Reprint ISSN 0973-9424

International Journal Of Mathematical Sciences And Engineering Applications

(IJMSEA)

ASCENT PUBLICATION @ ASCENT PUNE, INDIA

www.ascent-journals.com

International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 13 No. I (June, 2019), pp. 77-85

ON LINEAR GENERATING RELATIONS INVOLVING I-FUNCTIONS OF r-VARIABLES

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Abstract

The object of this paper is to derive four linear generating relations involving the I- functions of r-variables. Special cases include the result proved by Lawrynovicz [2].

1. Introduction

Notations and Results used :

$(a)_n$ stands for $a(a+1)\cdots(a+n-1)$

${}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p$ stands for $(a_1;\alpha_1^{(1)},\cdots,\alpha_1^{(r)};A_1),(a_2;\alpha_2^{(1)},\cdots,\alpha_2^{(r)};A_2),$
$\cdots,(a_p;\alpha_p^{(1)},\cdots,(a_p;\alpha_p^{(1)},\cdots,(\alpha_p^{(r)};A_p)$

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)},n\geq 1. \tag{1.1}$$

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Key Words : *I- function of r-variables, H-function of r-variables.*

2000 AMS Subject Classification : 45 A 05.

$$\frac{\Gamma(1-\alpha-n)}{\Gamma(\alpha)} = \frac{(-1)^n}{(\alpha)_n} \tag{1.2}$$

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n (t)^n}{n!} = (1-t)^{-\alpha} \tag{1.3}$$

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n (-t)^n}{n!} = (1+t)^{-\alpha}. \tag{1.4}$$

The generalized Fox's H-function, namely I-function of r-variables introduced by Prathima, Nambisan and Santha Kumari [3,p.38] is defined and represented as:

$$I[z_1, \cdots, z_r] = I^{0,N:m_1,n_1,\cdots,m_r,n_r}_{P,Q:p_1,q_1,\cdots,p_r,q_r}$$

$$\left[\begin{array}{c|c} z_1 & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P :{}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ z_r & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q :{}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;{}_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \theta_1(s_1)\cdots\theta_r(s_r)\phi(s_1,\cdots,s_r) z_1^{s_1}\cdots z_r^{s_r} ds_1\cdots ds_r, \tag{1.5}$$

where $\phi(s_1, \cdots, s_r)$ and $\theta_i(s_i), i = 1, 2, \cdots, r$ are given by

$$\varphi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{N} \Gamma^{A_j}(1-a_j+\sum_{j=1}^{r}\alpha_j^{(i)}s_i)}{\prod_{j=1}^{Q}\Gamma^{B_j}(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=N+1}^{P}\Gamma^{A_j}(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i)} \tag{1.6}$$

$$\theta_i(s_1) = \frac{\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)} \tag{1.7}$$

Also $z_i \neq 0$ $(i = 1, \cdots, r), \omega = \sqrt{-1}, m_j, n_j, p_j, q_j$ $(j = 1, \cdots, r)$, N, P, Q are non-negative integers such that $0 \leq N \leq P$, $Q \geq 0$, $0 \leq m_j \leq q_j$, $0 \leq n_j \leq p_j$ $(j = 1, 2, \cdots, r)$ (not all zero simultaneously), $\alpha_j^{(i)}$ $(j = 1, 2, \cdots, P, i = 1, 2, \cdots, r)$, $\beta_j^{(i)}$ $(j = 1, 2, \cdots, Q, i = 1, 2, \cdots, r)$, $\gamma_j^{(i)}$ $(j = 1, 2, \cdots, p_i, i = 1, 2, \cdots, r)$ and $\delta_j^{(i)}$ $(j = 1, 2, \cdots, q_i, i = 1, 2, \cdots, r)$ are positive numbers. a_j $(j = 1, 2, \cdots, P), b_j$ $(j = 1, 2, \cdots, Q)$, $c_j^{(i)}$ $(j = 1, 2, \cdots, p_i, i = 1, 2, \cdots, r)$ and $d_j^{(i)}$ $(j = 1, 2, \cdots, q_i; i = 1, 2, \cdots, r)$ are complex numbers. The exponents A_j $(j = 1, 2, \cdots, P)$, B_j $(j = 1, 2, \cdots, Q)$, $C_j^{(i)}$ $(j =$

$1, 2, \cdots, p_i, i = 1, 2, \cdots, r)$ and $D_j^{(i)}$ $(j = 1, 2, \cdots, q, i = 1, 2, \cdots, r)$ of various gamma functions may take non integer values. The I-function of r-variables is analytic if

$$\Psi_i = \sum_{j=1}^{P} A_j \alpha_j^{(i)} - \sum_{j=1}^{Q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leq 0, 1, 2, \cdots, r.$$

The integral (1.5) converges absolutely if $|arg(z_i)| < \frac{1}{2}\Delta_i \pi, i = 1, 2, \cdots, r$ where

$$\begin{aligned} \Delta_i \quad = \quad & -\sum_{j=n+1}^{P} A_j \alpha_j^{(i)} - \sum_{j=1}^{Q} B_j \beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} \delta_j^{(i)} \\ & + \sum_{j=1}^{n_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \gamma_j^{i} > 0 \end{aligned} \tag{1.8}$$

2. Result-I

$$\eta^{\frac{(a_k-1)}{\alpha_k}} \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \frac{y^s}{s!} \frac{(\eta^{1/\alpha_k} - 1)^t}{t!} I^{0,n:m_1,n_1',\cdots;m_r,n_r}_{p,q:p_1+2,q_1',\cdots;p_r,q_r} \left[\begin{array}{c|c} x_1 & I_1 \\ \vdots & \\ x_r & I_2 \end{array} \right] = (1+y)^{-\lambda} I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1+2,q_1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} x_1 \eta (1+y)^{-\alpha} & {}_1(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_p :_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}, (a_k, \alpha_k; 1), \\ x_2 & \\ \vdots & (1-\lambda, \alpha, 1); \cdots ;_1 (c_j^{(r)}.\gamma_j^{(r)} C_j^{(r)})_{p_r} \ {}_1(b_j; b_j^{(1)}, \cdots, \beta_j^{(r)}; B_j)_q : \\ x_r & \\ & {}_1(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \cdots, (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r} \end{array} \right] \tag{2.1}$$

where

$$\begin{aligned} I_1 \quad = \quad & {}_1(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_p :_1 (c_j^{(1)}.\gamma_j^{(1)}.; C_j^{(1)})_{p_1}, (a_k - t, \alpha_k; 1), \\ & (1 - \lambda - s, \alpha; 1); \cdots; \ {}_1(c_j^{(r)}, \gamma_j^{(r)}, C_j^{(r)})_{p_r} \\ I_2 \quad = \quad & {}_1(b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}; B_j)_q : \ {}_1(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \cdots, (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r} \end{aligned}$$

provided

(i) $k > n, \eta > 0$

(ii) $Re(\eta^{1/\alpha_k}) > 1/2$

(iii) $arg(\eta x_1) = \alpha_k \ arg(\eta^{1/\alpha_k}) + arg(x_1)$

(iv) $arg(\eta^{1/\alpha_k}) < \pi$ and

(v) $\Delta_i > 0, |argx_i| < \frac{1}{2}\pi\Delta_i, i = 1, 2, \cdots, r$

where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}.\end{aligned}$$

Proof : Expressing the I-function of r- variables on the left hand side of (2.1) as a contour integral using (1.5), the left hand side of (2.1) becomes:

$$\eta^{\frac{(a_k-1)}{\alpha_k}} \sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \frac{y^s}{s!} \frac{\left(\eta^{-1/\alpha_k} - 1\right)^t}{t!} \times \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi_1(t_1)\cdots\phi_r(t_r)x_1^{t_1}\cdots x_r^{t_r}$$
$$\times \frac{1}{\Gamma(a_k - t - \alpha_k t_1)\Gamma(1 - \lambda - s - \alpha t_1)} dt_1 \cdots dt_r \tag{2.2}$$

where

$$\varphi(t_1, \cdots, t_r) = \frac{\prod_{j=1}^{n} \Gamma^{A_j}(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)} t_i)}{\prod_{j=1}^{q} \Gamma^{B_j}(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} t_i) \prod_{j=n+1}^{p} \Gamma^{A_j}(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} t_i)}$$

and

$$\theta_i(s_1) = \frac{\prod_{j=1}^{m_i} \Gamma_j^{D^{(i)}}(d_j^{(i)} - \delta_j^{(i)} t_i) \prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}(1 - c_j^{(i)} + \gamma_j^{(i)} t_i)}{\prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}(1 - d_j^{(i)} + \delta_j^{(i)} t_i) \prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}(c_j^{(i)} - \gamma_j^{(i)} t_i)}$$

Using (1.2) the equation (2.2) reduces to :

$$\eta^{\frac{(a_k-1)}{\alpha_k}} \sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \frac{y^s}{s!} \frac{\left(\eta^{-1/\alpha_k} - 1\right)^t}{t!} \times \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \varphi_1(t_1\cdots t_2)\theta_1(t_1)\cdots\theta_r(t_r)x_1^{t_1}\cdots x_r^{t_r}$$
$$\times \frac{(1 - a_k + \alpha_k t_1)_t}{(-1)^t\Gamma(a_k - \alpha_k t_1)} \frac{(\lambda + \alpha t_1)_s}{(-1)^s\Gamma(1 - \lambda - \alpha t_1)} dt_1 \cdots dt_r \tag{2.3}$$

Now using (1.4) in (2.3), it becomes:

$$\eta^{\frac{(a_k-1)}{\alpha_k}}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\varphi(t_1,\cdots,t_r)\theta_1(t_1)\cdots\theta_r(t_r)x_1^{t_1}\cdots x_r^{t_r}$$
$$\frac{\eta^{\frac{1-a_k+a_kt_1}{\alpha_k}}}{\Gamma(a_k-\alpha_kt_1)}\frac{(1+y)^{-(\lambda+\alpha t_1)}}{\Gamma(1-\lambda-\alpha t_1)}dt_1\cdots dt_r$$
$$=\frac{(1+y)^{-\lambda}}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\varphi(t_1,\cdots,t_2)\theta_1(t_1)\cdots\theta_r(t_r)x_1^{t_1}\cdots x_r^{t_r}$$
$$\frac{\eta^{t_1}}{\Gamma(a_k-\alpha_kt_1)}\frac{1+y)^{-\alpha t_1}}{\Gamma(1-\lambda-\alpha t_1)}dt_1\cdots dt_r$$
$$=\frac{(1+y)^{-\lambda}}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\varphi(t_1,\cdots,t_r)\theta_1(t_1)\cdots\theta_r(t_r)$$
$$\frac{[x_1\eta(1+y)^{-\alpha}]^{t_1}x_2^{t_2}\cdots x_r^{t_r}}{\Gamma(a_k-\alpha_kt_1)\Gamma(1-\lambda-\alpha t_1)}dt_1\cdots dt_r. \tag{2.4}$$

On applying (1.5) in (2.4) the right hand side of (2.1) is obtained.

Special Cases

When $r=2$, (2.1) reduces to:

Corollary 2.1 :

$$\eta^{\frac{(a_k-1)}{\alpha_k}}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\frac{y^s}{s!}\frac{(\eta^{-1/\alpha_k}-1)^t}{t!}I^{0,n:m_1,n_1;m_2,n_2}_{p,q:p_1+2,q_1;p_2,q_2}\left[\begin{array}{c|c} x_1 & I_1 \\ & \\ x_r & I_2\end{array}\right]=(1+y)^{-\lambda}I^{0,n:m_1,n_1;m_2,n_2}_{p,q:p_1+2,q_1;p_2,q_2}$$
$$\left[\begin{array}{c|c} x_1\eta(1+y)^{-\alpha} & {}_1(a_j;\alpha_j^{(1)},\alpha_j^{(2)};A_j)_p:{}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1},(a_k,\alpha_k;1), \\ & (1-\lambda,\alpha,1);{}_1(c_j^{(2)}.\gamma_j^{(2)}C_j^{(2)})_{p_2}\ {}_1(b_j;b_j^{(1)},\beta_j^{(2)};B_j)_q: \\ x_2 & {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};(d_j^{(2)},\delta_j^{(2)};D_j^{(2)})_{q_2}\end{array}\right] \tag{2.5}$$

where

$$\begin{aligned} I_1 &= {}_1(a_j;\alpha_j^{(1)},\alpha_j^{(2)};A_j)_p:{}_1(c_j^{(1)}.\gamma_j^{(1)};C_j^{(1)})_{p_1},(a_k-t,\alpha_k;1), \\ &\quad(1-\lambda-s,\alpha;1);\ {}_1(c_j^{(2)},\gamma_j^{(2)},C_j^{(2)})_{p_2} \\ I_2 &= {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)};B_j)_q:\ {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};(d_j^{(2)},\delta_j^{(2)};D_j^{(2)})_{q_2}\end{aligned}$$

provided

(i) $k>n,\eta>0$

(ii) $Re(\eta^{1/\alpha_k}) > 1/2$

(iii) $arg(\eta x_1) = \alpha_k\ arg(\eta^{1/\alpha_k}) + arg(x_1)$

(iv) $arg(\eta^{1/\alpha_k}) < \pi$ and

(v) $\Delta_i > 0, |arg x_i| < \frac{1}{2}\pi\Delta_i, i = 1, 2$

where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}.\end{aligned}$$

Proof : Proof is similar to that of (2.1).

Remark : On specializing the parameters the equation (2.5) reduces to the result proved by Lawrynovicz [2].

Result -II :

$$\eta^{\frac{(b_1)}{\beta_1}} \sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \frac{y^s}{s!}\frac{(1-\eta^{1/\beta_1})^t}{t!} I^{0,n:m_1+1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1+1;\cdots;p_r,q_r}\left[\begin{array}{c|c} x_1 & I_1 \\ \vdots & \\ x_r & I_2 \end{array}\right] = (1-y)^{-\lambda} I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1+1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} x_1\eta(1-y)^{-\alpha} & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :_1 (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ x_2 & \\ \vdots & (\lambda,\alpha,1);(b_1,\beta_1;1);_1(b_j;\beta_j^{(1)};\cdots;\beta_j^{(r)};B_j)_q :_2 (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1}; \\ x_r & \cdots;_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right] \tag{2.6}$$

where

$$\begin{aligned} I_1 &= {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :_1 (c_j^{(1)},\gamma_j^{(1)}.;C_j^{(1)})_{p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ I_2 &= (\lambda+s,\alpha;1)(t+b_1,\beta_1;1);(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_q :\ {}_2(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots,(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{aligned}$$

provided

(i)η is arbitrary for $m = 1$ and for $m > 1, |\eta^{1/\beta_1} - 1| < 1$

(ii) $arg(\eta x_1) = \beta_1 arg(\eta^{1/\beta_1}) + arg(x_1)$

(iii) $arg(\eta^{1/\beta_1}) < \frac{\pi}{2}$

(iv) $arg(\eta^{1/\alpha_k}) < \pi$ and

(v) $\Delta_i > 0, |arg x_i| < \frac{1}{2}\pi\Delta_i, i = 1, 2, \cdots, r$

where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}.\end{aligned}$$

Proof : Proof is similar to that of (2.1).

Result III :

$$\eta^{\frac{(a_k-1)}{\alpha_1}} \sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \frac{(-y)^s}{s!}\frac{(1-\eta^{1/\alpha_1})^t}{t!} I^{0,n:m_1,n_1+1;\cdots;m_r,n_r}_{p,q:p_1+1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} x_1 & I_1 \\ \vdots & \\ x_r & I_2 \end{array}\right]$$

$$= (1+y)^{(\lambda-1)} I^{0,n:m_1,n_1+1;\cdots;m_r,n_r}_{p,q:p_1+1,q_1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} x_1\eta(1+y)^{-\alpha} & (\lambda,\alpha,1);(a_1,\alpha_1;1)\ {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p : \\ x_2 & {}_2(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ x_r & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_q :{}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right] \quad (2.7)$$

where

$$\begin{aligned} I_1 &= (\lambda-s,\alpha;1);(a_1-t,\alpha_1;1)_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :_2 (c_j^{(1)}.\gamma_j^{(1)};C_j^{(1)})_{p_1}; \\ &\quad \cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{p_r} \\ I_2 &= {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)};B_j)_q :\ {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{aligned}$$

provided

(i) $n > 0, \eta > 0$

(ii) $Re(\eta^{1/\alpha_1}) > 1/2$

(iii) $arg(\eta x_1) = \alpha_1\ arg(\eta^{1/\alpha_1}) + arg(x_1)$

(iv) $arg(\eta^{1/\alpha_1}) > \pi/2$ and

(v) $\Delta_i > 0, |arg x_i| < \frac{1}{2}\pi\Delta_i, i = 1, 2, \cdots, r$

where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}.\end{aligned}$$

Proof : Proof is similar to that of (2.1).

Result IV :

$$\eta^{\frac{(b_k)}{\beta_k}} \sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \frac{y^s}{s!}\frac{(\eta^{1/\beta_k}-1)^t}{t!} I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1+2;\cdots;p_r,q_r}\left[\begin{array}{c|c} x_1 & I_1 \\ \vdots & \\ x_r & I_2 \end{array}\right] = (1+y)^{-\lambda} I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1+2;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} x_1\eta(1+y)^{-\lambda} & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :_1 (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)})_{p_r}, \\ x_2 & \\ \vdots & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_q :_1 (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1},(b_k,\beta_k;1), \\ x_r & \\ & (\lambda,\alpha,1);\cdots;_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right] \quad (2.8)$$

where

$$\begin{aligned} I_1 &= {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :_1 (c_j^{(1)}.\gamma_j^{(1)};C_j^{(1)})_{p_1},;\cdots;_1(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{p_r} \\ I_2 &= {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_q :\ {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1},(b_k+t,\beta_k;1),(\lambda+s,\alpha;1); \\ &\quad cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{aligned}$$

provided

(i) $\eta > 0, k > m$

(ii) $|\eta^{1/\beta_k}) - 1| < 1$

(iii) $arg(\eta x_1) = \beta_1\ arg(\eta^{1/\beta_k}) + arg(x_1)$

(iv) $arg(\eta^{1/\beta_k}) < \pi/2$ and

(v) $\Delta_i > 0, |arg x_i| < \frac{1}{2}\pi\Delta_i, i = 1, 2, \cdots, r$

where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}.\end{aligned}$$

Proof : Proof is similar to that of (2.1).

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