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PROPAGATION OF SOLITONS OF ARBITRARY AMPLITUDE KINETIC ALFVEN WAVES IN DUSTY PLASMAS

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Abstract

A two fluid model has been employed to study the propagation of solitary kinetic Alfvén waves in presence of negatively charged dust particles. The set of basic equations governing the ions, electrons, dust and Maxwell's equation have been reduced to a single equation known as the Sagdeev Potential (SP) equation. An exact analytical expression for the SP or energy integral equation is obtained. Parametric ranges for the existence of arbitrary amplitude soliton are studied in detail. The SP is evaluated numerically in cases when solitary waves exist analytically. Study has been made related to the transition of the waves from shear to kinetic Alfvén waves and corresponding characters in terms of Mach numbers.

Key Words : *Kinetic Alfven Wave, SP, Mach number.*

1. Introduction

Alfvén waves (AWs) have been extensively studied because of their potential importance in the particle energization of magnetized plasmas and have been applied to laboratory [1, 2], astrophysical [3] and space [4] plasmas, tokamak plasma heating [5], the auroral electron acceleration [6], the solar coronal plasma heating [7] and the anomalous heating of heavy ions in the extended corona [8]. Therefore, the mechanism of generation and excitation of AWs is of great interest in wide areas. Kinetic Alfvén waves (KAWs) are of importance in the study of coupling between the ionosphere and magnetosphere [9]. These KAWs are dispersive waves and can generate a parallel electric field in a plasma with $\beta > m_e/m_i$ (the electron-to-ion mass ratio), the kinetic limit. Moreover, nonlinear interaction among the KAWs can also occur because of their polarization properties [10]. Rau and Tajima [11] examined the ion acceleration by a nonlinear compressional Alfvén wave propagating perpendicular to an arbitrarily strong external magnetic field. Ofman and Davila [12] investigated the nonlinear effects that drive the solitary waves associated with Alfvén waves in the context of coronal holes. It is worth mentioning here that ideal magnetohydrodynamic Alfvén waves are not of a dispersive nature. However, if the perpendicular wavelength is comparable to the gyroradius, the ions will no longer follow the magnetic lines of force, whereas the electrons, owing to their small Larmor radius, will still be attached to the field lines. As a result, charge separation follows and leads to what are termed as kinetic Alfvén waves.

Most of the solitary kinetic Alfvén wave (SKAW) studies were made for plasma with isothermal electrons, except a few. In reality, the polytropic index varies significantly under different physical conditions. It has now become a common concept that the acceleration of electrons and ions responsible for the aurora could be due to Alfvén waves [13]. Alfvénic solitons are finite-amplitude waves of permanent form which owe their existence to a balance between nonlinear wave steepening and dispersion of plasma medium. Dissipation mechanism such as collision and viscosity can drastically change the structure of the solitary Alfvén wave to shock-like structures [14, 15]. Initially the formation and propagation of solitary wave solutions for KAWs were investigated by Hasegawa and Mima [16]. Yu and Shukla [17] investigated that the existence of SKAWs propagating in an oblique direction w.r.t. the ambient magnetic field in a magnetized plasma with $\alpha \gg 1$ where $\alpha = \beta/2Q, \beta \rightarrow$ thermal pressure by electron/magnetic pressure by ambient magnetic field and $Q =$ electron to ion mass ratio. Kalita and Kalita [18] showed the existence of both super and subsonic rarefactive Alfvén solitons in a low-β ($\ll m_e/m_i$) plasma [18]. Moreover Kalita and Bhatta [19] have investigated the exact nonlinear localized Alfvén SW solutions in a low β plasma with hot electrons and finite electron inertia. The generation of electric fields which are parallel to the local magnetic field direction is found to be self-consistent and accordingly the formation and propagation of KA solitons have been studied through the Sagdeev Potential (SP) equation [20]. Woo et al. [21] investigated the effect of a small electron viscosity on KASWs in extremely low β plasma; they also demonstrated the formation of a kinetic Alfvén double layer for a large amplitude KASW. The solitary waves (SWs) (double layers (DLs)) are the localized symmetric (asymmetric) potential structures with no net potential drop (a net potential drop). Such localized structures were detected from the auroral acceleration region, for the first time by the S3-3 satellite [22]. Wu et al. [23] identified kinetic Alfvén solitary waves (KASWs) accompanied by both dip-type and hump-type density structures in the Freja observations. Louarn et al. [24] also examined the localized strong electromagnetic perturbations that were observed by the F4 experiment in the Freja satellite; Huang et al. [25] later identified these perturbations as KASW eigenmodes. Using data from the POLAR and CLUSTER satellites, Chen [26] reported an observation of large amplitude electromagnetic fluctuations and the associated energization of particles in the local magnetospheric cusp region. KASWs have also been observed during numerous space missions. More attention has been given to solitary kinetic Alfvén waves because of the data published from Freja satellite observations [27, 28]. Recently, spacecraft data provided new information about lower hybrid solitary structures. Lakhina et al. [29] have suggested that electron-acoustic solitons/double layers can explain the generation of magnetosheath electrostatic solitary structures and the broadband electrostatic noise in the plasma sheet boundary layer. Studies on the formation of large amplitude KAW solitons and double layers in plasmas have been studied by Devi et al. [31]. They have shown that the amplitude and width of soliton vary with the Mach number (M) and direction cosine k_z. Gogoi and Devi et al. [30] have studied the large amplitude electrostatic structures associated with low-frequency dust kinetic Alfvenic waves are investigated under the pressure gradient indicative of dust dynamic through SP equation [31]. In this paper we have studied the propagation of solitary kinetic Alfvén waves in presence of negatively charged dust particles through SP equation. The SP is evaluated numerically in cases when solitary waves exist analytically. Study has been made related to the transition of the waves from shear to kinetic Alfvén solitary waves and corresponding characters in terms of Mach numbers.

2. Basic Equations and Linear Dispersion Relation

Here we have considered a general type of ion motion under the effect of pressure gradient and inertia of electrons moving in the direction of the external magnetic field $\mathbf{B_0} = B_0\hat{z}$. We have adopted a fluid plasma model since the rate of Landau damping remains small under inertial effect of electrons. We consider ions to move in the (ZX-plane), Z-axis being the direction of external magnetic field. Electrons move along the magnetic field to establish charge equilibrium.

Basic equations are as follows:

For the electrons,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_{ez}) = 0 \tag{1}$$

$$\frac{\partial v_{ez}}{\partial t} + v_{ez}\frac{\partial v_{ez}}{\partial z} = \alpha\left(\frac{\partial \psi}{\partial z} - \frac{1}{n_e}\frac{\partial n_e}{\partial z}\right). \tag{2}$$

For the ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) + \frac{\partial}{\partial z}(n_i v_{iz}) = 0 \tag{3}$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial v_{ix}}{\partial x} + v_{iz}\frac{\partial v_{ix}}{\partial z} = -\alpha Q\frac{\partial \phi}{\partial z} + v_{iy} \tag{4}$$

$$\frac{\partial v_{iy}}{\partial t} + v_{ix}\frac{\partial v_{iy}}{\partial x} + v_{iz}\frac{\partial v_{iy}}{\partial z} = -v_{ix} \tag{5}$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial v_{iz}}{\partial x} + v_{iz}\frac{\partial v_{iz}}{\partial z} = -\alpha Q\frac{\partial \psi}{\partial z}. \tag{6}$$

From Maxwell's equations,

$$\frac{\partial^4}{\partial x^2 \partial z^2}(\phi - \psi) = \frac{1}{\alpha Q}\left[\frac{\partial^2 n_e}{\partial t^2} + \frac{\partial^2}{\partial z \partial t}(n_i v_{iz})\right]. \tag{7}$$

Charge neutrality, $\delta_e n_e + z\delta_d - n_i = 0$ (8)

Charge neutrality at equilibrium gives, $\delta_d = 1 - \delta_e$ (9)

Here $Q = m_e/m_i$ (electron to ion mass ratio), two potentials ϕ, ψ are included to justify a low $-\beta$ plasma model, z (dust charge number), and $\alpha = \beta/2Q$, $\delta_e = \frac{n_{e0}}{n_{i0}}$,

$\delta_d = \frac{z n_{d0}}{n_{i0}}$. We have normalized densities by the equilibrium plasma density n_0, time by the inverse of the ion cyclotron frequency Ω_{ci}^{-1}, velocities by Alfvén velocity $V_A = cB_0/(4\pi n_0 m_i)^{1/2}$, space by $\rho_s = c/\omega_{pi}$ (the ratio between the velocity of light and the ion plasma frequency), electric fields by $\frac{T_e \Omega_{ci}}{eV_A}$ and magnetic field by B_0. Using linearized equations (1) - (8) and to find the dispersion relation we have used the Poisson's equation as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = -4\pi e(n_i - n_e) \tag{10}$$

and after some algebraic manipulation, by using the variables,

$$V_A = \frac{eB_0}{\sqrt{4\pi n_{i0} m_i}}, \quad \Omega_c = \frac{eB_0}{m_i}, \quad C_s^2 = \frac{T_e}{m_i}, \quad Q = \frac{m_e}{m_i}$$

$$\omega_{pi}^2 = \frac{4\pi n_{i0} e^2}{m_i} \quad \omega_{pe}^2 = \frac{4\pi n_{e0} e^2}{m_e}, \quad \frac{\omega_{pi}^2}{\Omega_c^2} = \frac{c^2}{V_A^2}, \quad \frac{\omega_{pe}^2}{\omega_{pi}^2} = Q$$

we have obtained the dispersion relation as follows:

$$\left[\frac{\omega_{pe}^2}{c^2}\left(Q - \frac{m_e \omega^2}{B}\right) - 1 + k_z^2\right]\left[\frac{1}{k_z^2} - \frac{Q\omega_{pe}^2}{(\omega^2 - \Omega_c^2)k_z^2}\right] = \left[\omega_{pe}^2\left(\frac{Q}{\omega^2} - \frac{m_e}{B}\right) - 1\right] \tag{11}$$

where $B = (ek_z^2 - \omega^2 m_i)$.
From equation (11), we note that the dispersion relation obtained here depends on parameters k_z, m_e and others plasma parameters.

3. Derivation of the Sagdeev's Potential Equation

To obtain Sagdeev equation, we consider a stationary wave in the moving frame defined by $\xi = xk_x + zk_z - Mt$ with $k_x^2 + k_z^2 = 1$ where M is Mach number of the wave in the unit of the Alfvén velocity V_A. Using this set up for stationary frame, equations (1) - (7) can be reduced to the followings

$$v_{ez} = \frac{Mn_e - M}{k_z v_{ez}} \tag{12}$$

$$n_e = e^{\psi}\left[\exp A\left(1 - \frac{1}{n_e^2}\right)\right]. \tag{13}$$

Differentiating (13) w.r.t. "ξ" we get,

$$\Rightarrow \frac{\partial \psi}{\partial \xi} = \left[\frac{1}{n_e} - \frac{2A}{n_e^3}\right]\frac{\partial n_e}{\partial \xi} \tag{14}$$

where the parameter A is given by $A = \frac{M^2}{2\alpha k_z^2}$

$$k_x v_{ix} + k_z v_{iz} = M\left(1 - \frac{1}{n_i}\right) \tag{15}$$

$$-\left(\frac{M}{n_i}\right)\frac{\partial v_{ix}}{\partial \xi} = -\alpha Q k_x \frac{\partial \phi}{\partial \xi} + v_{iy} \quad (16$$

$$-\left(\frac{M}{n_i}\right)\frac{\partial v_{iy}}{\partial \xi} = -v_{ix} \tag{17}$$

$$-\left(\frac{M}{n_i}\right)\frac{\partial v_{iz}}{\partial \xi} = -\alpha Q k_z \frac{\partial \psi}{\partial \xi} \tag{18}$$

$$k_x^2 k_z^2 \frac{\partial^4(\phi - \psi)}{\partial \xi^4} = \frac{1}{\alpha Q}\left[M^2 \frac{\partial^2 n_e}{\partial \xi^2} - M k_z \frac{\partial^2}{\partial \xi^2}(n_i v_{iz})\right]. \tag{19}$$

Under the boundary conditions $v_{ix} = v_{iz} = v_{ez} = 0$; $\phi = \psi = 0$ at $n_i = n_e = 1$ when $\xi \to \infty$. Using (14), (18) becomes,

$$\Rightarrow \left(\frac{M}{n_i}\right)\frac{\partial v_{iz}}{\partial \xi} = \alpha Q k_z \left[\frac{1}{n_e} - \frac{2A}{n_e^3}\right]\frac{\partial n_e}{\partial \xi}. \tag{20}$$

From now onwards we shall use the quasi neutrality condition, $n_i = \delta_e n_e + z\delta_d$ in the above equations. Also we have used charge neutrality condition $n_i = n_e$. Let, $1 - n_e = N$, then $n_e = 1 - N$, write, $1 - N = p$.

Equation (20) becomes,

$$v_{iz} = \frac{\alpha Q k_z}{M}\left[-N\delta_e + \frac{N}{1-N} 2A\delta_e + z\delta_d ln(1-N) + Az\delta_d \frac{N(2-N)}{(1-N)^2}\right]. \tag{21}$$

Using boundary conditions $n_e \to 1, 1 - N = p \to 1, \xi \to \infty$.

Integrating (19) twice with respect to ξ under same boundary condition with some algebraic manipulation, we get the following equation:

$$\begin{aligned}
&\frac{1}{\alpha Q}\left[\left[\frac{\delta_e M^2}{(\delta_e p + z\delta_d)^3} - \alpha Q\left[\frac{1}{p} - \frac{2A}{p_3}\right]\right]\frac{dp}{d\xi}\frac{d}{d\xi}\left[\left[\frac{\delta_e M^2}{(\delta_e p + z\delta_d)^3} - \alpha Q\left[\frac{1}{p} - \frac{2A}{p^3}\right]\right]\right]\frac{dp}{d\xi}\right] \\
&= \left[-\frac{(\delta_e p + z\delta_d)}{M\alpha Q}\left[M - \frac{M}{(\delta_e p + z\delta_d)} - \frac{\alpha Q k_z^2}{M}\left[\delta_e(p-1) + \frac{(1-p)}{p}2A\delta_e + z\delta_d ln\ p + Az\delta_d\right.\right.\right. \\
&\left.\left.\left.\frac{(1-p)(p+1)}{p^2}\right]\right] + \frac{1}{\alpha Q k_z^2}\left[-M^2(1-p) - k_z M(\delta_e p + z\delta_d)\frac{\alpha Q k_z}{M}\left[-(1-p)\delta_e + \frac{(1-p)}{p}\right.\right. \\
&\left.\left.\left.2A\delta_e + z\delta_d lnp + Az\delta_d\frac{(1-p)^2}{p^2}\right]\right]\right]\left[\left[\frac{\delta_e M^2}{(\delta_e p + z\delta_d)^3} - \alpha Q\left[\frac{1}{p} - \frac{2A}{p^3}\right]\right]\frac{dp}{d\xi}\right]
\end{aligned} \tag{22}$$

Multiplying both sides of (22), by the term in the parentheses on the left hand side and after some algebra, we get an expression of the form

$$\frac{1}{2}\left(\frac{dp}{d\xi}\right)^2 + K(p,\alpha,Q,M,k_z,\delta_e) = 0 \tag{23}$$

which is the Sagdeev's potential equation, with

$$\begin{aligned}
K(p,\alpha,Q,M,k_z,\delta_e) &= \frac{1}{\frac{1}{\alpha Q}\left[\frac{\delta_e M^2}{(\delta_e p+z\delta_d)^3} - \alpha Q\left[\frac{1}{p}-\frac{2A}{p^3}\right]\right]^2}\left[\frac{M^2}{\alpha Q}\left\{\frac{1}{(\delta_e+z\delta_d)} - \right.\right. \\
&\left.\frac{1}{(\delta_e p+z\delta_d)}\right\} + \delta_e\left\{1+2A+Az-p-\frac{2A}{p}-\frac{Az\delta_e}{p^2}\right\} - z\delta_d\log p + \frac{M^2}{2\alpha Q} \\
&\left\{\frac{1}{(\delta_e p+z\delta_d)^2}-\frac{1}{(\delta_e+z\delta_d)^2}\right\} + \log p + A\left(\frac{1}{p^2}-1\right) + k_z^2\delta_e^2\left\{\frac{1}{(\delta_e+z\delta_d)^2}-\frac{1}{(\delta_e p+z\delta_d)^2}\right\} \\
&+\frac{k_z^2\delta_e p}{(\delta_e p+z\delta_d)} - \frac{k_z^2\delta_e}{(\delta_e+z\delta_d)} + k_z^2\{\log(\delta_e+z\delta_d)-\log(\delta_e p+z\delta_d)\} - 2Ak_z^2\delta_e\left\{\frac{1}{(\delta_e p+z\delta_d)}\right. \\
&\left.-\frac{1}{(\delta_e+z\delta_d)^2}\right\} - 2Ak_z^2\delta_e^2\left\{\frac{1}{(z\delta_d)^2}\{\log p-\log(\delta_e p+z\delta_d)+\log(\delta_e+z\delta_d)\}\right. \\
&\left.\left.+\frac{1}{(z\delta_d)}\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\}\right\}\right\} + \frac{k_z^2\alpha Q}{M^2}\left\{\frac{\delta_e^2}{2}(p^2-1)+(z\delta_d-\delta_e)\right. \\
&\left.\left\{\left(2A\delta_e\frac{1}{p}-\delta_e p\right)-(2A\delta_e-\delta_e)\right\} - \log p\,\delta_e(z\delta_d+2A\delta_e) - Az\delta_e\delta_d\left(\frac{1}{p^2}-1\right)\right\} \\
&+\frac{k_z^2}{M^2}2A\delta_e\alpha Q\left\{\delta_e\log p-\delta_e(p-1)-z\delta_d\left(\frac{1}{p}-1\right)-z\delta_d\log p+A\delta_e\left(\frac{1}{p^2}-1\right)\right. \\
&\left.-2A\delta_e\left(\frac{1}{p}-1\right)+\frac{2Az\delta_d}{3}\left(\frac{1}{p^3}-1\right)-Az\delta_d\left(\frac{1}{p^2}-1\right)\right\} + k_z^2 z\delta_d\left\{\log p\frac{1}{(\delta_e p+z\delta_d)}\right. \\
&\left.-\frac{1}{z\delta_d}\{\log p-\log(\delta_e p+z\delta_d)+\log(\delta_e+z\delta_d)\}\right\} + \frac{k_z^2}{M^2}z\alpha Q\delta_d\left\{\delta_e p(\log p-1)\right. \\
&\left.+2A\delta_e\frac{1}{p}(\log p+1)+\frac{1}{2}z\delta_d(\log p)^2+\frac{Az\delta_d}{p^2}\left(\log p+\frac{1}{2}\right)-2A\delta_e-\frac{1}{2}Az\delta_d\right\} + k_z^2\delta_e Az \\
&\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\} - k_z^2\delta_e^2 Az\left\{\frac{2\delta_e}{(z\delta_d)^3}(\log(\delta_e p+z\delta_d)-\log p)-\frac{1}{(z\delta_d)^2}\right. \\
&\left.\left(\frac{1}{p}-1\right)-\frac{\delta_e}{(z\delta_d)^2}\frac{1}{(\delta_e p+z\delta_d)}-\frac{2\delta_e}{(z\delta_d)^3}\log(\delta_e+z\delta_d)\frac{\delta_e}{(z\delta_d)^2}\frac{1}{(\delta_e+z\delta_d)}\right\} + \frac{k_z^2}{M^2}
\end{aligned}$$

$$
\begin{aligned}
&Az\alpha Q\delta_e\left\{\delta_e\left(2-p-\frac{1}{p}\right)+2A\delta_e\left(\frac{1}{3p^3}-\frac{1}{p}+\frac{2}{3}\right)-z\delta_d\left(\frac{1}{2p^2}-\frac{1}{2}\right)-z\delta_d\log p\right.\\
&\left.+Az\delta_d\left(\frac{1}{2p^4}-\frac{1}{p^2}+\frac{1}{2}\right)\right\}+\frac{M^4}{2k_z^2\alpha Q}\left\{\frac{1}{(\delta_e p+z\delta_d)^3}-\frac{1}{(\delta_e+z\delta_d)^3}\right\}+\frac{M^4}{k_z^2}\\
&\left\{\log p+\frac{A}{p^2}-A\right\}-\frac{M^4}{2k_z^2\alpha Q}\left\{\frac{p}{(\delta_e p+z\delta_d)^2}+\frac{1}{\delta_e}\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)^2}\right.\\
&\left.-\frac{1}{\delta_e}\frac{1}{(\delta_e+z\delta_d)}\right\}-\frac{k_z^2}{M^2}\left\{p+\frac{2A}{p}-1-2A\right\}-\delta_e M^2\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\}\\
&+\frac{p\delta_e M^2}{(\delta_e p+z\delta_d)}-\frac{\delta_e M^2}{(\delta_e+z\delta_d)}-M^2\{\log(\delta_e p+z\delta_d)-\log(\delta_e+z\delta_d)\}-\alpha Q\delta_e\left\{\delta_e(p-1)\right.\\
&+2A\delta_e\left(\frac{1}{p}-1\right)-\frac{\delta_e}{2}(p^2-1)+2A\delta_e\log p+z\delta_d(\log p+1)+Az\delta_d\left(\frac{1}{p^2}-1\right)+2Az\delta_d\\
&\left.\left(1-\frac{1}{p}\right)-z\delta_d p\right\}-2A\delta_e M^2\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\}-2A\delta_e^2M^2\left\{\frac{1}{(z\delta_d)^2}\{\log p-\right.\\
&\left.\log(\delta_e p+z\delta_d)\}+\frac{1}{(z\delta_d)}\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\}+\frac{1}{(z\delta_d)^2}\log(\delta_e+z\delta_d)\right\}+2A\alpha Q\delta_e\\
&\left\{A\delta_e\left(\frac{1}{p^2}-1\right)\delta_e(\log p-p-A)-2A\delta_e\left(\frac{1}{p}-1\right)-z\delta_d\log p+2Az\delta_d\left(\frac{1}{3p^3}-\frac{1}{3}\right)\right.\\
&\left.-Az\delta_d\left(\frac{1}{p^2}-1\right)\right\}+z\delta_d M^2\left\{\log p\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{z\delta_d}(\log p-\log(\delta_e p+z\delta_d)\right.\\
&\left.+\log(\delta_e+z\delta_d))\right\}+\alpha Qz\delta_d\left\{\delta_e p(\log p-1)+2A\delta_e\left(\frac{\log p}{p}+\frac{1}{p}\right)+z\delta_d\frac{(\log p)^2}{p}\right.\\
&\left.+2Az\delta_d\left(\frac{1}{4p^2}+\frac{\log p}{2p^2}\right)-2A\delta_e-\frac{1}{2}Az\delta_d\right\}+Az\delta_e^2M^2\left\{\left\{\frac{1}{\left(p^2+\frac{\delta_d}{\delta_e}zp\right)}\frac{1}{\left(2p+z\frac{\delta_d}{\delta_e}\right)}\right\}\right.\\
&\left.-\left\{\frac{1}{\left(1+\frac{\delta_d}{\delta_e}z\right)}\frac{1}{\left(2+z\frac{\delta_d}{\delta_e}\right)}\right\}\right\}-Az\delta_e M^2\left\{\frac{1}{(\delta_e p+z\delta_d)}-\frac{1}{(\delta_e+z\delta_d)}\right\}+Az\delta_e\alpha Q\\
&\left\{\frac{2A\delta_e}{3p^3}-\frac{\delta_e}{p}-\delta_e p-2A\delta_e\left(\frac{1}{p}-1\right)-\frac{z\delta_d}{2}\left(\frac{1}{p^2}-1\right)+\frac{Az\delta_d}{2}\left(\frac{1}{p^4}+1\right)\right.\\
&\left.\left.-z\delta_d\log p-\frac{Az\delta_d}{p^2}-\frac{2}{3}A\delta_e+2\delta_e\right\}\right] \qquad (24)
\end{aligned}
$$

as the Sagdeev's potential. The boundary condition used in deriving equation (23) is given as $\frac{dp}{d\xi}=0$ at $p=1$ as $\xi\to\infty$. Equation (23) can be interpreted as an energy integral of an oscillatory particle of an unit mass with velocity $\frac{dp}{d\xi}$ and position p in a potential well $K(p)$. That is the above equation can be considered as a motion of a particle whose pseudoposition is p at pseudotime ξ with pseudovelocity $\frac{dp}{d\xi}$ in a pseudopotential well $K(p)$. That is why Sagdeev's potential is called pseudopotential.

4. Existence Conditions for Solitary Waves

The conditions for solitary waves are

(i) $K(p) = 0$ at $p = 0$ and $p = p_m$

(ii) $\left.\frac{\partial K(p)}{\partial p}\right|_{p=0} = 0$ and $\left.\frac{\partial^2 K(p)}{\partial p^2}\right|_{p=0} < 0$

(iii) $K(p) < 0$ for p lying between 0 and p_m. If p_m is positive then the solitary wave is called compressive solitary wave and if p_m is negative then the solitary wave is called a rarefactive solitary wave. p_m is called the amplitude of solitary wave.

5. Results and Discussions

The study of nonlinear plasma dynamics is of crucial importance for the understanding of many astrophysical/geophysical phenomena and satellite observations as well as for industrial applications such as controlled nuclear fusion devices, coating of surfaces etc. Some recent observational studies on data from Freja satellite showed that the low-frequency auroral electromagnetic fluctuations resulting in strong electric spikes, which can be interpreted as density pulses exhibiting kinetic Recently, Woo et al. [30] studied the double layer formation of an obliquely propagating solitary Alfvén wave by the electron viscosity. wave field characteristics. Due to the strong nonlinearity of the dispersive plasma medium, these may emerge out in the form of solitary kinetic Alfvén wave, double layers, etc. Satellite experiments have carried and continue to carry out extensive measurements which have furnished the scientific community with a wealth of data (velocity, magnetic field, plasma density etc. or also particle distribution functions) at a resolution which is not available in any Earth laboratory.

We have numerically analyzed the effect of various sets of plasma parameters on the structures of kinetic Alfvén waves and have investigated the formation of arbitrary amplitude compressive solitary waves. The existence of large amplitude solitary waves can be determined by plotting $K(p)$ against density p for different values of parameters. In Figure 1, the Sagdeev potential curves for different values of Mach number M have been plotted for subsonic positive solitons. It can be seen that the amplitude of compressive potential profile increases with the increase of the value of Mach number M. It is clear that the compressive potential pulses become spiky with the increase of Mach number M. So it is clear that the compressive solitons, depending on the plasma parameters can propagate in subsonic regimes in our present plasma model. We have varied $M \geq 0.021$ and noticed that the Sagdeev potential depth means the width of the pulse increases and vice versa.

Figure 2 shows that the pseudopotential for different two values of β. We observed that only compressive subsonic solitons exist and amplitudes are found to be decreasing as β increases. We have found the amplitude to decrease in such a way that at some values of β, the soliton condition is not satisfied. It may be clear that the compressive solitons cease to exist when the parameter β crosses a certain limit ($\beta \geq 0.03$), which of course depends on the other parameter. Compressive solitary waves are also found to exist for some parameter sets and for z less than 1. Figure 3 shows the formation of compressive subsonic soliton for three different values of δ_e . It is seen that amplitude increases as δ_e decreases (i.e., as dust contamination increases). Also, it is noticed that an increase of the depth of the Sagdeev potential makes the solitary pulse narrower. Also the upper limit of δ_e is found to be less than 0.34. At $\delta_e = 0.34$, the soliton condition is not satisfied. So, it is clear that the compressive solitons, depending on the parameter δ_e can propagate in subsonic regimes in our plasma model. In figure 4, the effect of the direction cosine k_z on the formation of subsonic compressive soliton has been investigated. It is found that the amplitude of the compressive pseudopotential increases with the increase of direction cosine k_z. So, it is observed that amplitude of the nonlinear structures depends on the external magnetic field k_z.

6. Conclusions

Though the basic development has already been started long ago to study the nonlinear phenomena plasma-acoustic wave, but space and astroplasmas need further study for new formulations. The present pursuit of nonlinear phenomena in plasma is an attempt through evolutionary sequences leading to a present - day standard model, to achieve better understanding of various interactions of macroscopic particles in the medium and will definitely add good results to the field of plasma studies. In our present work, the set of basic equations governing the ions, electrons, dust and Maxwell's equation have been reduced to a single equation known as the Sagdeev Potential (SP) equation. An exact analytical expression for the energy integral is obtained. Furthermore, numerical calculations reveal that the present plasma system supports compressive subsonic solitons. The dependence of the solitary excitation characteristics on the different plasma parameters has been investigated. Our present theoretical studies could be of interest for explaining some of the recent satellite observations (e.g., Freja, Cassini) in space and astrophysical scenarios.

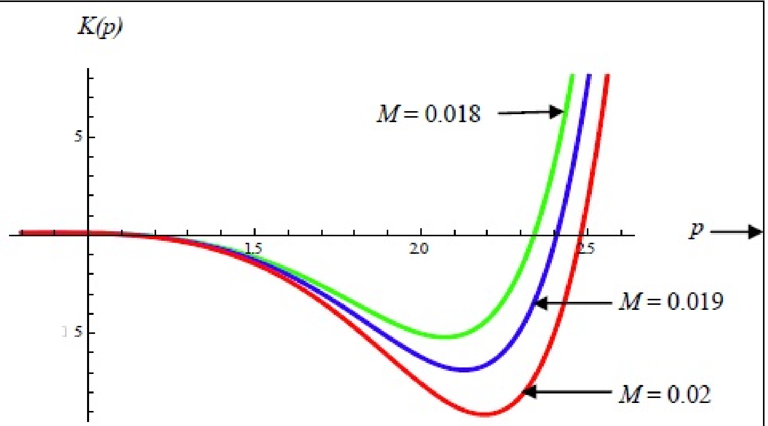

Fig.1. Exact solution of Sagdeev potential $K(p)$is plotted against density p for three different values of $M = 0.018,\ 0.019,\ 0.02$ and $z = 6.5$ (>1), $\delta_e = 0.009$, $Q = 0.00054$ (fixed), $k_z = 0.0002$, $\beta = 0.001$.

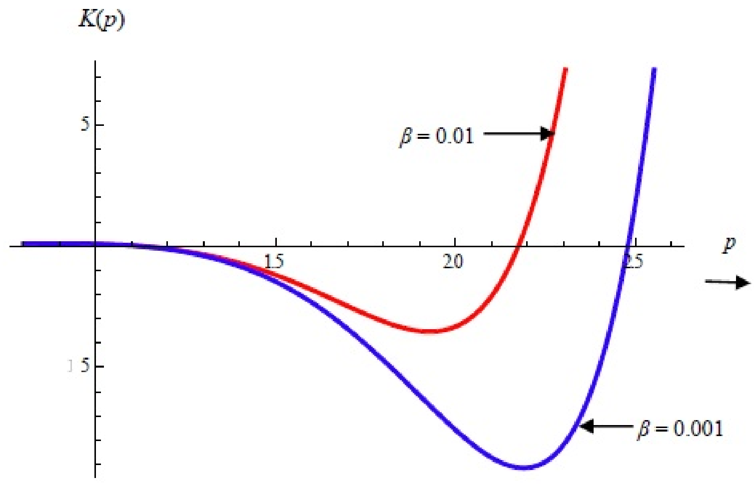

Fig.2. Sagdeev potential $K(p)$ is plotted against density p for $\beta = 0.01$ and 0.001 and z = 6.5 (>1), $M = 0.02$, $\delta_e = 0.009$, $Q = 0.00054$ (fixed), $k_z = 0.0002$.

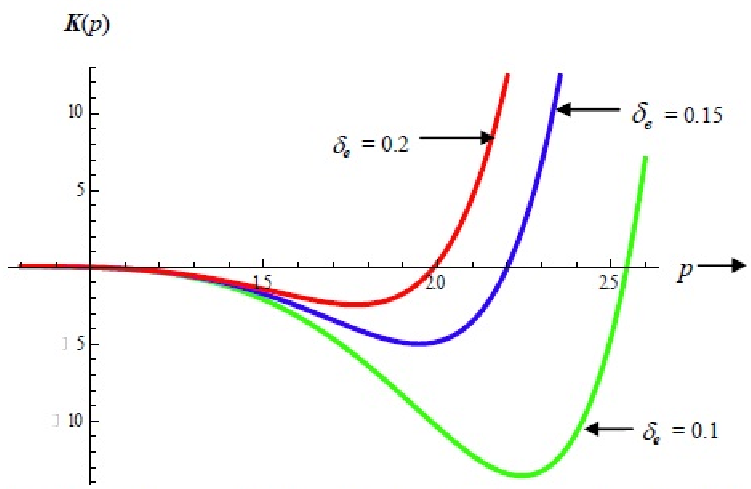

Fig.3. Sagdeev potential $K(p)$ is plotted against density p for $\delta_e = 0.1$, 0.15, 0.2 and $z = 0.7$ (<1), $M = 0.02$, $Q = 0.00054$ (fixed), $k_z = 0.0002$, $\beta = 0.001$.

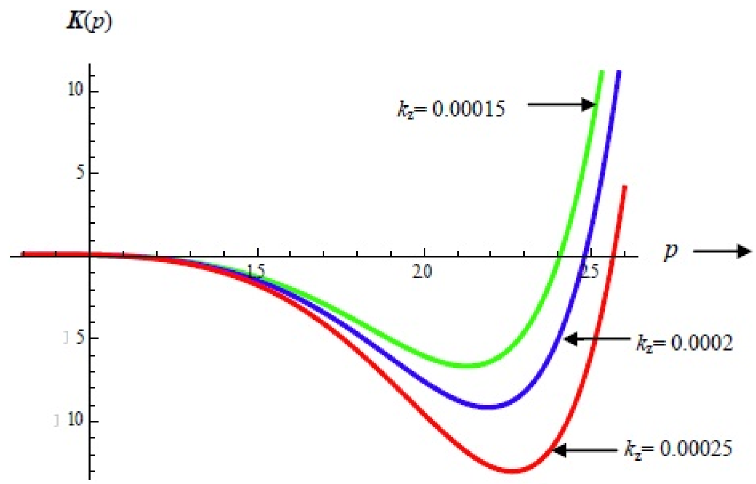

Fig.4. Sagdeev potential $K(p)$ is plotted against density p for $\beta = 0.001$, $z = 6.5$ (>1), $M = 0.02$, δ_e=0.009, $Q = 0.00054$ (fixed), k_z= 0.00015, 0.0002 and 0.00025.

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