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FIXED POINT THEOREMS FOR (α, ψ, ξ)-CONTRACTIVE MULTIVALUED MAPPINGS

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Abstract

We prove fixed point theorems for (α, ψ, ξ)-contractive multivalued mappings by changing the contractive conditions which generalize the results of Seong-Hoon Cho [10].

1. Introduction and Preliminaries

In 2012, Samet et al. [1] introduced the notions of $\alpha - \psi$ contractive mapping and α-admissible mappings in metric spaces and obtained corresponding fixed point results, which are generalizations of ordered fixed point results (see [1]). Since then, by using

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their idea, some authors investigated fixed point results in the field. Asl et al.[2] extended some of results in [1] to multivalued mappings by introducing the notions of $\alpha_* - \psi$-contractive mapping and α_*-admissible mapping.

Recently, Salimi et al. [3] modified the notions of $\alpha - \psi$ contractive mapping and α-admissible mappings by introducing another function η and then, they gave generalizations of the results of Samet et al.[1] and Karapinar and Samet [4]. Hussain et al.[5] extended these modified notions to multivalued mappings. That is, they introduced the notion of $\alpha - \eta$ contractive multifunctions and gave fixed point results for these multifunctions.

Very recently, Ali et al.[6] generalized and extended the notion of $\alpha - \psi$ contractive mappings by introducing the notion of (α, ψ, ξ)-contractive multivalued mappings and obtained fixed point theorems for these mappings in complete metric spaces.

Let (X, d) be a metric space. We denote by $CB(X)$ the class of nonempty closed and bounded subsets of X and by $CL(X)$ the class of nonempty closed subsets of X. Let $H(\cdot, \cdot)$ be the generalized Hausdorff distance on $CL(X)$, that is, for all $A, B \in CL(X)$,

$$H(A,B) = \begin{cases} \max\{\sup_{\alpha \in A} d(a,B) \sup_{b \in B} d(b,A), & \text{if the maximum exists,} \\ \\ \infty, & \text{otherwise} \end{cases} \tag{1}$$

where $d(a, B) = \inf\{d(a, b : b \in B\}$ is the distance from point a to subset B. For $A, B \in CL(X)$, let $D(A, B) = \sup_{x \in A} \inf_{y \in B} d(x, y)$. Then, we have $D(A, B) \leq H(A, B)$ for all $A, B \in CL(X)$.

From now on, we denote by

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Tx)d(y,Ty)}{1+d(x,y)}\right\} \tag{2}$$

for a multivalued map $T : X \to CL(X)$ and $x, y \in X$.

We denote by Ξ the class of all functions $\xi : [0, \infty) \to [0, \infty)$ such that

(1) ξ is continuous;

(2) ξ is nondecreasing on $[0, \infty)$;

(3) $\xi(t) = 0$ if and only if $t = 0$;

(4) ξ is subadditive.

Also, we denote by Ψ the family of all nondecreasing functions $\Psi : [0, \infty) \to [0, \infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$, where ψ^n is the n-th iterate of ψ.

Note that if $\Psi \in \Psi$, then $\Psi(0) = 0$ and $0 < \Psi(t) < t$ for all $t > 0$.

Let (X, d) be a metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function.

We consider the following conditions:

(1) For any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \aleph$ and $\lim_{n\to\infty} x_n = x$, we have

$$\alpha(x_n, x) \geq 1 \quad \forall \quad n \in \aleph \tag{3}$$

(2) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \aleph$ and a cluster point x of $\{x_n\}$, we have

$$\lim_{n\to\infty} \inf \alpha(x_n, x) \geq 1; \tag{4}$$

(3) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \aleph$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_n(k)\}$ of $\{x_n\}$ such that

$$\alpha(x_{n(k)}, x) \geq 1 \quad \forall \quad k \in \aleph. \tag{5}$$

Remark 1 : (1) imples (2) and (2) implies (1).

Note that if (X, d) is a metric space and $\xi \in \Xi$, then $(X, \xi \circ d)$ is a metric space.

Let (X, d) be a metric space, and let $T : X \to CL(X)$ be a multivalued mapping. Then, we say that

(1) T is called α_*-admissible [2] if

$$\alpha(x, y) \geq 1 \text{ implies } \alpha_*(Tx, Ty) \geq 1, \tag{6}$$

where $\alpha_*(Tx, Ty) = \inf\{\alpha(a, b) : a \in Tx, b \in Ty\}$;

(2) T is called α-admissible [7] if, for each $x \in X$ and $y \in Tx$ with $\alpha(x, y) \geq 1$, we have $\alpha(y, z) \geq 1$ for all $z \in Ty$.

Lemma 1.1 (see[10]) : Let (X, d) be a metric space and let $T : X \to CL(X)$ be a multivalued mapping. If T is α_*-admissible, then it is α-admissible.

Proof : Suppose that T is α_*-admissible mapping.

Let $x \in X$ and $y \in Tx$ be such that $\alpha(x,y) \geq 1$.

Let $z \in Ty$ be given.

Since T is α_*-admissible, $\alpha(y,z) \geq \alpha_*(Tx,Ty) \geq 1$.

Lemma 1.2 (see[10]) : Let (X,d) be a metric space and let $\xi \in \Xi$ and $B \in CL(X)$. If $a \in X$ and $\xi(d(a,B)) < c$, then there exists $b \in B$ such that $\xi(d(a,b)) < c$.

Proof : Let $\epsilon = c - \xi(d(a,B))$.

Since $\xi(d(a,B)) < c$ and $\xi \circ d$ is metric on X, there exists $b \in B$ such that $\xi(d(a,b)) < \xi(d(a,B)) + \epsilon$ by definition of infimum. Hence, $\xi(d(a,b)) < c$.

Let (X,d) be a metric space.

A function $f : X \to [0,\infty)$ is called upper semicontinuous if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim\limits_{n\to\infty} x_n = x$, we have $\lim\limits_{n\to\infty} f(x_n) \leq f(x)$.

A function $f : X \to [0,\infty)$ is called lower semicontinuous if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim\limits_{n\to\infty} x_n = x$, we have $f(x) \leq \lim\limits_{n\to\infty} f(x_n)$.

For a multivalued map $T : X \to CL(X)$, let $f_T : X \to [0,\infty)$ be a function defined by $f_T(x) = d(x,Tx)$.

2. Fixed Point Theorems

Theorem 2.1 (see[10]) : Let (X,d) be a complete metric space and let $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.

Assume that, for all $x,y \in X, \alpha(x,y) \geq 1$ implies

$$\xi(H(Tx,Ty)) \leq \psi(\xi(\xi(M(x,y))) + L\xi(d(y,Tx))$$

where

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}\{d(x,Ty) + d(y,Tx)\}\right\}$$

where $L \geq 0, \xi \in \Xi$ and $\psi \in \Psi$ is strictly increasing.

Also, suppose that the following are satisfied:

(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1) \geq 1$;

(2) either T is continuous or f_T is lower semicontinuous.

Then T has a fixed point in X.

Theorem 2.2 : Let (X, d) be a complete metric space and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible. Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \tag{7}$$

$$M(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Tx)d(y, Ty)}{1 + d(x, y)}\right\}$$

where $L \geq 0, \xi \in \Xi$ and $\psi \in \Psi$ is strictly increasing.

Also, suppose that the following are satisfied:

(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;

(2) either T is continuous or f_T is lower semicontinuous.

Then T has a fixed point in X.

Proof : Let $x_0 \in X$ and $x_1 \in Tx_0$ be such that $\alpha(x_0, x_1) \geq 1$. Let c be a real number with $\xi(d(x_0, x_1)) < \xi(c)$.

If $x_0 = x_1$, then x_1 is a fixed point. Let $x_0 \neq x_1$.

If $x_1 \in Tx_1$, then x_1 is a fixed point.

Let $x_1 \notin Tx_1$. Then $d(x_1, Tx_1) > 0$.

From (7) we obtain

$$\begin{aligned}
0 &\leq \xi(d(x_1, Tx_1)) \\
&\leq \xi(H(Tx_0, Tx_1) \\
&\leq \psi(\xi(\max\{d(x_0, x_1), d(x_0, Tx_0), d(x_1, Tx_1), \frac{d(x_0, Tx_0) \cdot d(x_1, Tx_1)}{1 + d(x_0, x_1)}\})) + L\xi(d(x_0, x_1)) \\
&\leq \psi(\xi(\max\{d(x_0, x_1), d(x_0, x_1), d(x_1, Tx_1), \frac{d(x_0, x_1) \cdot d(x_1, Tx_1)}{1 + d(x_0, x_1)}\})) + L\xi(d(x_1, x_1)) \\
&\leq \psi(\xi(\max\{d(x_0, x_1), d(x_1, Tx_1)\})). \qquad (8)
\end{aligned}$$

If $\max\{d(x_0, x_1), d(x_1, Tx_1)\} = d(x_1, Tx_1)$, then we have

$$0 < \xi(d(x_1, Tx_1)) \leq \psi(\xi(d(x_1, Tx_1))) < \xi(d(x_1, Tx_1))$$

which is a contraction.

Thus, $\max\{d(x_0,x_1),d(x_1,Tx_1)\}=d(x_0,x_1)$, and hence we have

$$0<\cdots<\psi(\xi(c)) \tag{9}$$

Hence,there exists $x_2\in Tx_1$ such that

$$\xi(d(x_1,x_2))<\psi(\xi(c)) \tag{10}$$

Since T is α-admissible, from condition (1)and $x_2\in Tx_1$, we have

$$\alpha(x_1,x_2)\geq 1. \tag{11}$$

If $x_2\in Tx_2$, then x_2 is a fixed point. Let $x_2\notin Tx_2$.
Then $d(x_2,Tx_2)>0$ and so $\xi(d(x_2,Tx_2))>0$.
From (7) we obtain

$$\begin{aligned}
0 &< \xi(d(x_2,Tx_2))\\
&\leq \xi(H(Tx_1,Tx_2)\\
&\leq \psi(\xi(\max\{d(x_1,x_2),d(x_1,Tx_1),d(x_2,Tx_2),\frac{d(x_1,Tx_1)\cdot d(x_2,Tx_2)}{1+d(x_1,x_2)}\}))\leq\cdots+L\xi(d(x_2,x_1))\\
&\leq \psi(\xi(\max\{d(x_1,x_2),d(x_1,x_2),d(x_2,Tx_2),\frac{d(x_1,x_2)\cdot d(x_2,Tx_2)}{1+d(x_1,x_2)}\}))+L\xi(d(x_2,x_2))\\
&\leq \psi(\xi(\max\{d(x_1,x_2),d(x_2,Tx_2)\})).
\end{aligned} \tag{12}$$

If $\max\{d(x_1,x_2),d(x_2,Tx_2)\}=d(x_2,Tx_2)$, then we have

$$\xi(d(x_2,Tx_2))\leq\psi(\xi(d(x_2,Tx_2)))<\xi(d(x_2,Tx_2))$$

which is a contraction.
Thus, $\max\{d(x_1,x_2),d(x_2,Tx_2)\}=d(x_1,x_2)$, and hence we have

$$\xi(d(x_2,Tx_2))\leq\psi(\xi(d(x_1,x_2)))<\psi^2(\xi(c)) \tag{13}$$

Hence, there exists $x_3\in Tx_2$ such that

$$\xi(d(x_2,x_3))<\psi^2(\xi(c)) \tag{14}$$

Since T is α-admissible, from $x_3\in Tx_2$, we have

$$\alpha(x_2,x_3)\geq 1 \tag{15}$$

By induction, we obtain a sequence $\{x_n\} \subset X$ such that, for all $n \in \aleph \cup \{0\}$, $\alpha(x_n, x_{n+1}) \geq 1$, $x_{n+1} \in Tx_n, x_n \neq x_{n+1}$

$$\xi(d(x_n, x_{n+1})) < \psi^n(\xi(c)). \tag{16}$$

Let $\epsilon > 0$ be given.
Since $\sum_{n=0}^{\infty} \psi^n(\xi(ep)) < \xi(\epsilon)\infty$, there exists $N \in \aleph$ such that

$$\sum_{n \geq N} \psi^n(\xi(c)) < \xi(\epsilon). \tag{17}$$

For all $m > n \geq N$, we have

$$\xi(d(x_n, x_m)) \leq \sum_{k=n}^{m-1} \psi^k(\xi(c)) < \sum_{n \geq N} \psi^n(\xi(c)) < \xi(\epsilon) \tag{18}$$

which implies $d(x_n, x_m) < \epsilon,\ \forall\ m > n \geq N$. Hence $\{x_n\}$ is a Cauchy sequence in X. It follows from the completeness of X that there exists

$$x_* = \lim_{n \to \infty} x_n \in X. \tag{19}$$

Suppose that T is continuous. We have

$$d(x_*, Tx_*) \leq d(x_*, x_{n+1}) + d(x_{n+1}, Tx_*) \leq d(x_*, x_{n+1}) + H(x_n, Tx_*). \tag{20}$$

By letting $n \to \alpha$ in the above inequality, we obtain $d(x_*, Tx_*) = 0$ and so $x_* \in Tx_*$. Assume that f_T is lower semicontinous. Then, $f_T(x_*) \leq \lim_{n \to \alpha} f_T(x_n)$. Hence

$$d(x_*, Tx_*) \leq \lim_{n \to \infty} d(x_n, Tx_n) \leq \lim_{n \to \infty} d(x_n, x_{n+1}) = 0.$$

Thus, $x_* \in Tx_*$. □

Corollary 2.1 : Let (X, d) be a complete metric space and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible. Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(\alpha(x, y))(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \qquad ((21)$$

$$M(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Tx)(d(y, Ty))}{1 + d(x, y)}\right\}$$

where $L \geq 0$, $\xi \in \Xi$ and $\psi \in \Psi$ is strictly increasing. Also, suppose that conditions (1) and (2)of Theorem 2.2 are satisfied.

Then T has a fixed point in X.

Remark 2.1 : If we have $\xi(t) = t$ for all $t > 0, L = 0,$

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}\{d(x,Ty) + d(y,Tx)\}\right\}$$

and T is continuous, then Corollary 2.1 reduces to Theorem 3.4 of [7].

Theorem 2.3 (see [10]) : Let (X,d) be a complete metric space and let $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.

Assume that, for all $x, y \in X, \alpha(x,y) \geq 1$ implies

$$\xi(H(Tx,Ty)) \leq \psi(\xi(M(x,y))) + L\xi(d(y,Tx)) \tag{22}$$

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}\{d(x,Ty) + d(y,Tx)\}\right\}$$

where $L \geq 0, \xi \in \Xi, \psi \in \Psi$ and is strictly increasing and upper semicontinuous function. Also, suppose that the following are satisfied.

(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;

(2) for a sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ forall $n \in \aleph \cup \{0\}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that, for all, $k \in \aleph \cup \{0\}$,

$$\alpha(x_{n(k)}, x) \geq 1. \tag{23}$$

Then T has a fixed point in X.

Theorem 2.4 : Let (X,d) be a complete metric space and let $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.

Assume that, for all $x, y \in X, \alpha(x,y) \geq 1$ implies

$$\xi(H(Tx,Ty)) \leq \psi(\xi(M(x,y))) + L\xi(d(y,Tx)) \tag{24}$$

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Tx) \cdot d(y,Ty)}{1 + d(x,y)}\right\}$$

where $L \geq 0, \xi \in \Xi, \psi \in \Psi$ is strictly increasing and upper semicontinuous function. Also, suppose that the following are satisfied.

(1) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;

(2) for a sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ forall $n \in \aleph \cup \{0\}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that, for all $k \in \aleph \cup \{0\}$,

$$\alpha(x_{n(k)}, x) \geq 1. \tag{25}$$

Then T has a fixed point in X.

Proof : Following the proof of Theorem 2.2, we obtain a sequence $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x_* \in X$ such that, for all $n \in \aleph \cup \{0\}$,

$$x_{n+1} \in Tx_n, x_n \neq x_{n+1}, \alpha(x_n, x_{n+1}) \geq 1. \tag{26}$$

From (2) there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that

$$(x_{n(k)}, x_*) \geq 1. \tag{27}$$

Thus, we have

$$\xi(d(x_{n(k)+1}, Tx_*)) = \xi(H(T_{xn(k)}, Tx_*)) \leq \psi(\xi(M(x_{n(k)}, x_*))) + L\xi(d(x_*, x_{n(k)+1})) \qquad ((28)$$

where

$$M(x_{n(k)}, x_*) = \max\left\{d(x_{n(k)}, x_*), d(x_{n(k)}, x_{n(k)+1}), d(x_*, Tx_*), \frac{d(x_{n(k)}, x_{n(k)+1}) \cdot d(x_*, Tx_*)}{1 + d(x_{n(k)}, x_*)}\right\} \tag{29}$$

we have

$$\lim_{k\to\infty} M(x_{n(k)}, x_*) = d(x_*, Tx_*) \tag{30}$$

and so

$$\lim_{k\to\infty} \xi(M(x_{n(k)}, x_*)) = \xi(d(x_*, Tx_*)). \tag{31}$$

Suppose that $d(x_*, Tx_*) \neq 0$.

Since ψ is upper semicontinuous.

$$\lim_{k\to\infty} \psi(\xi(M(x_{n(k)}, x_*))) \leq \psi(\xi(d(x_*, Tx_*))). \tag{32}$$

Letting $k \to \infty$ in inequality (27) and using continuity of ξ, we obtain

$$\begin{aligned} 0 &< \xi(d(x_*, Tx_*)) \\ &\leq \lim_{k\to\infty} \psi(\xi(M(x_{n+k}, x_*))) + \lim_{n\to\infty} L\xi(d(x_*, x_{n(k)+1})) \qquad (33) \\ &\leq \psi\xi(d(x_*, Tx_*)) < \xi(d(x_*, Tx_*)) \end{aligned}$$

which is a contraction. Hence, $d(x_*, Tx_*) = 0$, and hence x_* is a fixed point of T.
The following example shows that upper semicontinuity of ψ cannot be dropped in Theorem 2.4.

Example : Let $X = [0, \infty)$ and let $d(x, y) = |x - y|$ for all $x, y \geq 0$.
Define a mapping $T : X \to CL(X)$ by

$$Tx = \begin{cases} \left\{\frac{2}{3}, 1\right\} & (x = 0), \\ \left\{\frac{5}{6}x\right\} & (0 < x \leq 1), \\ \{3x\} & (x > 1). \end{cases} \tag{34}$$

Let $\xi(t) = t$ for all $t \geq 0$, and let

$$\psi(t) = \begin{cases} \left\{\frac{6}{7}t\right\} & (t \geq 1), \\ \left\{\frac{4}{5}t\right\} & (0 \leq t < 1). \end{cases} \tag{35}$$

Then $\xi \in \Xi$ and $\psi \in \Psi$ is a strictly increasing function.
Let $\alpha, \eta : X \times X \to [0, \infty]$ be defined by

$$\alpha(x, y) = \begin{cases} 6, & 0 \leq x,\ y \leq 1, \\ 0, & \text{otherwise} \end{cases} \tag{36}$$

Obviously condition (2)of Theorem 2.4 is satisfied. Condition (1) of Theorem 2.4 is satisfied with $x_0 = \frac{1}{6}$ we show that (7) is stisfied.
Let $x, y \in X$ be such that $\alpha(x, y) \geq 1$. Then $0 \leq x, y \leq 1$.
If $x = 0$ then obviously (7) is satisfied.
Let $x \neq y$. If $x = 0$ and $0 < y \leq 1$, then we obtain

$$\xi(H(Tx, Ty)) = H(\{\frac{2}{3}, 1\}, \frac{5}{6}y) \leq \frac{1}{6} \leq \psi(d(x, Tx)) \leq \psi(\xi(M(x, y))). \tag{37}$$

Let $0 < x \leq 1$ and $0 < y \leq 1$. Then, we have

$$\begin{aligned} \xi(H(Tx, Ty)) &= d(Tx, Ty) = d(\frac{5}{6}x, \frac{5}{6}y) \\ &= \frac{5}{6}|x - y| \\ &= \psi(\xi(M(x, y))) \\ &\leq \psi(\xi(M(x, y)). \end{aligned}$$

We now show that T is α-admissible.
Let $x \in X$ be given and let $y \in Tx$ be such that $\alpha(x, y) \geq 1$. Then $0 \leq x, y \leq 1$.
Obviously $\alpha(y, z) \geq 1$ for all $z \in Ty$ whenever $0 < y \leq 1$.

If $y = 0$, then $Ty = \{\frac{2}{3}, 1\}$. Hence, for all $z \in Ty, \alpha(y, z) \geq 1$. Hence, T is α-admisible. Thus, all hypothesis of Theorem 2.4 are satisfied. However, T has no fixed points.
Note that ψ is not upper semicontinuous.

Corollary 2.2 : Let (X, d) be a complete metric space and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible. Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \tag{38}$$

$$M(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Tx)d(y, Ty)}{1 + d(x, y)}\right\}$$

where $L \geq 0, \xi \in \Xi, \psi \in \Psi$ is strictly increasing and upper semicontinuous function. Also, suppose that conditions (1)and (2) of Theorem 2.4 are satisfied. Then T has a fixed point in X.

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