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CERTAIN PROPERTIES OF INTUITIONISTIC OF FUZZY MATRICES

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Abstract

In this paper, we present several properties on algebraic sum . and algebraic product . of intuitionistic fuzzy matrices. Also some results on existing operators along with these operators are presented.

1. Introduction

Atanassov [1] introduced the new concept of intuitionistic fuzzy set(IFS) in 1983 which is an extenstion of fuzzy set(FS) initiated by Zadeh [20]. Meenakshi [9] studied the theoretical developments of fuzzy matrices. Using the concept of IFS, Im et.al [6] studied intuitionistic fuzzy matrix(IFM). Simultaneousely Khan et.al [8] defined the

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intuitionistic fuzzy matrix and Pal [13] conceived the intuitionistic fuzzy determinant, studied some properties on it. IFM is a generalization of Fuzzy matrix introduced by Thomason[18] and has been useful in dealing with areas such as decision making, clusturing analysis, relational equations etc. Atanassov [3], using the definition of index matrix, has paved way for intuitionistic fuzzy index matrix and has further extending it to temporal intuitionistic fuzzy index matrix. IFM is also very useful in the discussion of intuitionistic fuzzy relation [5, 10]. Xu [19] studied intuitionistic fuzzy value and also IFMs. He defined intuitionistic fuzzy similarity relation and also utilized it in clusturing analysis. A lot of research activities have been carried out over the years on IFMs in [7, 11, 14, 15, 21].

Atanassov introduced model operators in [2] which are meaningless in fuzzy set theory and found a promising direction in research. Muthuraji et.al [12] studied some properties of model operators in IFM. Also they obtained a decomposition of an IFM. Shyamal and Pal [17] introduced two binary operators for fuzzy matrices and studied their algebraic properties. Sriram and Boobalan [4] investigated the algebraic properties and studied the properties of IFMs in the case where these operations are combined with the well known operations. In this paper, we present several properties on algebraic sum . and algebraic product . of intuitionistic fuzzy matrix. Also some results on existing operators along with these operators are presented.

2. Preliminieries

In this section, we refer to some basic definitions of intuitionistic fuzzy matrix that are necessary for this paper.

Definition 2.1 [2] : Let a set $X = \{x_1, x_2, \cdots, x_n\}$ be fixed, then an intuitionistic fuzzy set can be defined as $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i) \rightarrow / x_i \in X\}$, which assigns to each element x_i a membership degree $\mu_A(x_i)$ and a non membership degree $\nu_A(x_i)$, with condition:

$$0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1 \text{ for all } x_i \in X.$$

Definition 2.2 [16] : An intuitionistic fuzzy matrix is a matrix of pairs $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$ of non negative real numbers satisfying $\mu_{a_{ij}} + \nu_{a_{ij}} \leq 1$ for all i and j.

Definition 2.3 [11] : The $m \times n$ zero intuitionistic fuzzy matrix O is an intuitionistic fuzzy matrix all of whose entries are $\langle 0, 1 \rangle$. The $m \times n$ universal intuitionistic fuzzy

matrix J is an intuitionistic fuzzy matrix all of whose entries are $\langle 1, 0\rangle$.
bf Definition 2.4 [17] : Let $A = (\langle\mu_{a_{ij}}, \nu_{a_{ij}}\rangle)$ and $B = (\langle m_{b_{ij}}, \nu_{b_{ij}}\rangle)$ be two intuitionistic fuzzy matrices of order $m \times n$. Then

(i) $A \oplus B = (\langle\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}}\rangle)$ is called the algebraic sum of A and B.

(ii) $A \otimes B = (\langle\mu_{a_{ij}}\mu_{b_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} nu_{b_{ij}}\rangle)$ is called the algebraic product of A and B.

Definition 2.5 [16] : In the following we define some special types of intuitionistic fuzzy matrices. For an intuitionistic fuzzy matrix of order n,

(i) A is symmetric if and only if $A^T = A$.

(ii) A is reflexive if and only if $\langle\mu_{a_{ii}}\nu_{a_{ii}}\rangle = \langle 1, 0\rangle$ for all i.

(iii) A is irreflexive if and only if $\langle\mu_{a_{ii}}, \nu_{a_{ii}}\rangle = \langle 0, 1\rangle$ for all i.

(iv) A is identity if $\langle\mu_{a_{ij}}, \nu_{a_{ij}}\rangle = \langle 0, 1\rangle$ for all $i \neq j$ and $\langle\mu_{a_{ii}}, \nu_{a_{ii}}\rangle = \langle 1, 0$ for all i.

An identity intuitionistic fuzzy matrix of n is denoted by I_n.
Definition 2.6 [16] : Let A and B be two intuitionistic fuzzy matrices such that $A = (\langle\mu_{a_{ij}}, \nu_{a_{ij}}\rangle).B = (\langle\nu_{b_{ij}}, \nu_{b_{ij}}\rangle)$ then we write $A \leq B$ if and only if $\mu_{a_{ij}} \leq \nu_{a_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$ for all i, j.
Definition 2.7 [16] : Let A and B are two intuitionistic fuzzy matrices, such that $A = (\langle\mu_{a_{ij}}, \nu_{a_{ij}}\rangle) \quad B = (\langle\mu_{b_{ij}}, \nu_{b_{ij}}\rangle)$ then

$$A \vee B = (\langle\max(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{b_{ij}})\rangle)$$

$$A \wedge B = (\langle\min(\mu_{a_{ij}}, \mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{b_{ij}})\rangle).$$

Definition 2.8 [16] : The complement of an intuitionistic fuzzy matrix A which is denoted by A^c and is defined by $A^c = (\langle\nu_{a_{ij}}, \mu_{a_{ij}}\rangle)$ for all i, j.
Definition 2.9 [16] : If $A = (\langle\mu_{a_{ij}}, \nu_{a_{ij}}\rangle)$ is an $m \times n$ intuitionistic fuzzy matrix, then the $n \times m$ intuitionistic fuzzy matrix is $A^T = (\langle\mu_{a'_{ij}}, \nu_{a'_{ij}}\rangle)$ where $\mu_{a'_{ij}} = \mu_{a_{ji}}, \nu_{a'_{ij}} = \nu_{a_{ji}}, 1]lei \leq m, 1]lej]len$ is called the transpose of A.

3. Main Results

In this section, some properties of intuitionistic fuzzy matrices over the operators algebraic Sum $\oplus$, algebraic product $\otimes$ and some pre-defined operators are presented.

Proposition 3.1 : If A and B are two intuitionistic fuzzy matrices of same order, then

(i) If A and B are symmetric, then so are $A \oplus B$ and $A \otimes B$.

(ii) If A and B are nearly irreflexive, then so are $A \oplus B$ and $A \times B$.

Proof : (i) Let $A = (\langle \mu_{a_{ji}}, \nu_{a_{ij}} \rangle)$ and $B = (\langle \mu_{b_{ji}}, \nu_{b_{ij}} \rangle)$ be two symmetric intuitionistic fuzzy matrices.

Therefore $\mu_{a_{ij}} = \mu_{a_{ji}}, \nu_{a_{ij}} = \nu_{a_{ji}}$.

Let c_{ij} be the ij^{th} element of $A \oplus B$. Then

$$\begin{aligned} c_{ij} &= \langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle \\ &= \langle \mu_{a_{ji}} + \mu_{b_{ji}} - \mu_{a_{ji}}\mu_{b_{ji}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle \\ &= c_{ji}. \end{aligned}$$

Hence, $A \oplus B$ is symmetric.

Similarly, we prove $A \otimes B$ is symmetric.

(ii) Since A and B are nearly irreflexive

$$\text{i.e,} \quad \mu_{a_{ii}} \leq \mu_{a_{ij}}, \nu_{a_{ii}} \geq \nu_{a_{ij}}$$

$$\mu_{b_{ii}} \leq \mu_{b_{ij}}, \nu_{b_{ii}} \geq \nu_{b_{ij}}$$

Let c_{ij} be the ij^{th} element of $A \oplus B$

$$\begin{aligned} \mu_{c_{ij}} - \mu_{c_{ii}} &= (\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}) - (\mu_{a_{ii}} + \mu_{b_{ii}} - \mu_{a_{ii}}\mu_{b_{ii}}) \\ &= (1 - \mu_{a_{ii}})(1 - \mu_{b_{ii}}) - (1 - \mu_{a_{ij}})(1 - \mu_{b_{ii}}) \\ &\geq 0 \end{aligned}$$

as $1 - \mu_{a_{ii}} \geq 1 - \mu_{a_{ij}}$ and $1 - \mu_{b_{ii}} \geq 1 - \mu_{b_{ij}}$.

i.e. $\mu_{c_{ij}} \geq \mu_{c_{ii}}$.

$$\begin{aligned} \nu_{c_{ii}} - \nu_{c_{ij}} &= \nu_{a_{ii}}\nu_{b_{ii}} - \nu_{a_{ij}}\nu_{b_{ij}} \\ &\geq 0. \end{aligned}$$

$$\nu_{c_{ii}} \geq \nu_{c_{ij}}.$$

Hence, $A \oplus B$ is nearly irreflexive.

Similarly, we prove $A \otimes B$ is nearly irreflexive. □

Theorem 3.2 : Let A, B and C be three intuitionistic fuzzy matrices of order $n \times n$. Then

(i) $(A \oplus B)^T = A^T \oplus B^T$.

(ii) $(A \otimes B)^T = A^T \otimes B^T$.

(iii) If $A \leq B$ then $(A \oplus C) \leq (B \oplus C)$ and $(A \otimes C) \leq (B \otimes C)$.

Proof : (i) Let c_{ij} and d_{ij} be the ij^{th} elements of $A \oplus B$ and $A^T \oplus B^T$ respectively. Then

$$\begin{aligned} c_{ij} &= (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{aij}\nu_{b_{ij}} \rangle) \\ d_{ij} &= (\langle \mu_{a_{ji}} + \mu_{b_{ji}} - \mu_{a_{ji}}\mu_{b_{ji}}, \nu_{a_{ji}}\nu_{b_{ji}} \rangle) \\ &= c_{ji} \\ &= ij^{th} \text{ element of } (A \oplus B)^T. \end{aligned}$$

Therefore, $(A \oplus B)^T = A^T \oplus B^T$.

(ii) Let c_{ij} and d_{ij} be the ij^{th} elements of $A \otimes B$ and $A^T \otimes B^T$ respectively. Then

$$\begin{aligned} c_{ij} &= (\langle \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \rangle) \\ d_{ij} &= (\langle \mu_{a_{ji}}\mu_{b_{ji}}, \nu_{a_{ji}} + \nu_{b_{ji}} - \nu_{a_{ji}}\nu_{b_{ji}} \rangle) \\ &= c_{ji} \\ &= ij^{th} \text{ element of } (A \otimes B)^T. \end{aligned}$$

Therefore, $(A \otimes B)^T = A^T \otimes B^T$.

(iii) Let d_{ij}, e_{ij}, f_{ij} and g_{ij} be the ij^{th} elements of $A \oplus C, B \oplus C, A \otimes C$ and $B \times C$ respectively. Then

$$\begin{aligned} d_{ij} &= (\langle \mu_{a_{ij}} + \mu_{c_{ij}} - \mu_{a_{ij}}\mu_{c_{ij}}, \nu_{a_{ij}}\nu_{c_{ij}} \rangle) \\ e_{ij} &= (\langle \mu_{b_{ij}} + \mu_{c_{ij}} - \mu_{b_{ij}}\mu_{c_{ij}}, \nu_{b_{ij}}\nu_{c_{ij}} \rangle) \\ f_{ij} &= (\langle \mu_{a_{ij}} + \mu_{c_{ij}}, \nu_{a_{ij}} + \nu_{c_{ij}} - \nu_{a_{ij}}\nu_{c_{ij}} \rangle) \\ g_{ij} &= (\langle \mu_{b_{ij}}\mu_{c_{ij}}, \nu_{b_{ij}} + \nu_{c_{ij}} - \nu_{b_{ij}}\nu_{c_{ij}} \rangle) \end{aligned}$$

since $A \leq B, \mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$.
Then,

$$\mu_{a_{ij}}(1-\mu_{c_{ij}}) \leq \mu_{b_{ij}}(1-\mu_{c_{ij}}).\nu_{a_{ij}}\nu_{c_{ij}} \geq \nu_{b_{ij}}\nu_{c_{ij}}$$
$$\mu_{a_{ij}} + \mu_{c_{ij}} - \mu_{a_{ij}}\mu_{c_{ij}} \leq \mu_{b_{ij}} + \mu_{c_{ij}} - \mu_{b_{ij}}\mu_{c_{ij}}, \nu_{a_{ij}}\nu_{c_{ij}} \geq \nu_{b_{ij}}\nu_{c_{ij}}$$

i.e. $d_{ij} \leq e_{ij}$ for all i, j.
Hence, $(A \oplus C) \leq (B \oplus C)$.
Similarly, we prove $(A \otimes C) \leq (B \otimes C)$. □

Theorem 3.3 : For any $n \times n$ intuitionistic fuzzy matrix A,

(i) $I_n \oplus (A \oplus A^T)$ is reflexive and symmetric

(ii) $A \oplus A^T$ is nearly irreflexive and symmetric

(iii) $I_n \oplus (A \oplus A^T) = I_n \vee (A \oplus A^T)$.

Proof : (i) $A \oplus A^T = (\langle \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}, \nu_{a_{ij}}\nu_{a_{ji}} \rangle)$ and $I_n \oplus (A \oplus A^T) = [r_{ij}]$ where $r_{ij} = 1$ and

$$r_{ij} = (\langle \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}.\nu_{a_{ij}}\nu_{a_{ji}} \rangle)$$

Now,

$$\begin{aligned} r_{ji} &= (\langle \mu_{a_{ji}} + \mu_{a_{ij}} - \mu_{a_{ji}}\mu_{a_{ij}}.\nu_{a_{ji}}\nu_{a_{ij}} \rangle) \\ &= r_{ij} \end{aligned}$$

i.e, each diagonal element of $I_n \oplus (A \oplus A^T)$ is $\langle 1, 0 \rangle$ and all non-diagonal elements are $\mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}, \nu_{a_{ij}}\nu_{a_{ji}}$.
Hence, $I_n \oplus (A \oplus A^T)$ is reflexive and also symmetric.
(ii) Let $R = A \oplus A^T$.
i.e, $r_{ij} = (\langle \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}, \nu_{a_{ij}}\nu_{a_{ji}} \rangle) = r_{ji}$. Therefore, R is symmetric.
Again $r_{ii} = 2\mu_{a_{ij}}^2 - \mu_{a_{ij}}^2, \nu_{a_{ij}}^w$.
Since A is nearly irreflexive, $\mu_{a_{ii}} \leq \mu_{a_{ij}}$.
Therefore $1 - \mu_{a_{ii}} \geq 1 - \mu_{a_{ij}}$ and $\nu_{a_{ii}} \geq \nu_{a_{ij}}$

$$1 - \nu_{a_{ii}} \leq 1 - \nu_{a_{ij}}.$$

Now,$r_{ij} - r_{ii} = \nu_{a_{ij}}$.

(iii) $I_n \oplus (A \oplus A^T) = [r_{ij}]$ where $r_{ii} = (1,0)$

$$r_{ij} = (\langle \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}, \nu_{a_{ij}}\nu_{a_{ji}} \rangle), i \neq j$$

$$\begin{aligned} I_n \vee (A \oplus A^T) &= (\langle \max[(1,0), \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}], \min[(0,1), \nu_{a_{ij}}\nu_{a_{ji}}] \rangle) \\ &= (1,0), i = j \\ &= (\langle \mu_{a_{ij}} + \mu_{a_{ji}} - \mu_{a_{ij}}\mu_{a_{ji}}, \nu_{a_{ij}}\nu_{a_{ji}} \rangle), i \neq j \\ &= [r_{ij}]. \end{aligned}$$

Hence, $I_n \oplus (A \oplus A^T) = I_n \vee (A \oplus A^T)$. □

Proposition 3.4 : If A and B are two intuitionistic fuzzy matrices. Then

(i) $(A \oplus B)^c \leq A^c \oplus B^c$.

(ii) $(A \otimes B)^c \geq A^c \otimes B^c$.

Proof : (i) $(A \oplus B)^c = (\langle v_{a_{ij}}\nu_{b_{ij}} + \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} \rangle)$.

$$\begin{aligned} A^c \oplus B^c &= (\langle \nu_{a_{ij}}\mu_{a_{ij}} \rangle) \oplus (\langle \nu_{b_{ij}}\mu_{b_{ij}} \rangle) \\ &= (\langle \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}, \mu_{a_{ij}}\mu_{a_{ij}} \rangle) \end{aligned}$$

since $\nu_{a_{ij}}\nu_{n_{ij}} \leq \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}$

$$\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} \geq \mu_{a_{ij}}\mu_{a_{ij}}.$$

Hence , $(A \oplus B)^c \leq A^c \oplus B^c$.

(ii) $(A \otimes B)^c = (\langle \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}, \mu_{a_{ij}}\mu_{b_{ij}} \rangle)$

$$A^c \otimes B^c = (\langle \nu_{a_{ij}}\mu_{a_{ij}} \rangle) \otimes (\langle \nu_{b_{ij}}\mu_{b_{ij}} \rangle)$$

since $\nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \geq \nu_{a_{ij}}\nu_{b_{ij}}$

$$\mu_{a_{ij}}\mu_{a_{ij}} \leq \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}.$$

Hence , $(A \times B)^c \geq A^c \otimes B^c$. □

Theorem 3.5 : If A and B are two intuitionistic fuzzy matrices. Then

(i) $A \oplus B \geq (A \vee B)$

(ii) $A \otimes B \leq (A \vee B)$

Proof : (i) $A \oplus B = (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle)$

$$a \vee B = (\langle \max(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle)$$

$$m_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} = \begin{cases} \mu_{a_{ij}} + \mu_{b_{ij}}(1 - \mu_{a_{ij}}) \geq \mu_{a_{ij}} \\ \\ \mu_{b_{ij}} + \mu_{a_{ij}}(1 - \mu_{b_{ij}}) \geq \mu_{b_{ij}} \end{cases}$$

$$\begin{aligned} \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} &\geq \max(\mu_{a_{ij}}, \mu_{b_{ij}}) \\ \nu_{a_{ij}}\nu_{b_{ij}} &\leq \min(\nu_{a_{ij}}, \nu_{b_{ij}}). \end{aligned}$$

Hence $A \oplus B \geq (A \vee B)$.
(ii) $A \otimes B = (\langle \mu_{a_{ij}} + \mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \rangle)$

$$a \vee B = (\langle \max(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle)$$

$$\nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} = \begin{cases} \nu_{a_{ij}} + \nu_{b_{ij}}(1 - \nu_{a_{ij}}) \leq \nu_{a_{ij}} \\ \\ \nu_{b_{ij}} + \nu_{a_{ij}}(1 - \nu_{b_{ij}}) \leq \nu_{b_{ij}} \end{cases}$$

$$\min(\nu_{a_{ij}}, \nu_{b_{ij}}) \leq \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}$$

$$\max(\mu_{a_{ij}}, \mu_{b_{ij}}) \geq \mu_{a_{ij}}\mu_{b_{ij}}.$$

Hence $A \otimes B \leq (A \vee B)$. □

Theorem 3.6 : If A and B are two intuitionistic fuzzy matrices. Then

(i) $A \oplus B \geq (A \wedge B)$

(ii) $A \otimes B \leq (A \wedge B)$

Proof : (i) $A \oplus B = (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle)$

$$A \wedge B = (\min(\mu_{a_{ij}} + \mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{b_{ij}}$$

$$\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} = \begin{cases} \mu_{a_{ij}} + \mu_{b_{ij}}(1 - \mu_{a_{ij}}) \geq \mu_{a_{ij}} \\ \\ \mu_{b_{ij}} + \mu_{a_{ij}}(1 - \mu_{b_{ij}}) \geq \mu_{b_{ij}} \end{cases}$$

$$\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} \geq \min(\mu_{a_{ij}}, \mu_{b_{ij}})$$

$$\nu_{a_{ij}}\nu_{b_{ij}} \leq \max(\nu_{a_{ij}}, \nu_{b_{ij}}$$

Hence $A \oplus B \geq (A \wedge B)$.

(ii) $A \otimes B = (\langle \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}$

$$\nu_{a_{ij}} + \nu_{b_{ij}} - num_{a_{ij}}\nu_{b_{ij}} = \begin{cases} \nu_{a_{ij}} + \nu_{b_{ij}}(1-\nu_{a_{ij}}) \leq \nu_{a_{ij}} \\ \\ \nu_{b_{ij}} + \nu_{a_{ij}}(1-\nu_{b_{ij}}) \leq \nu_{b_{ij}} \end{cases}$$

$$\max(\nu_{a_{ij}}, \nu_{b_{ij}}) \leq \nu_{a_{ij}}\nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}}$$

$$\nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{n_{ij}} \geq \max(\nu_{a_{ij}}\nu_{b_{ij}}0$$

Hence $A \otimes B \leq (a \vee B)$. □

Proposition 3.7 : If A and B are two intuitionistic fuzzy matrices, then $A \otimes B \leq A \oplus B$.

Proof : $A \oplus B = (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}})$ for all i, j.

$$A \otimes B = (\langle \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \quad \text{for all } i, j.$$

Assume that $\mu_{a_{ij}}\mu_{b_{ij}} \leq \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}$

$$\text{i.e.}\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}} \geq 0.$$

$$\mu_{a_{ij}}(1-\mu_{b_{ij}}) + \mu_{bij}(1-\mu_{a_{ij}}) \geq 0$$

which is true as $0 \leq \mu_{a_{ij}} \leq 1$ and $0 \leq \mu_{b_{ij}} \leq 1$.

Also assume that $\nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \geq \nu_{a_{ij}}\nu_{b_{ij}}$

$$\text{i.e.}\nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \geq 0.$$

$$\nu_{a_{ij}}(1-\nu_{b_{ij}}) + \nu_{\beta ij}(1-\nu_{a_{ij}}) \geq 0$$

which is true as $0 \leq \nu_{a_{ij}} \leq 1$ and $0 \leq \nu_{b_{ij}} \leq 1$.

Hence $a \otimes B \leq A \oplus B$.

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