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I- FUNCTIONS AND HEAT CONDUCTION IN A SQUARE PLATE

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Abstract

The object of this paper is to obtain a solution of a heat conduction problem in a square plate by using the help of I-functions of several variables. Special cases include the results proved by Ambika A [1] and S. S. Srivastava and Ritu Srivastava [6 ,p.78-80].

1. Introduction

Notations and Results used :

$(a)_n$ stands for $a(a+1)\cdots(a+n-1)$

$(a_n) = \frac{\Gamma(a+n)}{\Gamma(a)}, m \geq 1$

${}_1(a_j;\alpha_j, A_j)_p$ stands for $(a_1;\alpha_1, A_1), (a_2, \alpha_2, A_2), \cdots, (a_p;\alpha_p, A_p)$.

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The I- function of r-variables introduced by Prathima, Nambisan and Santha Kumari [4,p.38] is defined and represented as:

$$I[z_1,\cdots,z_r]=I^{0,N:m_1,n_1;\cdots,;m_r,n_r}_{P,Q:p_{1,1};\cdots,p_r,q_r}\left[\begin{array}{c|c} z_1 & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:{}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ z_r & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_P:{}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;{}_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}\end{array}\right]$$
$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\theta_1(s_1)\cdots\theta_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}ds_1\cdots,ds_r. \tag{1.1}$$

where

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{N}\Gamma^{A_j}(1-a_j+\sum_{j=1}^{r}\alpha_j^{(i)}s_i)}{\prod_{j=1}^{Q}\Gamma^{B_j}(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=N+1}^{P}\Gamma^{A_j}(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i)} \tag{1.2}$$

$$\theta_i(s_1)=\frac{\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)} \tag{1.3}$$

Also $z_i\neq 0$ $(i=1,\cdots,r),\omega=\sqrt{-1},m_j,n_j,p_j,q_j$ $(j=1,\cdots,r)$, N,P,Q are non-negative integers such that $0\leq N\leq P$, $Q\geq 0$, $0\leq m_j\leq q_j$, $0\leq n_j\leq p_j$ $(j=1,2,\cdots,r)$ (not all zero simultaneously), $\alpha_j^{(i)}$ $(j=1,2,\cdots,P,i=1,2,\cdots,r)$, $\beta_j^{(i)}$ $(j=1,2,\cdots,Q,i=1,2,\cdots,r)$, $\gamma_j^{(i)}$ $(j=1,2,\cdots,p_i,i=1,2,\cdots,r)$ and $\delta_j^{(i)}$ $(j=1,2,\cdots,q_i,i=1,2,\cdots,r)$ are positive numbers. a_j $(j=1,2,\cdots,P),b_j$ $(j=1,2,\cdots,Q)$, $c_j^{(i)}$ $(j=1,2,\cdots,p_i,i=1,2,\cdots,r)$ and $d_j^{(i)}$ $(j=1,2,\cdots,q_i;i=1,2,\cdots,r)$ are complex numbers. The exponents A_j $(j=1,2,\cdots,P)$, B_j $(j=1,2,\cdots,Q)$, $C_j^{(i)}$ $(j=1,2,\cdots,p_i,i=1,2,\cdots,r)$ and $D_j^{(i)}$ $(j=1,2,\cdots,q,i=1,2,\cdots,r)$ of various gamma functions may take non integer values.

The I-function of r-variables is analytic if

$$\Psi_i=\sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leq 0,i=1,2,\cdots,r.$$

The integral (1.5) converges absolutely if $|arg(z_i)| < \frac{1}{2}\Delta_i\pi, i = 1, 2, \cdots, r$ where

$$\Delta_i = \Bigg(-\sum_{j=n+1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}\Bigg) > 0 \qquad (1.4)$$

On taking $D_j^{(i)} = 1(j = 1, 2, \cdots, m_i, i = 1, 2, \cdots, r)$ in (1.1), then I-function will be deonted by

$$\overline{I}[z_1, \cdots, z_r] = I_{P,Q:p_1,q_1;\cdots,p_r,q_r}^{0,N:m_1,n_1;\cdots,;m_r,n_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right]$$
$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1, \cdots, s_r)z_1^{s_1}\cdots z_r^{s_r}ds_1\cdots, ds_r. \quad (1.5)$$

where

$$I_1 = {}_1(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_p :_1 (c_j^{(1)}, \gamma_j^{(1)}.; C_j^{(1)})_{p_1}; \cdots ;\ _1(c_j^{(r)}, \gamma_j^{(r)}, C_j^{(r)})_{p_r}$$
$$I_2 = {}_1(b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}; B_j)_q :\ _1(d_j^{(1)}, \delta_j^{(1)}; 1)m_1;\ _{m_1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}$$
$$; \cdots ;_1 (d_j^r, \delta_j^r; 1)_{m_r};_{m_r+1} (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r}$$

and

$$\overline{\theta}_i(s_i) = \frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)} - \delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1 - c_j^{(i)} + \gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1 - d_j^{(i)} + \delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)} - \gamma_j^{(i)}s_i)}$$

$i = 1, 2, \cdots, r$.

The integral (1.5) converges absolutely if $|arg(z_i)| < \frac{1}{2}\Delta_i'\pi, i = 1, 2, \cdots, r$ where

$$\Delta_i' = \Bigg(-\sum_{j=n+1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)}\Bigg) > 0.$$

Churchill [2,p.125].

If a square plate has its faces and edges $x = 0$ and $x = \pi (0 < y < \pi)$, insulated its edges $y = 0$ and $y = \pi$ $(0 < x < \pi)$ are kept at temperatures zero and $f(x)$ respectively then its steady temperature $u(x, y)$ is given by

$$u(x,y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} \frac{a_n \sinh ny}{\sinh nx} \cos nx, \tag{1.6}$$

where

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx, \quad n = 0, 1, 2, \cdots \tag{1.7}$$

2. Heat Conduction in a Square Plate

Consider the Problem of determining $u(x, y)$, defined in (1.6) with $u(x, 0) = f(x)$ and

$$f(x) = (\sin x)^{m-1} \ \ I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r} \left[\begin{array}{cc} z_1(\sin x)^{\lambda_1} & I_1 \\ \vdots & \\ z_r(\sin x)^{\lambda_r} & I_2 \end{array} \right] \tag{2.1}$$

where

$$\begin{aligned} I_1 &= {}_1(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_p :_1 (c_j^{(1)}.\gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)}) \\ I_2 &= {}_1(b_j; \beta_j^{(1)}, \beta_j^{(2)}; B_j)_Q : \ {}_1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};_{m_1+1} (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1} \\ &\quad ; \cdots ;_1 (d_j^r, \delta_j^r; 1)_{m_r};_{m_r+1} (d_j^{(r)}, \delta_j^{(r)}, D_j^{(r)})_{q_r}. \end{aligned}$$

3. An Integral Required for the Sequel

$$\begin{aligned} &\int_0^{\pi} (\sin x)^{m-1} \cos nx \times \ \ I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r} \left[\begin{array}{cc} z_1(\sin x)^{\lambda_1} & I_1 \\ \vdots & \\ z_r(\sin x)^{\lambda_r} & I_2 \end{array} \right] dx \\ &= \frac{\pi \cos n\frac{\pi}{2}}{2^{m-1}} I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+2:p_1,q_1;\cdots;p_r,q_r} \left[\begin{array}{c|c} 2^{-\lambda_1} z_1 & I_3 \\ \vdots & \\ 2^{-\lambda_r} z_r & I_4 \end{array} \right] \quad (3.1) \end{aligned}$$

where

$$\begin{aligned}
I_1 &= {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_p :{}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r}\\
I_2 &= {}_1(b_j;\beta_j^{(1)},\beta_j^{(r)};B_j)_Q :\ {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};{}_{m_1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1}\\
&\quad ;\cdots;{}_1(d_j^{r},\delta_j^{r};1)_{m_r};{}_{m_r+1}(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{q_r}.\\
I_3 &= (1-m,\lambda_1\cdots\lambda_r)_1;(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P :\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r}\\
I_4 &= {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q : \left(\frac{1}{2}-\frac{m}{2}-\frac{n}{2},\frac{\lambda_1}{2},\cdots,\frac{\lambda_r}{2}\right)_1;\left(\frac{1}{2}-\frac{m}{2}+\frac{n}{2},\frac{\lambda_1}{2},\cdots,\frac{\lambda_r}{2}\right);\\
&\quad {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_1^{(j)},\delta_j^{(1)},D_j^{(1)})_{q_1};\cdots;(d_j^{(r)},\delta_j^{(r)},1)_{m_r};\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}
\end{aligned}$$

where

$$\begin{aligned}
\Delta_k &= \left(-\sum_{j=N+1}^{P}A_j\alpha_j^{(k)}-\sum_{j=1}^{Q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(i)}\right.\\
&\quad \left.-\sum_{j=m_k+1}^{q_{ik}}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}\right)>0
\end{aligned}$$

and

$$|arg(z_k)|<\frac{1}{2}\Delta_k\pi,\quad k=1,2,\cdots,r,\quad Re(m)>0.$$

Proof :

$$\begin{aligned}
&\int_0^\pi(\sin x)^{m-1}\cos nx\ \ I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{cc} z_1(\sin x)^{\lambda_1} & I_1\\ \vdots & \\ z_r(\sin x)^{\lambda_r} & I_2\end{array}\right]dx\\
&=\int_0^\pi(\sin x)^{m-1}\cos nx\\
&\times\left(\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)\right.\\
&\left. z_1^{s_1}(\sin x)^{\lambda_1 s_1}\cdots z_r^{s_r}(\sin x)^{\lambda_r s_r}ds_1\cdots ds_r\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots,z_r^{s_r} \\
&\quad\times\left(\int_0^{\pi}(\sin x)^{m-1+\lambda s_{11}+\cdots+\lambda_r s_r}\cos nx dx\right)ds_1\cdots ds_r \\
&\quad \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots,z_r^{s_r} \\
&\quad\times\left(\frac{\pi\cos n\frac{\pi}{2}\Gamma(\lambda_1 s_1+\cdots+\lambda_r s_r+m)}{2^{\lambda_1 s_1+\cdots+\lambda_r s_r+m-1}\frac{\Gamma(\lambda_1 s_1+\cdots+\lambda_r s_r+m+n+1)}{2}\frac{\Gamma(\lambda_1 s_1+\cdots+\lambda_r s_r+m-n+1)}{2}}\right)ds_1\cdots ds_r \\
&= \frac{\pi\cos n\frac{\pi}{2}}{2^{m-1}}I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+2:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} 2^{-\lambda_1}z_1 & I_3 \\ \vdots & \\ 2^{-\lambda_r}z_r & I_4\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
I_3 &= (1-m,\lambda_1\cdots\lambda_r)_1;(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\
I_4 &= {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q:\left(\frac{1}{2}-\frac{m}{2}-\frac{n}{2},\frac{\lambda_1}{2},\cdots,\frac{\lambda_r}{2}\right)_1;\left(\frac{1}{2}-\frac{m}{2}+\frac{n}{2},\frac{\lambda_1}{2},\cdots,\frac{\lambda_r}{2}\right); \\
&\quad {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_1^{(j)},\delta_j^{(1)},D_j^{(1)})_{q_1};\cdots;(d_j^{(r)},\delta_j^{(r)},1)_{m_r};\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}.
\end{aligned}
$$

4. Solution of the Problem

Combining (1.7) and (2.1), and making use of the integral (3.1), we derive

$$
\begin{aligned}
a_n &= \frac{2}{\pi}\int_0^{\pi}(\sin x)^{m-1}\cos nx\times\ I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{cc} z_1(\sin x)^{\lambda_1} & I_1 \\ \vdots & \\ z_r(\sin x)^{\lambda_r} & I_2\end{array}\right]dx \\
&= \frac{\pi\cos n\frac{\pi}{2}}{2^{m-1}}I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+2:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} 2^{-\lambda_1}z_1 & I_3 \\ \vdots & \\ 2^{-\lambda_r}z_r & I_4\end{array}\right] \qquad (4.1)
\end{aligned}
$$

On putting the value of a_n in (1.6) we get the following required solution of the problem.

$$
u(x,y)=\frac{a_0}{2\pi}y+\sum_{n=1}^{\infty}\frac{\pi\cos n\frac{\pi}{2}}{2^{m-2}}\frac{\sinh ny}{\sinh nx}\cos nx I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+2:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} 2^{-\lambda_1}z_1 & I_3 \\ \vdots & \\ 2^{-\lambda_r}z_r & I_4\end{array}\right] \qquad (4.2)
$$

provided that the condition stated with (3.1) are satisfied.

On taking $z_i = 0$ and $\lambda_i = 0$ for $i \geq 2$, (3.1) reduces to the result involving I-functions of 2-variables as follows:

Corollary 1 :

$$\int_0^{\pi} (\sin x)^{m-1} \cos nx \times I^{0,N:m_1,n_1;m_2,n_2}_{P,Q:p_1,q_1;p_2,q_2} \left[\begin{array}{c|c} z_1(\sin x)^{\lambda_1} & J_1 \\ \vdots & \\ z_2|(\sin x)^{\lambda_2} & J_2 \end{array} \right] dx$$

$$= \frac{\pi \cos n\frac{\pi}{2}}{2^{m-1}} I^{0,N+1:m_1,n_1;m_2,n_2}_{P+1,Q+2:p_1,q_1;p_2,q_2} \left[\begin{array}{c|c} 2^{-\lambda_1} z_1 & J_3 \\ \vdots & \\ 2^{-\lambda_r} z_r & J_4 \end{array} \right] \quad (4.3)$$

where

$$\begin{aligned} J_1 &= {}_1(a_j;\alpha_j^{(1)},\alpha_j^{(2)};A_j)_p :_1 (c_j^{(1)}.\gamma_j^{(1)};C_j^{(1)})_{p_1};_1 (c_j^{(2)},\gamma_j^{(2)};C_j^{(2)})_{p_2} \\ J_2 &= {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)};B_j)_Q : {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};_{m_1+1} (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1}; \\ &\quad {}_1(d_j^(2),\delta^(2)_j;1)_{m_2};_{m_2+1} (d_j^{(2)},\delta_j^{(2)},D_j^{(2)})_{q_2}. \\ J_3 &= (1-m,\lambda_1,\lambda_2)_1;(a_j;\alpha_j^{(1)},\alpha_j^{(2)};A_j)_P : {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\ {}_1(c_j^{(2)},\gamma_j^{(2)};C_j^{(2)})_{p_2} \\ J_4 &= {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)};B_j)_Q : \left(\frac{1}{2}-\frac{m}{2}-\frac{n}{2},\frac{\lambda_1}{2},\right);\left(\frac{1}{2}-\frac{m}{2}+\frac{n}{2},\frac{\lambda_1}{2},\frac{\lambda_2}{2}\right); \\ &\quad {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_1^{(j)},\delta_j^{(1)},D_j^{(1)})_{q_1};(d_j^{(2)},\delta_j^{(2)},1)_{m_2};\ {}_{m_2+1}(d_j^{(2)},\delta_j^{(2)};D_j^{(2)})_{q_2}. \end{aligned}$$

and

$$|arg(z_k)| < \frac{1}{2}\Delta_k \pi, \quad k = 1,2,\cdots,r, \quad Re(m) > 0.$$

$$\begin{aligned} \Delta_k &= \left(-\sum_{j=N+1}^{P} A_j\alpha_j^{(k)} - \sum_{j=1}^{Q} B_j\beta_j^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} \right. \\ &\quad \left. - \sum_{j=m_k+1}^{q_{ik}} D_j^{(k)}\delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)}\gamma_j^{(k)} \right) > 0. \end{aligned}$$

Remark 1 : On taking $z_2 = 0$ and $\lambda_2 = 0$, (4.3) reduces to the result proved by Ambika [1].

Remark 2 : On taking $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$ $(i, j, = 1, 2, \cdots, r)$, Remark 1 reduces to the result proved by S. S. Srivastava and Ritu Srivastava [6, p.79].

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