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ITERATIVE APPROXIMATION OF A SOLUTION OF A MULTI-VALUED VARIATIONAL-LIKE INCLUSION INVOLVING δ-STRONGLY MAXIMAL $P-\eta-$MONOTONE MAPPING IN REAL HILBERT SPACE

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Abstract

In this paper, we discuss some properties of a strongly $P-\eta-$proximal mapping of a δ-strongly maximal $P-\eta-$monotone mapping and prove that it is single-valued and Lipschitz continuous. Further, we consider a multi-valued variational-like inclusion problem (in short, MVLIP) in real Hilbert space and construct an iterative algorithm for MVLIP. Using strongly $P-\eta-$proximal mapping approach, we prove the existence of solution and discuss the convergence analysis of iterative algorithm for MVLIP. The technique and results presented in this paper can be viewed as extension of the techniques and corresponding results given in [2-8,10,13,14].

1. Introduction

In 1968, Brezis [1] initiated the study of the existence theory of a class of variational

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inequalities later known as variational inclusions, using proximal point mappings due to Moreau [11], have been widely studied in recent years. One of the most interesting and important problems in the theory of variational inclusions is the development of efficient and implementable iterative algorithms. Variational inclusions include variational, quasi-variational, variational-like inequalities as special cases. For application of variational inclusions, we refer to [1,6,9-12]. Various kinds of iterative method have been studied to find the approximate solutions for variational inclusions. Among these method, the proximal mapping method for solving variational inclusions has been widely used by many authors, see for example [2-8,10,13,14].

In 1994, Hassouni and Moudafi [6] introduced and studied a class of variational inclusions and developed a perturbed iterative algorithm for the variational inclusions. In recent years, a number of researchers namely, Chang et al. [2], Ding and Luo [4], Fang and Huang [5], Huang [7], Kazmi [9] and Noor [13] have obtained some important extensions of the results of Hassouni and Moudafi [6]. Recently, Chidume et al. [3], Huang and Fang [8], Kazmi and Khan [10] and Noor [14] have studied existence and convergence analysis of solutions for various classes of variational (-like) inclusions using P--proximal mappings and their generalizations.

Inspired by the work above, in this paper, we discuss some properties of a strongly $P-\eta-$proximal mapping of a δ-strongly maximal $P-\eta-$monotone mapping and prove that it is single-valued and Lipschitz continuous. Further, we consider a multi-valued variational-like inclusion problem (in short, MVLIP) in real Hilbert space and construct an iterative algorithm for MVLIP. Using strongly $P-\eta-$proximal mapping approach, we prove the existence of solution and discuss the convergence analysis of iterative algorithm for MVLIP. The results presented in this paper generalize and improve the results given in [2-8,10,13,14].

2. Strongly $P-\eta-$proximal Mapping

First, we need the following known concepts and results, which shall be used in the sequel.

Definition 2.1 [5, 8-10] : Let $\eta : H \times H \to H$ be a single-valued mapping. Then a multi-valued mapping $M : H \to 2^H$ is said to be:

(i) *η-monotone*, if

$$\langle u-v, \eta(x,y)\rangle \geq 0, \quad \forall \;\; x,y \in H, \quad \forall \; u \in M(x), \quad \forall \; v \in M(y);$$

(ii) strictly η-monotone, if

$$\langle u-v, \eta(x,y)\rangle \geq 0, \quad \forall \;\; x,y \in H, \quad \forall \; u \in M(x), \quad \forall \; v \in M(y),$$

and equality holds if and only if $x = y$.

(iii) *δ-strongly η-monotone*, if there exists a constant $\delta > 0$ such that

$$\langle u-v, \eta(x,y)\rangle \geq \delta\|x-y\|^2, \forall \;\; x,y \in H, \quad \forall \; u \in M(x), \quad \forall \; v \in M(y);$$

(iv) *maximal η-monotone*, if M is η-monotone and $(I+\rho M)(H) = H$ for any $\rho > 0$, where I is an identity mapping.

Definition 2.2 [8-10] : A mapping $\eta : H \times H \to H$ is said to be *ψ-Lipschitz continuous*, if there exists a constant $\psi > 0$ such that

$$\|\eta(x,y)\| \leq \psi\|x-y\|, \quad \forall \;\; x,y \in H.$$

Definition 2.3 [5,8-10] : Let $\eta : H \times H \to H$ and $P : H \to H$ be single-valued mappings. Then a multi-valued mapping $M : H \to 2^H$ is said to be *maximal $P-\eta-$monotone*, if M is η-monotone and $(P+\rho M)(H) = H$ for any $\rho > 0$.

Definition 2.4 [5,8-10] : Let $\eta : H \times H \to H$ and $P : H \to H$ be single-valued mappings. A multi-valued mapping $M : H \to 2^H$ is said to be *δ-strongly maximal $P-\eta$-monotone*, if M is δ-strongly η-monotone and $(P+\rho M)H = H$ for any $\rho > 0$.

Now, we state the following theorem which gives some properties of δ- strongly maximal $P-\eta$-monotone mapping and the proof of the theorem is on similar lines as the Theorem 2.1 is proved in [5,8-10].

Theorem 2.1 [8-10,14] : Let $\eta : H \times H \to H$ be a single-valued mapping and $P : H \to H$ be a strictly η-monotone mapping [see, 9-10]. Let $M : H \to 2^H$ be a δ-strongly maximal $P-\eta$-monotone multi-valued mapping. Then

(a) $\langle u-v, \eta(x,y) \geq 0, \quad \forall \; (v,y) \in Graph(M)$ implies $(u,x) \in Graph(M)$ where $Graph(M) = \{(u,x) \in H \times H : u \in M(x)\}$;

(b) the mapping $(P + \rho M)^{-1}$ is single-valued for all $\rho > 0$.

By Theorem 2.1, we define strongly $P - \eta$-proximal mapping for a δ-strongly maximal $P - \eta$-monotone mapping M as follows:

$$R^{M}_{P,\eta}(z) = (P + \rho M)^{-1}, \quad \forall \quad z \in H, \tag{2.1}$$

where $\rho > 0$ is a constant and $P : H \to H$ is a strictly η-monotone mapping.

Next, we state an important theorem which shows that strongly $P - \eta-$ proximal mapping is Lipschitz continuous.

Theorem 2.2 [8-10,13] : Let $P : H \to H$ be a δ-strongly η-monotone and $\eta : H \times H \to H$ be a ψ-Lipschitz continuous mappings. Let $M : H \to 2^H$ be a δ-strongly maximal $P - \eta$-monotone multi-valued mapping. Then strongly $P - \eta-$proximal mapping $R^{M}_{P,\eta}$ of M is $\frac{\psi}{\kappa+\rho\delta}$-Lipschitz continuous, that is,

$$\|R^{M}_{P,\eta}(x) - R^{M}_{P,\eta}(y)\| \leq \frac{\psi}{\kappa + \rho\delta}\|x - y\|, \quad \forall \quad x, y \in H.$$

3. Multi-valued Variational-like Inclusion Problem

Let H be a real Hilbert space and $CB(H)$ be the family of all nonempty, closed and bounded subsets of H. Let $P, g : H \to H, \eta : H \times H \to H, N : H \times H \times H \to H$ be single-valued and $T, A, S : H \to CB(H)$ be multi-valued mappings. Let $M : H \times H \to 2^H$ be a multi-valued mapping such that for each $z \in H, M(\cdot, z)$ is δ-strongly maximal $P - \eta$-monotone with domain $M(\cdot, z) \cap g(H) \neq \emptyset$. We consider the following multi-valued variational-like inclusion problem (in short, MVLIP):

Find $x \in H, u \in T(x), v \in A(x)$ and $w \in S(x)$ such that

$$0 \in N(u, v, w) + M(g(x), x). \tag{3.1}$$

We remark that for suitable choices of the mappings $g, \eta, M, N, P, T, A, S$ and the space H, MVLIP (3.1) reduces to various classes of variational inclusions as special cases, see for example [4,5,9-14].

We need the following definition, which shall be used in the sequel.

Definition 3.1 : Let $P, g : H \to H$ be single valued and $T, A, S : H \to CB(H)$ be multi-valued mappings. A mapping $N : H \times H \times H \to H$ is said to be:

(i) *s-strongly mixed $P \circ g$-monotone with respect to* T, A and S, if there exists a constant $s > 0$ such that

$$\langle N(u_1, v_1, w_1) - N(u_2, v_2, w_2),\ P \circ g(x) - P \circ g(y)\rangle \geq s\|x - y\|^2,$$

$\forall x, y \in H, u_1 \in T(x), u_2 \in T(y), v_1 \in A(x), v_2 \in A(y), w_1 \in S(x), w_2 \in S(y);$

(ii) (α, β, γ)-*mixed Lipschitz continuous*, if there exist constants $\alpha, \beta, \gamma > 0$ such that

$$\|N(x_1, y_1, z_1) - N(x_2, y_2, z_2)\| \leq \alpha\|x_1 - x_2\| + \beta\|y_1 - y_1\| + \gamma\|z_1 - z_2\|,$$

$\forall\ \ x_1, x_2, y_1, y_2, z_1, z_2 \in H.$

Remark 3.1 : The concept of s-strongly mixed $P \circ g$-monotonicity with respect to T, A and S and (α, β, γ)-mixed Lipschitz continuity of mapping $N(\cdot, \cdot, \cdot)$ are more general than the concepts used in [9-14]. If T is $\epsilon - \mathcal{H}$-Lipschitz continuous, then $s \leq \alpha\epsilon$, where $H(\cdot, \cdot)$ is the Hausdorff metric on $CB(H)$.

4. Iterative Algorithm and Convergence Analysis

First, we prove the following lemma, which will be used in the sequel, is an immediate consequence of the definition of $R^{M(\cdot,x)}_{P,\eta}$.

Lemma 4.1 : (x, u, v, w) with $x \in H, u \in T(x), v \in A(x)$ and $w \in S(x)$ is a solution of MVLIP (3.1) if and only if (x, u, v, w) satisfies the relation

$$g(x) = R^{M(\cdot,x)}_{P,\eta}(P \circ g(x) - \rho N(u, v, w)), \tag{4.1}$$

where $R^{M(\cdot,x)}_{P,\eta} \equiv (P + \rho M)(\cdot, x))^{-1}$, $P \circ g$ denotes P composition g, and $\rho > 0$ is a constant.

Proof : (x, u, v, w) is a solution of MVLIP (3.1) if and only if (x, u, v, w) satisfies

$$\begin{aligned}
&0 \in \rho N(u, v, w) + \rho M(g(x), x) \quad \text{for} \quad \rho > 0\\
&\Leftrightarrow P \circ g(x) \in \rho N(u, v, w) + P \circ g(x) + \rho M(g(x), x)\\
&\Leftrightarrow P \circ g(x) - \rho N(u, v, w) \in (P + \rho M(\cdot, x))g(x)\\
&\Leftrightarrow g(x) = R^{M(\cdot,x)}_{P,\eta}(P \circ g(x) - \rho N(u, v, w)).
\end{aligned}$$

This completes the proof.

By using Lemma 4.1 and Nadler's technique [12], we suggest and analyze the following iterative algorithm for finding the approximate solution of MVLIP (3.1):

Iterative Algorithm 4.1 : For given $x_0 \in H, u_0 \in T(x_0), v_0 \in A(x_0), w_0 \in S(x_0)$ compute an approximate solution (x_n, u_n, v_n, w_n) given by the iterative schemes:

$$x_{n+1} = (1-\lambda)x_n + \lambda\{x_n - g(x_n) + R_{P,\eta}^{M(\cdot,x_n)}(P \circ g(x_n) - \rho N(u_n, v_n, w_n))\}, \quad (4.2)$$

$$u_n \in T(x_n), \quad \|u_{n+1} - u_n\| \leq (1 + (1+n)^{-1})\mathcal{H}(T(x_{n+1}), T(x_n)), \quad (4.3)$$

$$v_n \in A(x_n), \quad \|v_{n+1} - v_n\| \leq (1 + (1+n)^{-1})\mathcal{H}(A(x_{n+1}), A(x_n)), \quad (4.4)$$

$$w_n \in S(x_n), \quad \|w_{n+1} - w_n\| \leq (1 + (1+n)^{-1})\mathcal{H}(S(x_{n+1}), S(x_n)), \quad (4.4)$$

where $n = 0, 1, 2, \cdots, 0 < \lambda < 1$ is a relaxation parameter, and $\rho > 0$ is a constant.

Next, we prove the existence of solution of MVLIP (3.1) and discuss the convergence analysis of Iterative Algorithm 4.1.

Theorem 4.1 : Let $\eta : H \times H \to H$ be ψ-Lipschitz continuous mapping; let $T, A, S : H \to CB(H)$ be $\epsilon - \mathcal{H}$-Lipschitz continuous, $\mu - \mathcal{H}$-Lipschitz continuous and $\xi - \mathcal{H}$-Lipschitz continuous mappings, respectively; let $g : H \to H$ be ν- strongly monotone and σ-Lipschitz continuous mapping; let $P : H \to H$ be κ-strongly η-monotone and $P \circ g$ be ω-Lipschitz continuous mappings; let the mapping $N : H \times H \times H \to H$ be s-strongly mixed $P \circ g$-monotone with respect to T, A and S and (α, β, γ)-mixed Lipschitz continuous. Let the mapping $M : H \times H \to 2^H$ be such that for each fixed $x \in H$, $M(\cdot, x)$ is δ-strongly maximal $P - \eta$-monotone and suppose that there exists $\tau > 0$ such that

$$\|R_{P,\eta}^{M(\cdot,x_1)}(z) - R_{P,\eta}^{M(\cdot,x_2)}(z)\| \leq \tau\|x_1 - x_2\|, \quad \forall\ x_1, x_2, z \in H, \quad (4.6)$$

and $\rho > 0$ satisfies the following condition:

$$\tau + \sqrt{1 - 2\nu + \sigma^2} + \frac{\psi}{\kappa + \rho\delta}\sqrt{\omega^2 - 2\rho s + \rho^2(\alpha\epsilon + \beta\mu + \gamma\xi)^2} < 1. \quad (4.7)$$

Then the sequences $\{x_n\}, \{u_n\}, \{v_n\}$ and $\{w_n\}$ generated by Iterative Algorithm 4.1 converge strongly to $x \in H, u \in T(x), v \in A(x)$ and $w \in S(x)$, respectively, and (x, u, v, w) is a solution of MVLIP (3.1).

Proof : By using Assumption (4.6), Iterative Algorithm 4.1 and Theorem 2.2, we estimate:

$$\begin{aligned}
&\|x_{n+2}-x_{n+1}\| = (1-\lambda)\|x_{n+1}-x_n\| + \lambda\|x_{n+1}-x_n-(g(x_{n+1})-g(x_n))\| \\
&+\lambda\|R_{P,\eta}^{M(\cdot,x_{n+1})}[P\circ g(x_{n+1})-\rho N(u_{n+1},v_{n+1},w_{n+1})] \\
&-R_{P,\eta}^{M(\cdot,x_n)}[p\circ g(x_n)-\rho N(u_n,v_n,w_n)]\| \\
&\le (1-\lambda)\|x_{n+1}-x_n\| + \lambda\|x_{n+1}-x_n-(g(x_{n+1})-g(x_n))\| \\
&+\lambda\|R_{P,\eta}^{M(\cdot,x_{n+1})}[P\circ g(x_{n+1})-\rho N(u_{n+1},v_{n+1},w_{n+1})] \\
&-R_{P,\eta}^{M(\cdot,x_n)}[P\circ g(x_{n+1})-\rho N(u_{n+1},v_{n+1},w_{n+1})]\| \\
&+\lambda\|R_{P,\eta}^{M(\cdot,x_n)}[P\circ g(x_{n+1})-\rho N(u_{n+1},v_{n+1},w_{n+1})] \\
&-R_{P,\eta}^{M(\cdot,x_n)}[P\circ g(x_n)-\rho N(u_n,v_n,w_n)]\| \\
&\le (1-\lambda)\|x_{n+1}-x_n\| + \lambda\|x_{n+1}-x_n-(g(x_{n+1}-g(x_n))\| + \lambda\tau\|x_{n+1}-x_n\| \\
&+\frac{\lambda\psi}{\kappa+\rho\delta}\|P\circ g(x_{n+1})-P\circ g(x_n)-\rho(N(u_{n+1},v_{n+1},w_{n+1})-N(u_n,v_n,w_n))\|. \quad (4.8)
\end{aligned}$$

Since g is ν-strongly monotone and σ-Lipschitz continuous, then we have

$$\begin{aligned}
\|_{n+1}-x_n-(g(x_{n+1}-g(x_n))\|^2 &= \|x_{n+1}-x_n\|^2 - 2\langle g(x_{n+1})-g(x_n), x_{n+1}-x_n\rangle \\
&\quad +\|g(x_{n+1})-g(x_n)\|^2 \\
&\le (1-2\nu+\sigma^2)\|\|x_{n+1}-x_n\|^2. \quad (4.9)
\end{aligned}$$

Since N is s-strongly mixed $P\circ g$-monotone with respect to T, A and S and (α,β,γ)-mixed Lipschitz continuous; T, A and S are $\epsilon-\mathcal{H}$-Lipschitz continuous, $\mu-\mathcal{H}$-Lipschitz continuous and $\xi-\mathcal{H}$-Lipschitz continuous, respectively; $P\circ g$ is ω-Lipschitz continuous, we have the following estimates:

$$\begin{aligned}
&\|N(u_{n+1},v_{n+1},w_{n+1})-N(u_n,v_n,w_n)\| \\
&\le \alpha\|u_{n+1}-u_n\| + \beta\|v_{n+1}-v_n\| + \gamma\|w_{n+1}-w_n\| \\
&\le (1+(1+n)^{-1})[\alpha\mathcal{H}(T(x_{n+1}),T(x_n)) + \beta\mathcal{H}(A(x_{n+1}),A(x_n)) + \gamma\mathcal{H}(S(x_{n+1}),S(x_n))] \\
&\le (1+(1+n)^{-1})(\alpha\epsilon+\beta\mu+\gamma\xi)\|x_{n+1}-x_n\|, \quad (4.10)
\end{aligned}$$

and

$$\begin{aligned}&\|P\circ g(x_{n+1})-P\circ g(x_n)-\rho(N(u_{n+1},v_{n+1},w_{n+1})-N(u_n,v_n,w_n))\|^2\\&=\|P\circ g(x_{n+1})-P\circ g(x_n)\|^2-2\rho\langle N(u_{n+1},v_{n+1},w_{n+1})-N(u_n,v_n,w_n),\\&P\circ g(x_{n+1})-P\circ g(x_n)\rangle+\rho^2\|N(u_{n+1},v_{n+1},w_{n+1})-N(u_n,v_n,w_n)\|^2\\&\le\omega^2\|x_{n+1}-x_n\|^2-\rho s\|x_{n+1}-x_n\|^2+\rho^2(1+(1+n)^{-1})^2(\alpha\epsilon+\beta\mu+\gamma\xi)^2\|x_{n+1}-x_n\|^2\\&=(\omega^2-2\rho s+\rho^2(1+(1+n)^{-1})^2(\alpha\epsilon+\beta\mu+\gamma\xi)^2)\|x_{n+1}-x_n\|^2.\end{aligned}\tag{4.11}$$

From (4.8)-(4.11), we have

$$\begin{aligned}\|x_{n+2}-x_{n+1}\| \quad\le\quad & \Big\{(1-\lambda)+\lambda((1-2\nu+\sigma^2)^{1/2}+\tau+\frac{\psi}{\kappa+\rho\delta}(\omega^2-2\rho s+\\&+\rho^2(1+(1+n)^{-1})^2(\alpha\epsilon+\beta\mu+\gamma\xi)^2)^{1/2})\Big\}\|x_{n+1}-x_n\|.\end{aligned}$$

Hence, we can write

$$\|x_{n+2}-x_{n+1}\|\le(1-\lambda(1-\theta_n)\|x_{n+1}-x_n\|,\tag{4.12}$$

where

$$\theta_n=\tau+\sqrt{1-2\nu+\sigma^2}+\frac{\psi}{\kappa+\rho\delta}\sqrt{\omega^2-2\rho s+\rho^2(1+(1+n)^{-1})^2(\alpha\epsilon+\beta\mu+\gamma\xi)^2}.\tag{4.13}$$

Letting $\theta_n\to\theta$ as $n\to\infty$, where

$$\theta:=\tau+\sqrt{1-2\nu+\sigma^2}+\frac{\psi}{\kappa+\rho\delta}\sqrt{\omega^2-2\rho s+\rho^2(\alpha\epsilon+\beta\mu+\gamma\xi)^2}.\tag{4.14}$$

By condition (4.7), it follows that $\theta\in(0,1)$. Hence $\theta_n<1$ for sufficiently large n. Therefore (4.12) implies that $\{x_n\}$ is a Cauchy sequence in H and hence there exists $x\in H$ such that $x_n\to x$ as $n\to\infty$. By $\epsilon-\mathcal{H}$-Lipschitz continuity of T and Iterative Algorithm 4.1, we have

$$\begin{aligned}\|u_{n+1}-u_n\| \quad&\le\quad (1+(1+n)^{-1})\mathcal{H}(T(x_{n+1}),T(x_n))\\&\le\quad (1+(1+n)^{-1})\epsilon\|x_{n+1}-x_n\|.\end{aligned}\tag{4.15}$$

Since $\{x_n\}$ is a Cauchy sequence in H, it follows from (4.15) that $\{u_n\}$ is a Cauchy sequence in H and hence there exists $u\in H$ such that $\{u_n\}\to u$ as $n\to\infty$. Similarly, Lipschitz continuity of A,S,g implies that $\{v_n\}$, $\{w_n\}$ and $\{g(x_n)\}$ are Cauchy sequences in H and hence there exist v, w and $g(x)$ in H such that $v_n \to v$, $w_n \to w$ and $g(x_n) \to g(x)$ as $n \to \infty$.

Next, we claim that $u \in T(x)$. Since $u_n \in T(x_n)$, we have

$$\begin{aligned} d(u, T(x)) &\leq \|u - u_n\| + d(u_n, T(x_n)) + \mathcal{H}(T(x_n), T(x)) \\ &\leq \|u - u_n\| + \epsilon\|x_n - x\| \to 0 \ \text{ as } n \to \infty \end{aligned}$$

and hence $u \in T(x)$. Similarly, we can show that $v \in A(x)$ and $w \in S(x)$. Further, Lipschitz continuity of the mappings $g, P \circ g$, $N(\cdot, \cdot, \cdot)$, $R_{P,\eta}^{M(\cdot,x)}$, Assumption 4.6 and Iterative Algorithm 4.1 gives that

$$x = (1 - \lambda)x + \lambda[x - g(x) + R_{P,\eta}^{M(\cdot,x)}(P \circ g(x) - \rho N(u, v, w))],$$

that is

$$g(x) = R_{P,\eta}^{M(\cdot,x)}(P \circ g(x) - \rho N(u, v, w)),$$

and hence, from Lemma 4.1, it follows that (x, u, v, w) is a solution of MVLIP (3.1). This completes the proof.

Remark 4.1 : For $\rho > 0$, it is clear that $s \leq \alpha\epsilon; \sigma > \sqrt{2\nu - 1}; 2\rho s < \omega^2$; $\kappa + \rho\delta \neq 0$. Further, $\theta \in (0, 1)$ and condition (4.7) of Theorem 4.1 can be easily verified by giving some suitable values of constants.

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