	2	1	2	2	18
$\|C_{Q_{18}}(CL_\alpha)\|$	36	18	18	36	18	18	2
Φ_1							0
Φ_2							0
Φ_3	$2Ar(C_{18})$						0
Φ_4							0
Φ_5							0
Φ_6							0
Φ_7	9	0	0	9	0	0	1

=

Γ-Classes	[1]	[x^6]	[x^2]	[x^9]	[x^3]	[x]	[y]
$\|CL_\alpha\|$	1	2	2	1	2	2	18
$\|C_{Q_{18}}(CL_\alpha)\|$	36	18	18	36	18	18	2
Φ_1	36	0	0	0	0	0	0
Φ_2	12	12	0	0	0	0	0
Φ_3	4	4	4	0	0	0	0
Φ_4	18	0	0	18	0	0	0
Φ_5	6	6	0	6	6	0	0
Φ_6	2	2	2	2	2	2	0
Φ_7	9	0	0	9	0	0	1

Table (3.8)

Theorem 3.8 [4] : Let H be a cyclic subgroup of G and $h_1, h_2, \cdots, h_m$ are chosen representatives for the m-conjugate classes of H contained in $CL(g)$ in G, then :

$$\phi'(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_1) & \text{if} \quad h_i \in H \cap CL(g) \\ \\ 0 & \text{if} \quad H \cap CL(g) = \phi. \end{cases}$$

Proposition 3.9 [3] : The number of all distinct Artin characters on group G is equal to the number of Γ-classes on G. Furthermore, Artin characters are constant on each Γ-classes.

4. The Main Results

In this section we give the general form of Artin's characters table of the group ($Q_{2m} \times C_5$), When $m = p^2, p > 2$, p is prime number. The group ($Q_{2m} \times C_5$) is the direct product group of the quaternion group Q_{2m} of order **4m** and the cyclic group C_5 of order 5, then the order of The group ($Q_{2m} \times C_5$) is **20m**.

Example 4.1 : Let $m = 9 = 3^2$ then $(Q_{2m} \times C_5) = (Q_{2.9} \times C_5) = (Q^2_{2.3} \times C_5) =$ $\{(1,I),(1,z),(1,z^2),(1,z^3),(1,z^4),(x,I),(x,z),(x,z^2),(x,z^3),(x,z^4)(x^2,,I),(x^2,z),(x^2,z^2),$ $(x^2,z^4),\cdots,(x^{17},I),(x^{17},z),(x^{17},z^2),(x^{17},z^3),(x^{17},z^4),,(y_I),(y,z),(y,z^2),(y,z^3),(y,z^4),$ $(xy,I),(xy,z),(xy,z^2),(xy,z^3),(Xy,z^4),(x^2y,I),(x^2y,z),(x^2y,z^2),(x^2y,z^3),(x^2y,z^4),$ $\cdots,(x^{17}y,I),(x^{17}y,z),(x^{17}y,z^2),(x^{17}y,z^3),(x^{17}y,z^4)\}$.

To find Artin's characters for this group, there are 14 cyclic subgroups, which are : $\langle 1,1\rangle,\langle x^6,I\rangle,\langle x^2I\rangle,\langle x^9,I\rangle,\langle x^3,I\rangle,\langle x,I\rangle,\langle y,I\rangle,\langle 1,z\rangle,\langle x^6,z\rangle,\langle x^2,z\rangle,\langle x^9,z\rangle,\langle x^3,z\rangle,\langle x,z\rangle,\langle y,z\rangle$, then there are 14 Γ-Classes, we have 14 distinct Artin's characters.

Let $g \in (Q_{10} \times C_5), g = (q,I)$ or $g = (q,z), q \in Q_{10}, I, z \in C_5$ and let φ the principal character of H, Φ_j Artin characters of $Q_{10}, 1 \leq j \leq 7$, then by using Theorem 3.8

$$\phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_1) & \text{if} \quad h_i \in H \cap CL(g) \\ \\ 0 & \text{if} \quad H \cap CL(g) = \phi. \end{cases}$$

Case I : If H is a cyclic subgroup of $(Q_{2m} \times \{I\})$, then:

$H_1 = \langle 1, I\rangle$.

If $g = (1,I), \Phi_{(1,1)}((1,I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}\varphi((I,1)) = \frac{180}{1}.1 = 180 = 5.36 = 5.\Phi(1)$

since $H \cap CL(g) = \{(1,I)\}$ and $\varphi(g) = 1$.

Otherwise $\Phi_{(1,1)}(g) = 0$ since $H \cap CL(g) = \phi$. $H_2 = \langle x^6, I\rangle$.

If $g = (1,I), \Phi_{(2,1)}(g) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}\varphi(g) = \frac{180}{30}.1 = 60 = 5.12 = 5.\Phi_2(1)$

since $H \cap CL(g) = \{(1,I)\}$ and $\varphi(g) = 1$. If $g = (x^6,I), \Phi_{(2,1)}((x^6,I)) = \frac{|C_{Q_{18}}\times C_5(g)}{|C_H(g)|}(\varphi(g)+$ $\varphi(g^{-1})) = \frac{90}{30}(1+1) = 60 = 5.12 = 5.\Phi_2(x^6)$ since $H \cap CL(g) = \{(g,g^{-1})\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(2,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_3 = \langle x^2, I\rangle$.

If $g = (1,I)$, $\Phi_{(3,1)}((1,I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{9}.1 = 20 = 5.4 = 5.\Phi_3(1)$ since $H \cap CL(g) = \{(1,I)\}$ and $\varphi(g) = 1$.

If $g = (x^6,I), \Phi_{(3,1)}((x^6,I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})) = \frac{90}{9}(1+1) = 20 = 5.4 =$ $5.\varphi_3(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^2,I), \Phi_{(3,1)}((x^2,I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})) = \frac{90}{9}(1+1) = 20 = 5.4 =$ $5.\varphi_3(x^2)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(3,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_4 = \langle x^9, I\rangle$

If $g = (1, I)$, $\Phi_{(4,1)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{2}.1 = 90 = 5.18 = 5.\Phi_4(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g = (x^9, I), \Phi_{(4,1)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{2}.1 = 90 = 5.18 = 5.\varphi_4(x^9)$ since $H \cap CL(g) = \{x^9, I)\}$ and $\varphi(g) = 1$.

Otherwise $\Phi_{(4,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_5 = \langle x^3, I\rangle$

$g = (1, I)$, $\Phi_{(5,1)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{6}.1 = 30 = 5.6 = 5.\Phi_5(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g = (x^6, I), \Phi_{(5,1)}((x^6, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1}) = \frac{90}{6}(1 + 1) = 30 = 5.6 = 5.\varphi_5(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1} = 1$.

If $g = (x^9, I), \Phi_{(5,1)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{6} = 30 = 5.6 = 5.\varphi_5(x^9)$ since $H \cap CL(g) = \{g)\}$ and $\varphi(g) = 1$.

If $g = (x^3, I), \Phi_{(5,1)}((x^3, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{6}(1 + 1) = 30 = 5.6 = 5.\varphi_5(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(5,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_6 = \langle x, I\rangle$

If $g = (1, I)$, $\Phi_{(6,1)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{18}.1 = 10 = 5.2 = 5.\Phi_6(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g = (x^6, I), \Phi_{(6,1)}((x^6, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1}) = \frac{90}{18}(1 + 1) = 10 = 5.2 = 5.\varphi_6(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1} = 1$.

If $g = (x^2, I), \Phi_{(6,1)}((x^2, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) + \varphi(g^{-1}) = \frac{90}{18}(1 + 1) = 10 = 5.2 = 5.\varphi_6(x^2)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^9, I), \Phi_{(6,1)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{18}.1 = 10 = 5.2 = 5.\varphi_6(x^9)$ since $H \cap CL(g) = \{(x^9, I)\}$ and $\varphi(g) = 1$.

If $g = (x^3, I), \Phi_{(6,1)}((x^3, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) + \varphi(g^{-1}) = \frac{90}{18}(1 + 1) = 10 = 5.2 = 5.\Phi_6(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x, I), \Phi_{(6,1)}((x, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{18}(1 + 1) = 10 = 5.2 = 5.\Phi_6(x)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(6,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_7 = \langle y, I\rangle$

If $g = (1, I)$, $\Phi_{(7,1)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{4}.1 = 45 = 5.9 = 5.\Phi_7(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g=(x^9,I), \Phi_{(7,1)}((x^9,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{4}.1=45=5.9=5.\Phi_7(x^9)$ since $H\cap CL(g)=\{(x^9,I)\}$ and $\varphi(g)=1$.

If $g=(y,I), \Phi_{(7,1)}((y,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))+\varphi(g^{-1})=\frac{10}{4}(1+1)=5=5.1=5.\Phi_7(y)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$.

Otherwise $\Phi_{(7,1)}(g)=0$ since $H\cap CL(g)=\phi$.

Case II :If H is a cyclic subgroup of $(Q_{2m}\times\{z\})$, then :

$H_1=\langle 1,z\rangle$

If $g=(1,I), \Phi_{(1,2)}((1,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{5}.1=36=\Phi_1(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$.

If $g=(1,z), \Phi_{(1,2)}((1,z))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{5}.1=36=\Phi_1(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$.

Otherwise $\Phi_{(1,2)}(g)=0$ since $H\cap CL(g)=\phi$.

$H_2=\langle x^6,z\rangle$

If $g=(1,I), \Phi_{(2,2)}((1,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{15}.1=12=\Phi_2(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$.

If $g=(1,z), \Phi_{(2,2)}((1,z))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{15}.1=12=\Phi_2(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$.

If $g=(x^6,I), \Phi_{(3,2)}((x^6,I))=(\varphi(g))\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{90}{45}.(1+1)=4=\Phi_3(x^6)$ since $H\cap CL(g)==\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$.

If $g=(x^6,z), \Phi_{(3,2)}((x^6,z))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{90}{45}.(1+1)=4=\Phi_3(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$.

Otherwise $\Phi_{(2,2)}(g)=0$ since $H\cap CL(g)=\phi$.

$H_3=\langle x^2,z\rangle$

If $g=(1,I), \Phi_{(3,2)}((1,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{45}.1=4=\Phi_3(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$.

If $g=(1,z), \Phi_{(3,2)}((1,z))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{180}{45}.1=4=\Phi_3(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$.

If $g=(x^6,I), \Phi_{(3,2)}((x^6,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{180}{45}.1=4=\Phi_3(x^6)$ since $H\cap CL(g)==\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$.

If $g=(x^6,z), \Phi_{(3,2)}((x^6,z))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{180}{45}.1=4=\Phi_3(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$.

If $g=(x^2,I), \Phi_{(3,2)}((x^2,I))=\frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{90}{45}.(1+1)=4=\Phi_3(x^2)$

since $H \cap CL(g) == \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
If $g = (x^2, z), \Phi_{(3,2)}((x^2, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{45}.(1+1) = 4 = \Phi_3(x^2)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
Otherwise $\Phi_{(3,2)}(g) = 0$ since $H \cap CL(g) = \phi$.
$H_4 = \langle x^9, z\rangle$
If $g = (1, I), \Phi_{(4,2)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{10}.1 = 18 = \Phi_4(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.
If $g = (1, z), \Phi_{(4,2)}((1, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{10}.1 = 18 = \Phi_4(1)$ since $H \cap CL(g) = \{(1, z)\}$ and $\varphi(g) = 1$.
If $g = (x^9, I), \Phi_{(4,2)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{10}.1 = 18 = \Phi_4(x^9)$ since $H \cap CL(g) = \{(x^9, I)\}$ and $\varphi(g) = 1$.
If $g = (x^9, z), \Phi_{(4,2)}((x^9, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{10}.1 = 18 = \Phi_4(x^9)$ since $H \cap CL(g) = \{(x^9, z)\}$ and $\varphi(g) = 1$.
Otherwise $\Phi_{(4,2)}(g) = 0$ since $H \cap CL(g) = \phi$.
$H_5 = \langle x^3, z\rangle$
If $g = (1, I), \Phi_{(5,2)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{30}.1 = 6 = \Phi_5(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.
If $g = (1, z), \Phi_{(5,2)}((1, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{30}.1 = 6 = \Phi_5(1)$ since $H \cap CL(g) = \{(1, z)\}$ and $\varphi(g) = 1$.
If $g = (x^6, I), \Phi_{(6,2)}((x^6, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1+1) = 2 = \Phi_6(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
If $g = (x^6, z), \Phi_{(6,2)}((x^6, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1+1) = 2 = \Phi_6(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
If $g = (x^9, I), \Phi_{(5,2)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{30}(1) = 6 = \Phi_5(x^9)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.
If $g = (x^9, z), \Phi_{(5,2)}((x^9, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{30}(1) = 6 = \Phi_5(x^9)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.
If $g = (x^3, I), \Phi_{(5,2)}((x^3, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + vp(g^{-1})) = \frac{90}{30}(1+1) = 6 = \Phi_5(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
If $g = (x^3, z), \Phi_{(5,2)}((x^3, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{30}(1+1) = 6 = \Phi_5(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
Otherwise $\Phi_{(5,2)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_6 = \langle x, z\rangle$

If $g = (1, I), \Phi_{(6,2)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{90}.1 = 2 = \Phi_6(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g = (1, z), \Phi_{(6,2)}((1, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{90}.1 = 2 = \Phi_6(1)$ since $H \cap CL(g) = \{(1, z)\}$ and $\varphi(g) = 1$.

If $g = (x^6, I), \Phi_{(6,2)}((x^6, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{180}{90}(1 + 1) = 2 = \Phi_6(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^6, z), \Phi_{(6,2)}((x^6, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{180}{90}(1 + 1) = 2 = \Phi_6(x^6)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^2, I), \Phi_{(6,2)}((x^2, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x^2)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = (g^{-1}) = 1$.

If $g = (x^2, z), \Phi_{(6,2)}((x^2, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x^2)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^9, I), \Phi_{(6,2)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{90}(1) = 2 = \Phi_6(x^9)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

If $g = (x^9, z), \Phi_{(6,2)}((x^9, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{90}(1) = 2 = \Phi_6(x^9)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

If $g = (x^3, I), \Phi_{(6,2)}((x^3, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x^3, z), \Phi_{(6,2)}((x^3, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x^3)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x, I), \Phi_{(6,2)}((x, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

If $g = (x, z), \Phi_{(6,2)}((x, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{90}{90}(1 + 1) = 2 = \Phi_6(x)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(6,2)}(g) = 0$ since $H \cap CL(g) = \phi$.

$H_7 = \langle y, z\rangle$

If $g = (1, I), \Phi_{(7,2)}((1, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{20}.1 = 9 = \Phi_7(1)$ since $H \cap CL(g) = \{(1, I)\}$ and $\varphi(g) = 1$.

If $g = (1, z), \Phi_{(7,2)}((1, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{20}.1 = 9 = \Phi_7(1)$ since $H \cap CL(g) = \{(1, z)\}$ and $\varphi(g) = 1$.

If $g = (x^9, I), \Phi_{(7,2)}((x^9, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)) = \frac{180}{20}(1) = 9 = \Phi_7(x^9)$ since $H \cap$

$CL(g) = \{g\}$ and $\varphi(g) = 1$.
If $g = (x^9, z), \Phi_{(7,2)}((x^9, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}\varphi(g) = \frac{180}{20}(1) = 9 = \Phi_7(x^9)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.
If $g = (y, I), \Phi_{(7,2)}((y, I)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{10}{20}(1+1) = 1 = \Phi_7(y)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = (g^{-1}) = 1$.
If $g = (y, z), \Phi_{(7,2)}((y, z)) = \frac{|C_{Q_{18}\times C_5}(g)|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{10}{20}(1+1) = 1 = \Phi_7(y)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.
Otherwise $\Phi_{(7,l2)}(g) = 0$ since $H \cap CL(g) = \phi$. Then the Artin characters table of $(Q_{18} \times C_5)$ is given in the following table.

$Ar(Q_{18} \times C_5)=$

Γ- classes	$[1,I]$	$[x^6,I]$	$[x^2,I]$	$[x^9,I]$	$[x^3,I]$	$[x,I]$	$[y,I]$	$[1,z]$	$[x^6,z]$	$[x^2,z]$	$[x^9,z]$	$[x^3,z]$	$[x,z]$	$[y,z]$
$\|CL_\alpha\|$	1	2	2	1	2	2	18	1	2	2	1	2	2	18
$\|Q_{18}\times C_5\,(CL_\alpha)\|$	180	90	90	180	90	90	10	180	90	90	180	90	90	10
$\Phi_{(1,1)}$	180	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	60	60	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	20	20	20	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	90	0	0	90	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	30	30	0	30	30	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	10	10	10	10	10	10	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	45	0	0	45	0	0	5	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	36	0	0	0	0	0	0	36	0	0	0	0	0	0
$\Phi_{(2,2)}$	12	12	0	0	0	0	0	12	12	0	0	0	0	0
$\Phi_{(3,2)}$	4	4	4	0	0	0	0	4	4	4	0	0	0	0
$\Phi_{(4,2)}$	18	0	0	18	0	0	0	18	0	0	18	0	0	0
$\Phi_{(5,2)}$	6	6	0	6	6	0	0	6	6	0	6	6	0	0
$\Phi_{(6,2)}$	2	2	2	2	2	2	0	2	2	2	2	2	2	0
$\Phi_{(7,2)}$	9	0	0	9	0	0	1	9	0	0	9	0	0	1

Table(4,1)

Theorem 4.2 : The Artin's character table of the group (Q_{2m}, C_5) where $m = p^2$, $p > 2$, p is primenumber; is given as

$Ar(Q_{2m}\times C_5)=$

Γ- classes	$[1,I]$	$[x^{2p},I]$	$[x^2,I]$	$[x^{p^2},I]$	$[x^p,I]$	$[x\ \ ,I]$	$[y,I]$	$[1,z]$	$[x^{2p},z$	$[x^2,z]$	$[x^{p^2},z]$	$[x^p,z]$	$[x,z]$	$[y,z]$
$\lvert CL_\alpha \rvert$	1	2	2	1	2	2	$2p^2$	1	2	2	1	2	2	$2p^2$
$\lvert Q_{18}\times C_5\,(CL_\alpha)\rvert$	$20p^2$	$10p^2$	$10p^2$	$20p^2$	$10p^2$	$10p^2$	10	$20p^2$	$10p^2$	$10p^2$	$20p^2$	$10p^2$	$10p^2$	10
$\Phi_{(1,1)}$														
$\Phi_{(2,1)}$														
$\Phi_{(3,1)}$														
$\Phi_{(4,1)}$			$5Ar(Q_{2p^2})$								$5Ar(Q_{2p^2})$			
$\Phi_{(5,1)}$														
$\Phi_{(6,1)}$														
$\Phi_{(7,1)}$														
$\Phi_{(1,2)}$														
$\Phi_{(2,2)}$														
$\Phi_{(3,2)}$			$Ar(Q_{2p^2})$					$Ar(Q_{2p^2})$						
$\Phi_{(4,2)}$														
$\Phi_{(5,2)}$														
$\Phi_{(6,2)}$														
$\Phi_{(7,2)}$														

Table (4.2)
Which is [14]×[14] matrix square.

Proof : Let $g \in (Q_{2p^2} \times C_5); g = (q, I)$ or $g = q(z), q \in Q_{2m}, I, z \in C_5$.
Case I : If H is a cyclic subgroup of $(Q_{2p^2} \times \{I\})$,then:

$$1 - H = \langle (x, I)\rangle \quad 2 - H = \langle (y, I)\rangle$$

and φ the principal character of H, Φ_j Artin characters of $Q_{2p^2}, 1 \leq j \leq l+1$ then by using theorem 3.8

$$\phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & \text{if} \quad h_i \in H \cap CL(g) \\ \\ 0 & \text{if} \quad H \cap CL(g) = \phi. \end{cases}$$

(i) **If** $g = (1, I)$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2p^2}\times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{20p^2}{|C_H((1,I))|}.1 = \frac{5|Q_{2p^2}(1)}{|C_{\langle x\rangle(1)}|} = 5\Phi_j(1)$$

since $H \cap CL(1, I) = \{(1, I)\}$ and $\varphi(g) = 1$.
(ii) **If** $g = x^{p^2}, I), g \in H$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2p^2}\times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{20p^2}{|C_H((x^{p^2},1))|}.1 = \frac{5|Q_{2p^2}(x^{p^2})|}{|C_{\langle x\rangle(x^{p^2})}|}.1 = 5\Phi_j(p^2)$$

since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

(iii) **If** $g \neq (x^{p^2}, I)$ **and** $g \in H$

$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})) = \frac{10p^2}{|C_H((g))|}(1+1) = \frac{20p^2}{|C_H(g)|} = \frac{5|Q_{Q_{2p^2}}(q)|}{|C_{\langle x \rangle}(q)|} = 5\Phi_j(q)$ since $H \cap CL(g) = \{g, g^{-1}\}, g = (q, I), q \in Q_{2p^2}, q \neq x^{p^2}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

(iv) **If** $g \notin H$, $\Phi_{(j,1)}(g) = 0 = 5.0 = 5\Phi_j(q)$ since $H \cap CL(g) = \phi$ and $q \in Q_{2p^2}$

$2 - IF\ H = \langle (y, I) \rangle = \{(1, I), (y, I), (y^2, I), (y^3, I)\}$.

(i) If $g = (1, I)$, $\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{20p^2}{4}.1 = 5\Phi_j(1)$ since $H \cap CL(1, I) = \{(1, I)\}$ and $\varphi(g) = 1$.

(ii) **If** $g = (x^{p^2}, I) = (y^2, I)$ **and** $g \in H$

$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}(\varphi(g)) = \frac{20p^2}{4}.1 = 5.p^2 = 5\Phi_{(i+1)}(x^{p^2})$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

(iii) **If** $g \neq (x^{p^2}, I)$ **and** $g \in H$**, i.e.** $\{g = (y, I)$ **or** $g = (y^3, I)$.

$\Phi_{(l+1,1)}(g)\frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}(\varphi(g) + \varphi(g^{-1})) = \frac{10}{4}(1 + 1) = \frac{20}{4} = 5.1 = 5\Phi_{j+1}(y)$ since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(l+1,1)}(g) = 0$ since $H \cap CL(g) = \phi$.

Case II : If H is a cyclic subgroup of $(Q_{2p^2} \times \{z\})$ then:

$$1 - H = \langle (x, z) \rangle \qquad 2 - H = \langle (y, z) \rangle$$

and φ the principal character of H, Φ Artin characters of $Q_{2m}, 1 \leq j \leq l + 1$ then by using Theorem 3.8

$$\phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_i) & \text{if} \quad h_i \in H \cap CL(g) \\ \\ 0 & \text{if} \quad H \cap CL(g) = \phi. \end{cases}$$

$1 - H = \langle x, z \rangle$.

(i) If $g = (1, I), (1, z)$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{20p^2}{|C_H((1, I))|}.1 = \frac{5|Q_{2p^2}(1)}{5|C_{\langle x \rangle (1)}|} = \varphi(1) = \Phi_j(1)$$

since $H \cap CL(g) = \{(1, 1), (1, z)\}$ and $\varphi(g) = 1$.

(ii) **If** $g = (1, I), (1, z), (x^{p^2}, I), (x^{p^2}, z); g \in H$**.**

If $g = (1, I),)(1, z)$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|} = \frac{20p^2}{|C_H((1, I))|}.1 = \frac{5|Q_{2p^2}(1)|}{5|C_{\langle x \rangle (1)}|}\varphi = \Phi_j(1)$$

since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

If $g = (x^{p^2}, I), (x^{p^2}, z)$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{5|Q_{2p^2}|(x^{p^2})|}{5|C_{\langle x \rangle}(x^{p^2})|}\varphi(x^{p^2}) = \Phi_j(x^{p^2})$$

since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

(iii) **If** $g \neq (x^{p^2}, I), (x^{p^2}, z)$ **and** $g \in H$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|} = (\varphi(g)+\varphi(g^{-1})) = \frac{10}{|C_H((g))|}(1+1) = \frac{5|Q_{2p^2}(q)|}{5|C_{\langle x \rangle}(q)|}\varphi(q) = \Phi_j(q)$$

since $H \cap CL(g) = \{g, g^{-1}\}, \varphi(g) = \varphi(g^{-1}) = 1$ and $(g) = (q, z), q \in Q_{2p^2}; q \neq x^{p^2}$.

(iv) **If** $g \notin H$.

$\Phi_{(j,2)}(g) = 0 = \Phi_j(q)$ since $H \cap CL(g) = \phi$ and $q \in Q_{2p^2}$

2-If $H = \langle (y, I) \rangle = \{(1, I), (y, I), (y^2, I), (y^3, I), (1, z), (y, z), (y^2, z), (y^3, z), (1, z^2), (y, z^2), (y^2, z^2), (y^3, z^2)(1, z^3), (y, z^3), (y^2, z^3), (y^3, z^3), (1, z^4), (y, z^4), (y^2, z^4), (y^3, z^4)\}$.

(i) **If** $g = (1, I), (1, z)$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}(\varphi(g) = \frac{20p^2}{20}.1 = p^2 = \Phi_{l+1}(g)$$

since $H \cap CL(g) = \{(1, I), (1, z)\}$ and $\varphi(g) = 1$.

(ii) **If** $g = (y^2, I) = (x^{p^2}, I), (y^2, z), (y^2, z^2), (y^2, z^3), (y^2, z)$ **and** $g \in H$

$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|}\varphi(g) = \frac{20p^2}{20}.1 = p^2 = \Phi_{l+1}(g)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g) = 1$.

(iii) **If** $g \neq (x^{p^2}, I)$ **and** $g \in H$**, i.e.** $g = \{(y, I), (y, z), (y, z^2), (y, z^3), (y, z^4)\}$ or $g = (y^3, I), (y^3, z), (y^3, z^2), (y^3, z^3), (y^3, z^4)\}$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2p^2} \times C_5(g)}|}{|C_H(g)|} = (\varphi(g) + \varphi(g^{-1})) = \frac{10}{20}(1 + 1) = 1 = \Phi_{j+1}(y)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$.

Otherwise $\Phi_{(l+1,2)}(g) = 0$ since $H \cap CL(g) = \phi$.

Example 4.3 : To construct $Ar(Q_{98} \times C_5) = Ar(Q^2_{2.7} \times C_5), p = 7$, we use Theorem 3.5 as the following :

Ar(Q_{98})=

Γ-Classes	[1]	$[x^{14}]$	$[x^2]$	$[x^{49}]$	$[x^7]$	$[x]$	$[y]$
$\vert CL_\alpha \vert$	1	2	2	1	2	2	98
$\vert C_G(CL_\alpha)\vert$	196	98	98	98	98	98	196
Φ_1	196	0	0	0	0	0	0
Φ_2	28	28	0	0	0	0	0
Φ_3	4	4	4	0	0	0	0
Φ_4	98	0	0	98	0	0	0
Φ_5	14	14	0	14	14	0	0
Φ_6	2	2	2	2	2	2	0
Φ_7	49	0	0	49	0	0	1

Table(4,3)

Ar($Q_{98} \times C_5$)

Γ-Classes	[1,I]	$[x^{14},I]$	$[x^2,I]$	$[x^{49},I]$	$[x^7,I]$	$[x,I]$	$[y,z]$	[1,z]	$[x^{14},z]$	$[x^2,z]$	$[x^{49},z]$	$[x^7,z]$	$[x,z]$	$[y,z]$
$\vert CL_\alpha \vert$	1	2	2	1	2	2	98	1	2	2	1	2	2	98
$\vert C_G(CL_\alpha)\vert$	980	490	490	980	490	490	10	980	490	490	980	490	490	10
$\Phi_{(1,1)}$	980	0	0		0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	140	140	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	20	20	20	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	490	0	0	490	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	60	60	0	60	60	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	10	10	10	10	10	10	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	245	0	0	245	0	0	5	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	196	0	0	0	0	0	0	196	0	0	0	0	0	0
$\Phi_{(2,2)}$	28	28	0	0	0	0	0	28	28	0	0	0	0	0
$\Phi_{(3,2)}$	4	4	4	0	0	0	0	4	4	4	0	0	0	0
$\Phi_{(4,2)}$	98	0	0	98	0	0	0	98	0	0	98	0	0	0
$\Phi_{(5,2)}$	14	14	0	14	14	0	0	14	14	0	14	14	0	0
$\Phi_{(6,2)}$	2	2	2	2	2	2	0	2	2	2	2	2	2	0
$\Phi_{(7,2)}$	49	0	0	49	0	0	1	49	0	0	49	0	0	1

Table(4,4)

Then by using Theorem 4.2 Artin characters table of the group $(Q_{98} \times C_5)$ is :

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