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INTUITIONISTIC FUZZY FILTERS ON $\beta-$ALGEBRAS

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Abstract

In this paper, we define the notion of an Intuitionistic fuzzy filter on $\beta-$algebras and investigate some of their properties and results.

1. Introduction

In 2002, J. Neggers and H. S. Kim [3], introduced a new notion of algebra: namely $\beta-$algebra.The theory of fuzzy sets proposed by L. A. Zadeh [7] in 1965 is generalized in 1986 by K. T. Atanassov [1] an Intuitionistic fuzzy sets. The study of fuzzy algebraic structures was initiated with the concept of the fuzzy subgroup by A. Rosenfeld [5]. Then many researchers have been engaged in extending the concepts and results of abstract algebra. The notion of filters was introduced by Henri Cartan in 1937. In 1991, C. S. Hoo [2] introduced the concept of the filters in BCI-algebras. Also in 2013,

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A. Rezaei and A. Bourmand [4] introduced the notion of generalized fuzzy filters of BE-algebras. In our earlier papers [6], we introduced the notions of $\beta-$filter, Fuzzy $\beta-$filter in $\beta-$ algebras. In this paper, we discuss the concept of Intuitionistic fuzzy filters in $\beta-$algebras and prove some of their properties and theorems.

2. Preliminares

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1 : A $\beta-$algebra is a non-empty set X with a constant 0 and two binary operations $+\ -$ satisfying the following axioms:

1. $x - 0 = x$
2. $(0 - x) + x = 0$
3. $(x - y) - z = x - (z + y)\ \forall\ x, y, z\ \in\ X.$

Definition 2.2 : A filter of X, is a nonempty subset S such that $x \in S$ and $y \in S \implies x \triangle y \in S$, where $x \triangle y = x * (x * y)$ and $0 \notin S$.

Definition 2.3 : Let X and Y be two $\beta-$algebras. A mapping $f : X \to Y$ is said to be a $\beta-$homomorphism, if

$$f(x + y) = f(x) + f(y) and f(x - y) = f(x) - f(y) \text{ for all } x, y \in X.$$

Definition 2.4 : Let X be a $\beta-$algebra and A a $\beta-$subalgebra. A is said to be a $\beta-$filter on X, if for all $x, y \in A$, $x \triangle y = x + (x + y)$and $x \bigtriangledown y = x - (x - y) \in A$.

Definition 2.5 : Let X be a $\beta-$algebra and A a fuzzy $\beta-$subalgebra. A is said to be a fuzzy $\beta-$filter on X, if it satisfies for all $x, y \in A$

1. $\mu_A(x \triangle y) \geq min\{\mu_A(x), \mu_A(x + y)\}$ and
   $\mu_A(x \bigtriangledown y) \geq min\{\mu_A(x), \mu_A(x - y)\}$
2. $\mu_A(y) \geq \mu_A(x)$ if $x \leq y$.

3. Intuitionistic Fuzzy $\beta-$Filter

In this section, we introduce the notion of Intuitionistic fuzzy $\beta-$filter on a $\beta-$algebra. We begin with the definition.

Definition 3.1 : Let X be a $\beta-$algebra and let A be an Intuitionistic fuzzy $\beta-$subalgebra. A is said to be an Intuitionistic fuzzy $\beta-$filter on X, if it satisfies for all $x, y \in A$,

1. $\mu_A(x \triangle y) \geq min\,\{\mu_A(x), \mu_A(x+y)\}$ and
   $\nu_A(x \triangle y) \leq max\,\{\nu_A(x), \nu_A(x+y)\}$
2. $\mu_A(x \bigtriangledown y) \geq min\,\{\mu_A(x), \mu_A(x-y)\}$ and
   $\nu_A(x \bigtriangledown y) \leq max\,\{\nu_A(x), \nu_A(x-y)\}$
3. $\mu_A(y) \geq \mu_A(x)$ and $\nu_A(y) \leq \nu_A(x)$, if $x \leq y$.

Example 3.1 : Let $X = \{0, 1, 2, 3\}$ be a $\beta-$algebra with constant 0 and two binary operations + and - defined on X with the Cayley's table

$+$	0	1	2	3
0	0	0	0	0
1	1	0	3	1
2	2	0	2	3
3	3	1	3	3

$-$	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, $A = \{2, 3\}$ is $\beta-$fliter and A is an Intuitionistic fuzzy $\beta-$subalgbera, defined by :

$$\mu_A(x) = \begin{cases} 0.4, \ if \ x = 2 \\ 0.5, \ if \ x = 3 \end{cases} \quad and \quad \nu_A(x) = \begin{cases} 0.3, \ if \ x = 2 \\ 0.2, \ if \ x = 3 \end{cases}$$

One can observe that A is an Intuitionistic fuzzy $\beta-$filter on X.

Example 3.2 : Let $X = \{0, 1, 2, 3\}$ be a $\beta-$algebra with constant 0 and two binary operations + and - defined on X with the Cayley's table

$+$	0	1	2	3
0	0	0	0	0
1	1	1	1	3
2	0	1	2	3
3	3	1	2	3

$-$	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, $A = \{1, 2, 3\}$ is $\beta-$fliter and A is an Intuitionistic fuzzy $\beta-$subalgbera, defined by:

$$\mu_A(x) = \begin{cases} 0.7, \ if \ x = 1, 2 \\ 0.8, \ if \ x = 3 \end{cases} \quad and \quad \nu_A(x) = \begin{cases} 0.3, \ if \ x = 1, 2 \\ 0.2, \ if \ x = 3 \end{cases}$$

Then we can observe that A is not an Intuitionistic fuzzy $\beta-$filter on X.

Lemma 3.1 : If A and Bbe an Intuitionistic fuzzy $\beta-$filters of X, then $A \cap B$ is also an Intuitionistic fuzzy $\beta-$filter of X.

Proof:

$$\begin{aligned}
\mu_{(A\cap B)}(x \bigtriangleup y) &= min\left\{\mu_A(x \bigtriangleup y), \mu_B(x \bigtriangleup y)\right\} \\
&\geq min\left\{min\left\{\mu_A(x), \mu_A(x+y)\right\}, min\left\{\mu_B(x), \mu_B(x+y)\right\}\right\} \\
&\geq min\left\{min\left\{\mu_A(x), \mu_B(x)\right\}, min\left\{\mu_A(x+y), \mu_B(x+y)\right\}\right\} \\
&= min\left\{\mu_{A\cap B}(x), \mu_{A\cap B}(x+y)\right\}
\end{aligned}$$

Similarly, $\mu_{(A\cap B)}(x \bigtriangledown y) \geq min\left\{\mu_{(A\cap B)}(x), \mu_{(A\cap B)}(x-y)\right\}$
Also, we can prove that,
$\nu_{(A\cap B)}(x \bigtriangleup y) \geq max\left\{\nu_{A\cap B}(x), \nu_{A\cap B}(x+y)\right\}$
and $\nu_{(A\cap B)}(x \bigtriangledown y) \leq max\left\{\nu_{(A\cap B)}(x), \nu_{(A\cap B)}(x-y)\right\}$
Hence $A \cap B$ is also a fuzzy $\beta-$filter of X.

Lemma 3.2 : Every Intuitionistic fuzzy $\beta-$filter is also an Intuitionistic fuzzy$\beta-$subalgebra. The proof directly follows from our definition of Intuitionistic fuzzy $\beta-$ filter.

The following example shows that the converse part of the above lemma need not be true, in general

Example 3.3 : Let $X = \{0, 1, 2, 3\}$ be a $\beta-$algebra with constant 0 and two binary operations + and - defined on X with the Cayley's table

+	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	1	1	2	0
3	3	3	1	1

−	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, $A = \{1, 3\}$ is $\beta-$ filter and A is an Intuitionistic fuzzy $\beta-$subalgbera, defined by $\mu_A(x) = \begin{cases} 0.4, & if\ x = 1 \\ 0.3, & if\ x = 3 \end{cases}$ and $\nu_A(x) = \begin{cases} 0.3, & if\ x = 1 \\ 0.2, & if\ x = 3 \end{cases}$ One can observe that A is not an intuitionistic fuzzy $\beta-$filter on X, $\because \mu_A(2) \geq \mu_A(1)$, if $1 \geq 2 \implies 0.3 \ngeq 0.4$

Theorem 3.1 : If A is an Intuitionitic fuzzy filter of X, then $\mu_A(x \bigtriangleup y) \geq \mu_A(x)$ and $\nu_A(x \bigtriangledown y) \leq \nu_A(x)$, where $x \leq y$.

Proof : Assume that A is an Intuitionistic fuzzy filter of X.

Let $x, y \in X$. Then we get,

$$\begin{aligned}
\mu_A(x \triangle y) &= \mu_A(x + (x + y)) \\
&\geq min\{\mu_A(x), \mu_A(x + y)\} \\
&= min\{\mu_A(x), min\{\mu_A(x), \mu_A(y)\}\} \quad \textit{using the lemma } 3.1 \\
&= min\{\mu_A(x), \mu_A(x)\} \quad \textit{Since } x \leq y \implies \mu_A(y) \geq \mu_A(x) \\
&= \mu_A(x)
\end{aligned}$$

Similarly,we can prove that, $\nu_A(x \bigtriangledown y) \leq \nu_A(x)$.

Definition 3.2 : Let A be an Intuitionistic fuzzy $\beta-$ filter in a $\beta-$subalgebra X. For$s, t \in [0, 1]$, the set

$\mu_{A_{s,t}} = \{x \in X \mid \mu_A(x) \geq s \textit{ and } \nu_A(x) \leq t\}$ is called a level of filter A in X.

Theorem 3.2 : An intuitionistic fuzzy subset A of $\beta-$algebra X is an intuitionistic fuzzy $\beta-$filter iff for any $s, t \in [0, 1]$ the s,t-level subset $A_{s,t} = \{x \in X \mid \mu_A(x) \geq s \textit{ and } \nu_A(x) \leq t\}$ is either a $\beta-$filter or $A_{s,t} \neq \phi$.

Proof : Assume that the level subset of A in X, $A_{s,t} \neq \phi$.

Then $x, y \in A_{s,t}$, $\mu_A(x) \geq s$ and $\mu_A(y) \geq s$.

Now,

$$\begin{aligned}
\mu_A(x \triangle y) &= \mu_A(x + (x + y)) \\
&\geq min\{\mu_A(x), \mu_A(x + y)\} \\
&\geq min\{\mu_A(x), min\{\mu_A(x), \mu_A(y)\}\} \\
&= s
\end{aligned}$$

Similarly, we can prove that, $\mu_A(x \bigtriangledown y) \geq s$

Analogously, one can prove that,$\nu_A(x \bigtriangledown y) \in A_{s,t}$ and $\nu_A(x \triangle y) \in A_{s,t}$.

So $x \triangle y \in A_{s,t}$ and $x \bigtriangledown y \in A_{s,t}$. Hence $A_{s,t}$ is a $\beta-$ filter of X.

Conversely, assume that $A_{s,t}$ is a $\beta-$ filter of X.

$\implies$ For all $x, y \in X$, $x \triangle y$ and $x \bigtriangledown y \in A_{s,t}$.

$\implies$ $\mu_A(x \triangle y) \geq s$ and $\mu_A(x \bigtriangledown y) \geq s$.

Take $s = min\{\mu_A(x), \mu_A(x + y)\}$for any $x, y \in X$.

$\mu_A(x \triangle y) = \mu_A(x + (x + y) \geq s = min\{\mu_A(x), \mu_A(x + y)\}$

Similarly, we can prove that, $\mu_A(x \bigtriangledown y) \geq min\{\mu_A(x), \mu_A(x - y)\}$

Analogously, one can prove that, $\nu_A(x \bigtriangledown y) \leq max \{\nu_A(x), \nu_A(x-y)\}$ and $\nu_A(x \bigtriangleup y) \leq max \{\nu_A(x), \nu_A(x+y)\}$. Thus proving that A is an intuitionistic fuzzy $\beta-$ filter.

Theorem 3.3 : Let f be an onto $\beta-$ algebra homomorphism from X to Y. If B is an intuitionistic fuzzy $\beta-$filter of Y, then $f^{-1}(B)$ is also an intuitionistic fuzzy $\beta-$filter on X.

Proof : Let B be an intutionistic fuzzy $\beta-$filter of Y.
For $x, y \in X$, then

$$\begin{aligned} f^{-1}(\mu_B(x \bigtriangleup y)) &= f^{-1}(\mu_B(x+(x+y))) \\ &= \mu_B(f(x+(x+y))) \\ &= \mu_B(f(x)+f(x+y)) \\ &\geq min \{\mu_B(f(x)), \mu_B(f(x+y))\} \\ &= min \left\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(x+y))\right\} \end{aligned}$$

Also we can prove that,
$f^{-1}(\mu_B(x \bigtriangledown y)) \geq min \left\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(x-y))\right\}$.
Similarly, we can prove that,
$f^{-1}(\nu_B(x \bigtriangleup y)) \leq max \left\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(x+y))\right\}$
and $f^{-1}(\nu_B(x \bigtriangledown y)) \leq max \left\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(x-y))\right\}$.
Let $x, y \in X$ be such that $x \geq y$.
Since B is an intuitionistic fuzzy $\beta-$ filter, we have $\mu_B(f(y)) \geq \mu_B(f(x)) = f^{-1}(\mu_B(x))$ such that $f^{-1}(\mu_B(y)) \geq f^{-1}(\mu_B(x))$ and $\nu_B(f(y)) \leq \nu_B(f(x)) = f^{-1}(\nu_B(x))$ such that $f^{-1}(\nu_B(y)) \leq f^{-1}(\nu_B(x))$. Thus we can conclude that $f^{-1}(B)$ is an intuitionistic fuzzy $\beta-$filter on X.

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