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# ON SOME NEW STRONGER FORMS OF FUZZY $g^{**}$ CONTINUOUS FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

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**Abstract**

The aim of this paper is to introduce and study some stronger forms of fuzzy $g^{**}$-continuous functions namely, strongly fuzzy $g^{**}$-continuous, perfectly fuzzy $g^{**}$-continuous and completely fuzzy $g^{**}$-continuous functions and their properties.

## 1. Introduction

Prof. L. A. Zadeh's [23] in 1965 introduced of the concept of 'fuzzy subset', in the year 1968, C L. Chang [6] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, C. K. Wong [22], R. H. Warren [21], R. Lowen[10], A. S. Mashhour [12], K. K. Azad [2], M. N. Mukherjee [13,14], G. Balasubramanian and P. Sundaram [3] and many others have contributed to the development of fuzzy topological spaces.

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Key Words : *Strongly $fg^{**}$-continuous, perfectly $fg^{**}$-continuous, completely $fg^{**}$-continuous.*

The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by C. L. Chang [6] and R. H.Warren [21] are included. Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of C. L. Chang [6] , R. H. Warren [20], and C. K. Wong [22] are presented. And some basic preliminaries are included. N. Levine [9] introduced generalized closed sets ($g$-closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology.
Dr. Sadanand Patil [15,16 and 17], S. P. Arya and R. Gupta [1], R. N. Bhaounik and Anjan Mukharjee [3], M. N. Mukharjee and B. Ghosh [13] and so many researchers have introduced and studied some stronger forms of fuzzy continuous functions like, Strongly, Perfectly and Completely fuzzy continuous functions and their mappings.

## 2. Preliminaries

Throughout this paper $(X,T),(Y,\sigma)$ and $(Z,\eta)$ or (simply $X,Y$ and $Z$) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset $A$ of a space $(X,T)$. $cl(A), int(A)$ and $C(A)$ denotes the closure, interior and the compliment of A respectively.

**Definition 2.01** : A fuzzy set $A$ of a fts $(X,T)$ is called:

(1) A semi-open fuzzy set, if $A \leq cl(int(A))$ and a semi-closed fuzzy set, if $int(cl(A)) \leq$ 0 [15]

(2) A pre-open fuzzy set, if $A \leq int(cl(A))f$ and a pre-closed fuzzy set, if $cl(int(A)) \leq A$ [15]

(3) A $\alpha$-open fuzzy set, if $A \leq int(cl(int(A)))$ and $\alpha$ a -closed fuzzy set, if $cl(int(cl(A))) \leq$ $A$ [16].

The semi closure (respectively pre-closure, $\alpha$-closure) of a fuzzy set $A$ in a fts $(X,T)$ is the intersection of all semi closed (respectively pre closed fuzzy set, $\alpha$-closed fuzzy set) fuzzy sets containing $A$ and is denoted by $scl(A)$ (respectively $pcl(A), \alpha cl(A)$).

**Definition 2.02** : A fuzzy set $A$ of a fts $(X,T)$ is called:

(1) A generalized closed ($g$-closed) fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and $U$ is open fuzzy set in $(X,T)$. [3]

(2) A generalized pre-closed ($gp$-closed) fuzzy set, if $pcl(A) \leq U$, whenever $A \leq U$ and $U$ is open fuzzy set in $(X,T)$. [15]

(3) A $\alpha$-generalized closed ($\alpha g$-closed) fuzzy set, if $\alpha cl(A) \leq U$, whenever $A \leq U$ and $U$ is open fuzzy set in $(X,T)$. [15, 16 and 17]

(4) A generalized $\alpha$-closed ($g\alpha$-closed) fuzzy set, if $\alpha cl(A) \leq U$, whenever $A \leq U$ and $U$ is open fuzzy set in $(X,T)$. [15,16 and 17]

(5) A generalized semi pre closed ($gsp$-closed) fuzzy set, if $spcl(A) \leq U$, whenever $A \leq U$ and $U$ is open fuzzy set in $(X,T)$. [15, 16 and 17]

(6) A $g^*$-closed fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and $U$ is $g$-open fuzzy set in $(X,T)$. [9]

(7) A $g^\#$-closed fuzzy set, if $cl(A) \leq U$, whenever $A \in U$ and $U$ is $\alpha g$-open fuzzy set in $(X,T)$. [15, 16]

(8) A $g^{\#\#}$-closed fuzzy set, if $\alpha cl(A) \leq U$, whenever $A \leq U$ and $U$ is $\alpha g$-open fuzzy set in $(X,T)$. [19]

(9) A $g^{**}$-closed fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and $U$ is $g^*$-open fuzzy set in $(X,T)$. [ ]

Complement of $g$-closed fuzzy (respectively $gp$-closed fuzzy set, $\alpha g$-closed fuzzy set, $g\alpha$-closed fuzzy set, $gsp$-closed fuzzy set, $g^*$-closed fuzzy set and $g^\#$-closed fuzzy set) sets are called $g$-open (respectively $gp$-open fuzzy set, $\alpha g$-open fuzzy set, $g\alpha$-open fuzzy set, $gsp$-open fuzzy set, $g^*$-open fuzzy set, $g^\#$-open fuzzy , $g^{**}$-open and $g^{\#\#}$-open fuzzy set) sets.

**Definition 2.03** : Let $X$ and $Y$ be two fuzzy topological Spaces, A function $f : X \to Y$ is called:

(1) A fuzzy continuous ($f$-continuous) if $f^{-1}(A)$ is closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [3]

(2) A fuzzy $\alpha$-continuous ($f\alpha$-continuous) if $f^{-1}(A)$ is $\alpha$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [15]

(3) A fuzzy generalized-continuous ($fg$-continuous) if $f^{-1}(A)$ is $g$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [15]

(4) A fuzzy generalized $\alpha$-continuous ($fg\alpha$-continuous) if $f^{-1}(A)$ is $g\alpha$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [3]

(5) A fuzzy $\alpha$-generalized continuous ($f\alpha g$-continuous) if $f^{-1}(A)$ is $\alpha g$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [15]

(6) A fuzzy $g^*$-continuous ($fg^*$-continuous) if $f^{-1}(A)$ is $g^*$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [15]

(7) A fuzzy $g^{\#}$-continuous ($fg^{\#}$-continuous) if $f^{-1}(A)$ is $g^{\#}$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [16]

(8) A fuzzy $g^{\#\#}$-continuous ($fg^{\#\#}$-continuous) if $f^{-1}(A)$ is $g^{\#\#}$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [19]

(9) A fuzzy $g^{**}$-continuous ($fg^{**}$-continuous) if $f^{-1}(A)$ is $g^{**}$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [19]

(10) Some New Fuzzy $g^{**}$-open Sets, Fuzzy $g^{**}$-Irresolute and Fuzzy $g^{**}$-Homeomorphism Mappings in Fuzzy Topological Spaces, International Journal of Science, Engineering and Management, ISSN: 2456-1304, Volume 01, Isue 05, Sept 2016(28-36). [20].

(11) A fuzzy $g^{\#\#}$-irresolute ($fg^{\#\#}$-irresolute) if $f^{-1}(A)$ is $g^{\#\#}$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [19]

(12) A fuzzy $g^{**}$-irresolute ($fg^{**}$-irresolute) if $f^{-1}(A)$ is $g^{**}$-closed fuzzy set in $X$, for every closed fuzzy set $A$ of $Y$. [ ]

(13) A fuzzy strongly continuous (strongly $f$-continuous) if $f^{-1}(V)$ is closed fuzzy set in $X$, for every closed set in $Y$. [1]

(14) A fuzzy strongly $g$-continuous (strongly $fg$-continuous) if $f^{-1}(V)$ is open fuzzy set in $X$, for every $g$-open set in $Y$. [13]

**Definition 2.04** : A map $f : X \to Y$ is called:

(1) fuzzy-open ($f$-open) iff $f(V)$ is open-fuzzy set in $Y$ for every open fuzzy set in $X$ [15]

(2) fuzzy $g$-open ($fg$-open) iff $f(V)$ is $g$-open-fuzzy set in $Y$ for every open fuzzy set in $X$ [15]

(3) fuzzy $g^*$-open ($fg^*$-open ) iff $f(V)$ is $g^*$-open-fuzzy set in $Y$ for every open fuzzy set in $X$ [16]

(4) fuzzy $g^{**}$-open ($fg^{**}$-open ) iff $f(V)$ is $g^{**}$-open-fuzzy set in $Y$ for every open fuzzy set in $X$ [ ]

(5) fuzzy $g^{\#}$-open ($fg^{\#}$-open ) iff $f(V)$ is $g^{\#}$-open-fuzzy set in $Y$ for every open fuzzy set in $X$ [16]

(6) fuzzy $g^{\#\#}$-open ($fg^{\#\#}$-open) iff $f(V)$ is $g^{\#\#}$-open-fuzzy set in $Y$ for every open fuzzy set in $X$ [19]

## 3. Strongly $g^{**}$-Continuous Function in Fuzzy Topologifal Spaces

**Definition 3.01** : A function $f : X \to Y$ is said to be strongly fuzzy $g^{**}$-continuous (briefly strongly $fg^{**}$-continuous) iff the inverse image of every $g^{**}$-open fuzzy set in $Y$ is open fuzzy set in $X$. Now we introduce the following.

**Theorem 3.02** : A function $f : X \to Y$ is strongly $fg^{**}$-continuous iff the inverse image of every $g^{**}$-closed fuzzy set in $Y$ is closed fuzzy set in $X$.

**Proof** : The proof follows from the definition.

**Theorem 3.03** : Every strongly $fg^{**}$-continuous function is a $f$-continuous function.

**Proof** : Let $f : X \to Y$ be strongly $fg^{**}$-continuous function. Let $V$ be open fuzzy set in $Y$, and $V$ is $g^{**}$-open set in $Y$. Then $f^{-1}(V)$ is open fuzzy set in $X$. Hence $f$ is $f$-continuous function. The converse of the above theorem need not be true as seen from the following example.

**Example 3.04** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B$ and $C$ be defined as follows. $A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}$, $B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}$, $C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}$. Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$ then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then $f$ is $f$-continuous as $B$ is open fuzzy set in $Y$ and $f^{-1}(B) = B$ is open fuzzy set in $X$. But

$f$ is not strongly $fg^{**}$-continuous as the fuzzy set $C$ is $g^{**}$-closed fuzzy set in $Y$ and $f^{-1}(C) = C$ is not closed fuzzy set in $X$.

**Theorem 3.05** : Every $f$-strongly continuous function is a strongly $fg^{**}$-continuous function.

**Proof** : Let $f : X \to Y$ be $f$-strongly continuous function. Let $V$ be $g^{**}$-open fuzzy set in $Y$. And then $f^{-1}(V)$ is both open and closed fuzzy set in $X$ as $f$ is $f$-strongly continuous function. Hence $f$ is strongly $fg^{**}$ -continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.06** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A_1, A_2, A_3, A_4, A_5$ and $A_6$ be defined as follows. $A_1 = \{(a, 1),(b, 0),(c, 0)\}$, $A_2 = \{(a, 0),(b, 1),(c, 0)\}$, $A_3 = \{(a, 0),(b, 0),(c, 1)\}$, $A_4 = \{(a, 1),(b, 1),(c, 0)\}$, $A_5 = \{(a, 1),(b, 0),(c, 1)\}$ and $A_6 = \{(a, 0),(b, 1),(c, 1)\}$. Consider $T = \{0, 1, A_1, A_2, A_4\}$ and $\sigma = \{0, 1, A_4\}$. Then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = b, f(b) = a$ and $f(c) = c$. Then $f$ is strongly $fg^{**}$-continuous but not $f$-strongly continuous as $A_1$ in $Y$ is such that $f^{-1}(A_1) = A_2$ is open fuzzy set in $X$ not closed fuzzy set in $X$.

**Theorem 3.07** : Let $f : X \to Y$ be strongly $fg^{**}$-continuous and $g : Y \to Z$ is strongly $fg^{**}$-continuous. Then the composition map $gof : X \to Z$ is strongly $fg^{**}$-continuous function.

**Proof** : Let $V$ be $g^{**}$-open fuzzy set in $Z$. Then $g^{-1}(V)$ is open fuzzy set in $Y$, since $g$ is strongly $fg^{**}$-continuous. Therefore $g^{-1}(V)$ is $g^{**}$-open fuzzy set in $Y$. Also since $f$ is strongly $fg^{**}$-continuous, $f^{-1}(g^{-1}(V) = (gof)^{-1}(V)$ is open fuzzy set in $X$. Hence $gof$ is strongly $fg^{**}$-continuous function.

**Theorem 3.08**: Let $f : X \to Y$, $g : Y \to Z$ be maps such that $f$ is strongly $fg^{**}$-continuous and $g$ is $fg^{**}$-continuous then $gof : X \to Z$ is f-continuous.

**Proof** : Let $F$ be a closed fuzzy set in $Z$. Then $g^{-1}(F)$ is $g^{**}$-closed fuzzy set in $Y$. Since $g$ is $fg^{**}$-continuous. And since $f$ is strongly $fg^{**}$-continuous, $f^{-1}(g^{-1}(F) = (gof)^{-1}(F)$ is closed fuzzy set in $X$. Hence $gof$ is $f$-continuous.

**Theorem 3.09** : If $f : X \to Y$ be strongly $fg^{**}$-continuous and $g : Y \to Z$ is $fg^{**}$-irresolute, then the composition map $gof : X \to Z$ is strongly $fg^{**}$-continuous.

**Proof** : Omitted.

## 4. Perfectly $g^{**}$-Continuous Function in Fuzzy Topological Spaces

**Definition 4.01** : A function $f : X \to Y$ called perfectly fuzzy $g^{**}$-continuous (briefly perfectly $fg^{**}$-continuous) if the inverse image of every $g^{**}$-open fuzzy set in $Y$ is both open and closed fuzzy set in $X$.

**Theorem 4.02** : A map $f : X \to Y$ is perfectly $fg^{**}$-continuous iff the inverse image of every $g^{**}$-closed fuzzy set in $Y$ is both open and closed fuzzy set in $X$.

**Proof** : The proof follows from the definition.

**Theorem 4.03** : Every perfectly $fg^{**}$-continuous function is $f$-continuous function.

**Proof** : Let $f : X \to Y$ be perfectly $fg^{**}$-continuous. Let $V$ be open fuzzy set in $Y$, and $V$ is $g^{**}$-open fuzzy set in $Y$. Since $f$ is perfectly $fg^{**}$-continuous, then $f^{-1}(V)$ is both open and closed fuzzy set in $X$. That is $f^{-1}(V)$ is open fuzzy set in $X$. Hence $f$ is $f$-continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 4.04** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B$ and $C$ be defined as follows. $A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}$, $B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}$, $C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}$. Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. Then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then $f$ is $f$-continuous but not perfectly $fg^{**}$-continuous as the fuzzy set $1 - C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{**}$-open fuzzy set in $Y$ and $f^{-1}(1 - C) = 1 - C$ which is not both open and closed fuzzy set in $X$.

**Theorem 4.05** : Every perfectly $fg^{**}$-continuous function is a $f$-perfectly continuous function.

**Proof** : let $f : X \to Y$ be perfectly $fg^{**}$-continuous. Let $V$ be open fuzzy set in $Y$, then $V$ be $g^{**}$-open fuzzy set in $Y$. Since $f$ is perfectly $fg^{**}$-continuous. Then $f^{-1}(V)$ is both open and closed fuzzy set in $X$. And hence $f$ is $f$-perfectly continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 4.06** : Example : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B$ and $C$ be defined as follows.

$A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}, B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}, C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}$. Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. Then $(X, T) \to$ and $(Y, \sigma)$ are $fts$. Define $f : XY$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then $f$ is $f$-perfectly function. As the fuzzy set in $B$ is open fuzzy set in $Y$, and its inverse image $f^{-1}(B) = B$ is both open and closed fuzzy set in $X$. But $f$ is not perfectly $fg^{**}$-continuous as the fuzzy set $1-C=\{(a,0.2),(b,0.5),(c,0.1)\}$ is $g^{**}$-open fuzzy set in $Y$ and $f^{-1}(1-C)=1-C$ which is not both open and closed fuzzy set in $X$.

**Theorem 4.07** : Every perfectly $fg^{**}$-continuous function is strongly $fg^{**}$-continuous function.

**Proof** : Let $f:X\to Y$ be perfectly $fg^{**}$-continuous. Let $V$ be $g^{**}$-open fuzzy set in $Y$. Then $f^{-1}(V)$ is both open and closed fuzzy set in $X$. Therefore $f^{-1}(V)$ is open fuzzy set in $X$. Hence $f$ is strongly $fg^{**}$-continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 4.08** : Let $X=Y=\{a,b,c\}$ and the fuzzy sets $A_1,A_2,A_3,A_4,A_5$ and $A_6$ be defined as follows.

$A_1=\{(a,1),(b,0),(c,0)\}$, $A_2=\{(a,0),(b,1),(c,0)\}$, $A_3=\{(a,0),(b,0),(c,1)\}$, $A_4=\{(a,1),(b,1),(c,0)\}$, $A_5=\{(a,1),(b,0),(c,1)\}$ and $A_6=\{(a,0),(b,1),(c,1)\}$. Consider $T=\{0,1,A_1,A_2,A_4\}$ and $\sigma=\{0,1,A_4\}$. Then $(X,T)$ and $(Y,\sigma)$ are $fts$. Define $f:X\to Y$ by $f(a)=b$, $f(b)=a$ and $f(c)=c$. Then $f$ is strongly $fg^{**}$-continuous but not perfectly $fg^{**}$-continuous as the fuzzy set $A_3$ is $g^{**}$-closed fuzzy set in $Y$ and $f^{-1}(A_3)=A_3$ is not both open and closed fuzzy set in $X$.

**Theorem 4.09** : Let $f:X\to Y$, $g:Y\to Z$ be two perfectly $fg^{**}$-continuous function then $gof:X\to Z$ is perfectly $fg^{**}$-continuous function.

**Proof** : Let $V$ be $g^{**}$-open fuzzy set in $Z$. Then $g^{-1}(V)$ is both open and closed fuzzy set in $Y$, since $g$ is perfectly $fg^{**}$-continuous. Therefore $g^{-1}(V)$ is $g^{**}$-open fuzzy set in $Y$. Also since $f$ is perfectly $fg^{**}$-continuous. $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is both open and closed fuzzy set in $X$. Hence $gof$ is perfectly $fg^{**}$-continuous function.

**Theorem 4.10** : Let $f:X\to Y$ be perfectly $fg^{**}$-continuous and $g:Y\to Z$ be $g^{**}$-irresolute function then $gof:X\to Z$ is perfectly $fg^{**}$-continuous function.

**Proof** : Omitted.

## 5. Completely $g^{**}$-Continuous Function in Fuzzy Topological Spaces

**Definition 5.01** : A map $f:X\to Y$ is called completely fuzzy $g^{**}$-continuous (briefly completely $g^{**}$-continuous) if the inverse image of every $g^{**}$-open fuzzy set in $Y$ is regular-open fuzzy set in $X$.

**Theorem 5.02** : A map $f:X\to Y$ is completely $fg^{**}$-continuous. Iff the inverse image of every $g^{**}$-closed fuzzy set in $Y$ is regular-closed fuzzy set in $X$.

**Proof** : The proof follows from the definition.

**Theorem 5.03** : Every completely $fg^{**}$-continuous function is a $f$-continuous function.

**Proof** : Let $f : X \to Y$ be completely $fg^{**}$-continuous function. Let $V$ be open fuzzy set in $Y$. Then $V$ is $g^{**}$-open fuzzy set in $Y$. And then $f^{-1}(V)$ is both regular-open fuzzy set in $X$, and therefore $f^{-1}(V)$ is open fuzzy set in $X$. Hence $f$ is $f$-continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 5.04** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B$ and $C$ be defined as follows.

$A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}$, $B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}$, $C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}$.
Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. Then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then $f$ is $f$-continuous function but not completely $fg^{**}$-continuous as the fuzzy set $1 - C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{**}$-open fuzzy set in $Y$ and $f^{-1}(1 - C) = 1 - C$ which is not regular open fuzzy set in $X$.

**Theorem 5.05** : Every completely $fg^{**}$-continuous function is a $f$-completely continuous function.

**Proof** : Let $f : X \to Y$ be completely $fg^{**}$-continuous. Let $V$ be open fuzzy set in $Y$. Then $V$ be $g^{**}$-open fuzzy set in $Y$. Then $f^{-1}(V)$ is regular-open fuzzy set in $X$. Hence $f$ is $f$-completely continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 5.06** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B$ and $C$ be defined as follows.

$A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}$, $B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}$, $C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}$.
Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. Then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then $f$ is $f$-completely continuous function as the fuzzy set $B$ is open fuzzy set in $Y$, and its inverse image $f^{-1}(B) = B$ is regular-open fuzzy set in $X$. But not completely $fg^{**}$-continuous as the fuzzy set $1 - C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{**}$-open fuzzy set in $Y$ and $f^{-1}(1 - C) = 1 - C$ which is not regular open fuzzy set in $X$.

**Theorem 5.07** : Every completely $fg^{**}$-continuous function is strongly $fg^{**}$-continuous function.

**Proof** : Let $f : X \to Y$ be completely $fg^{**}$-continuous. Let $V$ be $g^{**}$-open fuzzy set in $Y$. Then $f^{-1}(V)$ is regular-open fuzzy set in $X$. Therefore $f^{-1}(V)$ is open fuzzy set in $X$. Hence $f$ is strongly $fg^{**}$-continuous function.

The converse of the above theorem need not be true as seen from the following example.

**Example 5.08** : Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A_1, A_2, A_3, A_4, A_5$ and $A_6$ be defined as follows.

$A_1 = \{(a, 1), (b, 0), (c, 0)\}$, $A_2 = \{(a, 0), (b, 1), (c, 0)\}$, $A_3 = \{(a, 0), (b, 0), (c, 1)\}$, $A_4 = \{(a, 1), (b, 1), (c, 0)\}$, $A_5 = \{(a, 1), (b, 0), (c, 1)\}$ and $A_6 = \{(a, 0), (b, 1), (c, 1)\}$. Consider $T = \{0, 1, A_1, A_2, A_4\}$ and $\sigma = \{0, 1, A_4\}$. Then $(X, T)$ and $(Y, \sigma)$ are $fts$. Define $f : X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then $f$ is strongly $fg^{**}$-continuous but not completely $fg^{**}$-continuous as the fuzzy set $A_3$ is $g^{**}$-closed fuzzy set in $Y$ and its inverse image $f^{-1}(A_3) = A_3$ is not regular- closed fuzzy set in $X$.

**Theorem 5.09** : If $f : X \to Y$ is completely $fg^{**}$-continuous and $g : Y \to Z$ is $fg^{**}$-irresolute function then $gof : X \to Z$ is completely $fg^{**}$-continuous function.

**Proof** : Let $V$ be $g^{**}$-open fuzzy set in $Z$. Then $g^{-1}(V)$ is $g^{**}$-open fuzzy set in $Y$, since $g$ is $fg^{**}$-irresolute function. Also since $f$ is completely $fg^{**}$-continuous. $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is regular-open fuzzy set in $X$. Hence $gof$ is completely $fg^{**}$-continuous function.

**Theorem 5.10** : If $f : X \to Y$ and $g : Y \to Z$ be two completely $fg^{**}$-continuous function then $gof : X \to Z$ is completely $fg^{**}$-continuous function.

**Proof** : Omitted.

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