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A REVIEW OF ANALYTICAL STUDY OF SOME ASPECTS OF MATHEMATICAL MODELING AND SIMULATION OF PROBABILITY DISTRIBUTIONS WITH IMPETUS OF RANDOM NUMBER GENERATIONS

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Abstract

Mathematical Modeling is all about describing deterministic and probabilistic system. In probabilistic system, the behaviour is determined by random events. Random number generations are vital in mathematical modeling and simulation. The cumulative distribution function of uniform distribution, exponential distributions etc. are playing an important role in mathematical modeling and simulation. In this paper, we will present a review of analytical study of some aspects of mathematical modeling of probability distribution in testing of hypothesis with some examples using simulation with impetus of random number generations and inverse transform techniques.

Key Words : *Modeling and Simulation, Probability Distribution, Random Numbers.*

2010 AMS Subject Classification : 60 A05, 60 C05, 60 E05, 68 R10.

1. Introduction

Introduction System is a set of principles or procedures according to which something is done. Model is a description of the action of building something and working of some system of interest. A model is similar to but then the system it describes. Simulation is the process of planning a model of an actual system and carry out experiments with this model for the purpose of either of understanding the behaviour of the system or of evaluating various movements for the operation of system. Mathematical model is a set of equations that entitled an actual system. That system could be any particular part of natural world . from the movement of molecules in a balloon, to the relation among neurons in your brain, to the relation among species in an ecosystem. In this paper, we present a review of analytical study of some aspects of mathematical modeling of probability distribution in testing of hypothesis with deterministic problem using simulation with impetus of random number generations and inverse transform techniques [1].

1.1 Probability Distribution

Let X be a continuous random variable which takes values x. The probability that the random variable X takes the value x is defined as the probability distribution of X. It is denoted by $f(x)$. Properties of probability distribution are $f(x) \geq 0$, for all values of x and $\Sigma f(x) = 1$.

1.2 Uniform Distribution

A random variable X is said to have a continuous rectangular (uniform) distribution over an interval (a, b) i.e. $(-\infty < a < b < \infty)$ if its probability density function [3] is given by,

$$f(x; a, b) = \begin{cases} \frac{1l}{b-a}, & \text{if } \ a < x < b \\ & \qquad \text{denoted by } u(a,b) \\ 0 & \text{otherwise} \end{cases}$$

The cumulative distribution function $F(x)$ is given by [1, 3]:

$$F(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a}, & a < x < b \\ 1 & x \geq b \end{cases}$$

Graphical relationship between uniform continuous random variable and its pdf:

Figure: 1.1

The mean of uniform distribution is $E(X) = \frac{1}{z}(a+b)$, where a and b are two parameters which are its maximum and minimum values. The variance of uniform distribution is $V(X) = \frac{1}{12}(b-a)^2$.

1.3 Exponential Distribution

A random variable X is said to have an exponential distribution with parameter $\theta > 0$ and its probability density function [1] is given by

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 1 - \exp(-\theta x), & \geq 0 \\ 0, & \text{otherwise} \end{cases} .$$

The mean of the exponential distribution is $E(X) = \frac{1}{\lambda}$ and the variance of the exponential distribution is

$$V(X) = E(X^2) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}.$$

1.4 Inverse Transform Technique

The procedure for the inverse-transformation illustrated by the exponential distribution is:

Step-1 : Calculate the cdf of the desiring random variable X. For the exponential distribution, the cdf is $F(X) = 1 - e^{-\lambda x}, x \geq 0$.

Step-2 : Set $F(X) = R$ on the range of X.

For the exponential distribution, it becomes $1 - e^{-\lambda x} = R$ on the Range $X \geq 0$. X is a random variable (with the exponential distribution in this case), so $1 - e^{-\lambda x}$ is also a random variable, here called R. It is clear that R has a uniform distribution over the interval [0,1].

Step-3 : Solve the equation $F(X) = R$ for X in terms of R. For the exponential distribution, the solution proceeds as follows: $X = -\frac{1}{\lambda} ln(1 - r)$. Equation is called a random variate generator for the exponential distribution. In general, equation is written as $X = F^{-1}(R_i)$.

Step-4 : Now generate uniform random numbers $R_1, R_2, R_3, \cdots$ and compute the aspired random variates by $X_i = F^{-1}(R_i)$. For the exponential case, $F^{-1}(R) = -\frac{1}{\lambda} ln(1 - R)$ by the equation, so $X_i = -\frac{1}{\lambda} ln(1 - R)$. For $i = 1, 2, \cdots$. One translation that is usually enrol in equation is so replace $1 - R_i$ by R_i to give $X_i = -\frac{1}{\lambda} ln\ R_i$. This possibility is proved to be right by the fact that both R_i and $1 - R_i$ are uniformly distributed on [0,1].

2. Illustrative Example

Monte Carlo was first introduced in World War II by a scientist working on an atom bomb since then it was named as Monte Carlo. Monte Carlo simulation is one of the frequently used examples of simulation. To generate a model this method uses a large number of random numbers. Even a complex or difficult system can be easily translated.

2.1 Area Under Curve

Here we will construct a probabilistic model of a deterministic system. Particularly we will use a simulation to estimate the area under the curve $y = \sqrt{1 - x^2}$ over the interval $[-1, 1]$. This curve forms the top half of a circle with radius 1, so the area under the curve is exactly $\pi/2$. This simulation could also be seen as a way of estimating the value of π. A graph of the curve is shown in figure along with a rectangle of height $h = 1$ and width $w = 2$ drawn around it. In the simulation, we will randomly pick points inside the rectangle and determine whether each one is above or below the curve. We will then estimate the area under the curve using the relationship

$$\text{Area under the curve} \approx \frac{\text{Number of points under the curve}}{\text{Total nuber of points}} * \text{ Area of rectangle.}$$

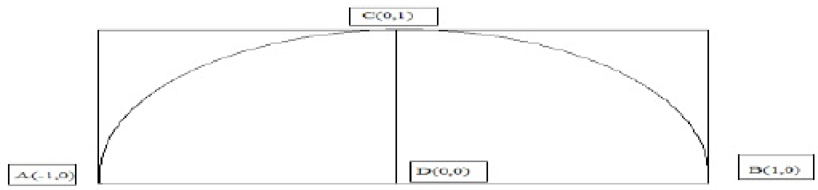

Figure: 1.2

1. Randomly pick 200 points inside the rectangle.
2. Determine whether each point lies under the curve.
3. Count the number of points under the curve.
4. Repeat for a total of 20 trials.

			MONTE CARLO SIMULATION								
	a=	-1		$x^2+y^2=1$							
	b=	1									
	h=	1									
Iteration	x- coordinate	y- coordinate	value of y from curve	under curve ?		Points under the curve:		Average:		Trial:	Area:
1	-0.508350403	0.297360448	0.861150316	1		151		1.559			1.51
2	-0.460709141	0.82451692	0.887551175	1		Area under the curve:				1	1.52
3	0.040195378	0.432130411	0.999191839	1		1.51				2	1.56
4	0.707221083	0.884600814	0.70699246	0						3	1.55
5	-0.779079812	0.281418868	0.626924753	1						4	1.59
6	-0.771792165	0.258381587	0.635874873	1						5	1.62
7	0.762425973	0.027027889	0.647075449	1						6	1.46
8	-0.272038357	0.930119709	0.962286409	1						7	1.54
9	-0.191754259	0.100342062	0.98144297	1						8	1.6
10	0.134455527	0.360977664	0.990919629	1						9	1.51
11	0.012251557	0.017350537	0.999924947	1						10	1.56
12	0.796293937	0.789730548	0.604909882	0						11	1.48
13	0.027758434	0.030024156	0.99961466	1						12	1.6
14	-0.167087922	0.489940793	0.985941999	1						13	1.56
15	0.410982603	0.396597618	0.911643187	1						14	1.53

Figure 1.3

3 Conclusion

In this paper we demonstrate the deterministic problem by mathematical modeling of probability distribution with testing of hypothesis with simulation .Analytical and simulation results are matched up to the desired degree of accuracy. This technique may be used where analytical solutions are not possible.

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