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COVERING OF PROTOCOL SPACES

YONGHONG LIU
SCHOOL OF AUTOMATION,
WUHAN UNIVERSITY OF TECHNOLOGY, WUHAN, CHINA

ABSTRACT. We define the protocol spaces with the help of the covering concept. For protocol sets, we represent the covering relation between sets, and we introduce the kinetic functions. We propose the covering axioms and discuss the properties of the covering. Finally, we propose the minimal covering problem.

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1. Introduction

Rosenfeld (See [**1**]) studied topological relationships on a digital image display, which is said to be of digital topology. Recently, the rapid development of AI technology, so it is very important to study the topological properties of the image. As a geometric property, covering means that if two points are in different sets, then the line segment connecting the two points is greater than or equal to zero. Therefore, our purpose and motivation in creating protocol spaces is to try to establish the covering theory. Why is protocol space worth studying? From a theoretical point of view, some results, such as protocol mapping theorem, covering theorem, etc. hold in protocol spaces.

In topology, the concept of covering is intuitive. So we introduce the covering relationship between sets in the protocol spaces. The protocol geometry mainly studies some basic and intuitive problems, such as image recognition problems and matching problems. The covering theory has important application in this regard. We believe that the study of such problems involves axiomatic sets theory, discrete geometry, mathematical logic, combination mathematics, abstract algebras, topological algebras, dynamical systems, and computer science.

Certainly, in the initial development of the field of protocol spaces, we work focused on defining and studying the covering of topological concept.

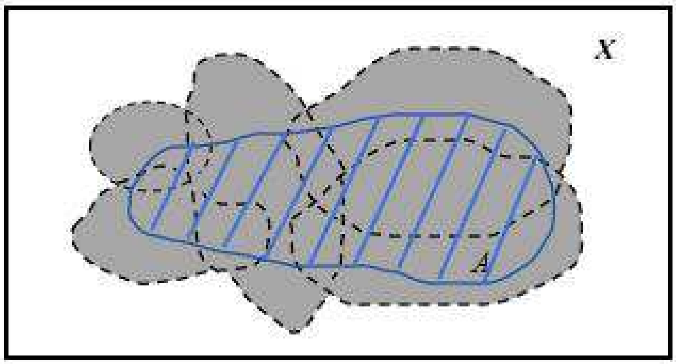

Figure 1. An open covering of A.

2. Protocol Spaces

In this section, we will begin the covering construction after making a few definitions.

Definition 1.([2]) Let A be a subset of a topological space X, and let $\mathcal{O}$ be a collection of subset of X.

i) The collection $\mathcal{O}$ is said to covering A or to be a covering of A if A is contained in the union of the sets in $\mathcal{O}$.

ii) If $\mathcal{O}$ covering A, and each set in $\mathcal{O}$ is an open, then we call $\mathcal{O}$ is just an open covering of A. (See Figure 1)

iii) If $\mathcal{O}$ covering A, and $\mathcal{O}'$ is a subcollection of $\mathcal{O}$ that also covering A, then $\mathcal{O}'$ is called a subcovering of $\mathcal{O}$.

Definition 2. Let $X = (X, \mathcal{O}_X)$ be a protocol space. A covering system A on X is a map $x \mapsto A_x$ assigning any subset $A_x \subseteq \text{Spec } \mathcal{O}_X, x$ to each point $x \in X$. For covering systems A, B on X we write $A \subseteq B$ to mean $A_x \subseteq B_x$ for all $x \in X$. If A is a covering to the closed subspace of X, then A is called a covering of Spec $\mathcal{O}_X$.

Definition 3. A protocol space X is covering if every open covering of X has a finite subcovering.

Remark 1. *Since Definition 3 is special case of Definition 1, suppose $A \subset X$. Then A is covering in X if and only if every of A by sets that are open in X has a finite subcovering.*

Example 1. Consider three circles S^1, and shown in Figure 2. Here the protocol Spec $\mathcal{O}_X(S^1)$ is a torus, $S^1 \times S^1$. In order to picture Spec $\mathcal{O}_X(S^1)$, we

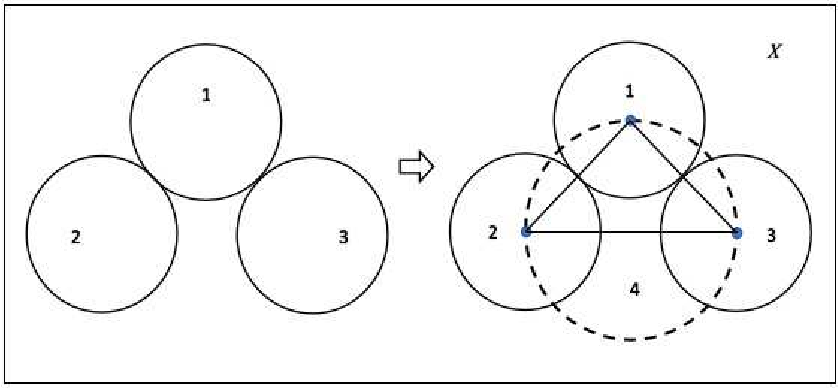

Figure 2. Spec $\mathcal{O}_X(S^1)$ is an open covering.

examine how the three subsets by an open covering to form Spec $\mathcal{O}_X(S^1)$.

To begin, we connect the center of the three circles S^1 to form a triangle. Therefore, there is a circle constructed by the triangle, then S^1, by itself, there is an open covering of $\mathcal{O}_X$, is a circle.

The protocol spaces satisfy two basic properties, such as:

Axiom 1.(The union axiom.) *Let $X = (X, \mathcal{O}_X)$ be a protocol space. If $C_1, ..., C_n$ are each covering in X, then $\bigcup_{j=1}^{n} C_j$ is covering in X.*

Axiom 2.(The intersection axiom.) *If X is Hausdorff, and $\{C_\alpha\}_{\alpha \in A}$ is a collection of sets that is covering in X, then $\bigcap_{\alpha \in A} C_\alpha$ is covering in X.*

Theorem 1. *Let X be a protocol space and map $f : X \to Y$ be continuous, and let A be covering in X. Then $f(A)$ is covering in Y.*

Proof. It suffices to show that $f(A)$ is covering in Y, let $\mathcal{O}$ be a covering of $f(A)$ by open sets in Y. Then $f^1(G)$ is open in X for every open set G in $\mathcal{O}$. We have

$$\mathcal{O}' = \{f^{-1}(G) \mid G \in \mathcal{O}\} \tag{1}$$

is a covering of A by an open set in X. Since A is covering, Remark 1 implies that there is a finite subcollection of $\mathcal{O}'$, say $\{f^{-1}(G_1), ..., f^{-1}(G_n)\}$, that covering A. Then the collection of an open set $\{ G_1, ..., G_n \}$ in $\mathcal{O}$ covering $f(A)$. Thus, $\mathcal{O}$ has a finite subcovering. The proof is complete. □

Definition 4. A vector bundle E on the protocol space X is a map $\pi : E \to X$ satisfies all of the following:

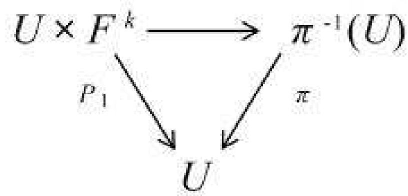

Figure 3. The diagram for the protocol space.

i) Suppose that for each $x \in X$. Then, fibre

$$E_x = \pi^{-1}(x) \tag{2}$$

is a finite dimensional vector space and k is a dimension.

ii) Suppose that for each $x \in X$, and it is in a neighborhood U of X. If there exists a homeomorphism

$$\varphi : U \times F^k \to \pi^{-1}(U) \tag{3}$$

making the diagram in Figure 3.
where all F^k are basic field and p_1 is the projection

$$p_1(x, \nu) = x. \tag{4}$$

For each $x \in U$ and map

$$\varphi_x : F^k \to E_x, \quad \nu \mapsto \varphi(x, \nu) \tag{5}$$

is linear isomorphic mapping.

Definition 5. Let X be a protocol space and suppose that a point $x \in U_i \cap U_j$, the kinetic function

$$\varphi_{ij} : U_i \cap U_j \to GL(F, k) \tag{6}$$

defined by

$$\varphi_{ij}(x) = \varphi_{ix}^{-1} \circ \varphi_{jx} \tag{7}$$

defined by

$$f : F^k \to F^k. \tag{8}$$

$GL(F, k)$ is called general linear group.

Theorem 2. *Let X be a protocol space and let $\mathcal{B} = \{U_i\}_{i \in I}$ be an open covering with kinetic function. Then $\{\varphi_{ij}\}$ is a vector bundle $\pi : E \to X$.*

Proof. Let $\mathcal{E} = \bigsqcup_i U_i \times F^k$ be a disjoint union, we define

$$(i, x, \nu) \sim (j, x', \nu') \tag{9}$$

is an equivalence relation. If
$$x = x', \tag{10}$$
and
$$\nu = \varphi_{ij}(x)\nu'. \tag{11}$$
Now, suppose that $E = \mathcal{E}/\sim$, and so $\pi[i, x, \nu] = x$, by Definition 4 we have
$$\varphi_i : U_i \times F^k \to E, \tag{12}$$
and
$$U_i \times F^k \to \mathcal{E} \to \mathcal{E}/\sim = E. \tag{13}$$
Thus $\pi : E \to X$, as required. $\square$

The Minimal Covering Problem. *Covering is a connected protocol space. The problem:*

Provide a method that can construct the minimal covering.

Although covering in Example 1 is not minimal covering.

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