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ON PRIME AND SEMI PRIME NEAR RINGS WITH DERIVATIONS

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Abstract

Let N be a semi prime right rings, A a subset of N such that $0 \in A$ and $AN \subseteq A$ and d be a derivation on N. The purpose of this paper is to show that if d acts as a homomorphism on A or anti homomorphism on A then $d(A) = 0$.

1. Introduction

Throughout this paper N will be right near ring. A derivation on N is defined to be an additive endomorphism satisfying the product rule $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$. According to Bell and Mason 1 a near ring N is said to be prime if $xNy = 0$ for all $x, y \in N \Rightarrow x = 0$ or $y = 0$ and N is said to be semi prime if $xNx = 0 \Rightarrow x = 0$ for all $x \in N$. Let S be a nonempty subset of N and d be a derivation on N. If $d(xy) = d(x)d(y)$ or $d(xy) = d(y)d(x)$ for all $x, y \in S$ then, d is said to act as an homomorphism or anti homomorphism on S respetively.

Bell and Kappe 3 proved that if d is derivation of semiprime ring R which is either an endomorphism or anti endomorphism, then d is $= 0$. They also showed that if d is derivation of prime ring R which acts as a homomorphism or anti homomorphism on I, where I is non zero right ideal, then $d = 0$ on R.

2. Results

The aim of this paper is to prove the above conclusions holds for near rings.

Theorem : Let N be a semi prime right near ring, and d a derivation on N. Let A be a subset of N such that $0 \in A$ and $AN \subset A$. If d acts as a homomorphism on A or as an anti homomorphism on A then $dA = 0$.

In order to prove theorem we need the following lemmas.

Lemma 1 : If N is a right near ring and d is a derivation on N, then $cyd(x)d(y)x = cyd(x) + cd(y)x$ for all $x, y, c \in N$.

Lemma 2 : Let N be a right near ring and d a derivation on N and A a multiplicative sub semi group of N which contains 0. If d acts as an anti homomorphism on A then $a_0 = 0, \ \forall \ a \in A$.

Proof : Since $a_0 = 0, \ \forall a \in A$ and d acts as an anti homomorphism on A then we have $d(a) = 0, \ \forall \ a \in A \ a_0 + d(a)0 = 0, \ forall a \in A$. Thus we get $a_0 = 0, \ \forall a \in A$.

Lemma 3 : Let N be a right near ring and A a multiplicative sub semi group of N then

(i) If d acts as a homomorphism on A then

$$d(y)xd(y) = yxd(y) = d(y)xy, \ \forall \ x, y \in A. \tag{3.1}$$

(ii) If d acts as an anti homomorphism on A then

$$d(y)xd(y) = d(y)yx = xyd(y), \ \forall \ x, y \in A. \tag{3.2}$$

Proof : (i) Let d acts as an homomorphism on A. Then

$$d(xy) = xd(y) + d(y) = d(x)d(y), \ \forall \ x, y \in A. \tag{3.3}$$

Now taking yx instead of x in 3.3 We have

$$yxd(y) + d(yx)y = d(yx)d(y) = d(y)d(xy) \ x, y \in A. \tag{3.4}$$

Now by Lemma 1 ,

$$d(y)d(xy) = d(y)xd(y) + d(y)d(x)y = d(y)xd(y)d(yx)y.$$

Using this relation in (3.4), we obtain $yxd(y) = d(y)xd(y), \;\; \forall\, x, y \in A$. Similarly taking yx instead of y in (3.3), we can prove the relation

$$d(y)xd(y) = d(y)xy, \;\; \forall\, x, y \in A.$$

(ii) Let d acts as an anti homomorphism on A, we have,

$$d(xy) = xd(y) + d(y) = d(y)d(x), \;\; \forall \;\; x, y \in A. \tag{3.5}$$

Putting xy for y in (3.5), we have

$$\begin{aligned} xd(xy) + d(x)xy &= d(xy)d(x) \\ &= xd(y) + d(x)d(x)yd(x) \\ &= xd(xy) + d(x)yd(x), \;\; \forall \;\; x, y \in A. \end{aligned}$$

From this relation we have $d(x)xy = d(x)yd(x) = 0, \;\; \forall \;\; x, y \in A$.
Similarly by taking xy instead of x in (3.5), one can prove that,

$$d(y)xd(x) = xyd(y), \;\; \forall \;\; x, y \in A.$$

Proof of the Theorem: Case (i). Let d acts an homomorphism on A. Then by Lemma 3(i), we have,

$$d(y)xd(y) = d(y)xy, \;\; \forall \;\; x, y \in A. \tag{3.6}$$

Right multiplying (3.6) by $d(z)$, where $z \in A$ and using the hypothesis d acts as an homomorphism on A together with Lemma 1. We obtain $d(y)xd(y)z = 0$, forall, $x, y, z \in A$. Taking xr instead of x, where $r \in N$, we have $d(y)xrd(y)z = 0, \;\; \forall x, y, z \in A$ and $r \in N$.
Hence $d(y)xd(y)x = 0$ for all $x, y \in A$ and by semiprimeness

$$d(y)x = 0, \;\; \forall \;\; x, y \in A. \tag{3.7}$$

Substuting yr for y (3.7), where $r \in N$,

$$\Rightarrow, \; yd(r)x + d(y)rx = 0, \;\; \forall \;\; x, y \in A, r \in N. \tag{3.8}$$

Now left mulyiply (3.8) by $d(z)$, where $z \in A$, we have

$$d(z)yd(r)x + d(z)d(y)rx = 0.$$

According to the relation (3.7) reduces to $d(zy)rx = 0$ for all $x, y, z \in A$ and $r \in N$. By semiprimeness, we get

$$zd(y) = 0 = zrd(y) \;\; \forall \;\; y, z \in A \;\; \text{and} \;\; r \in N. \tag{3.9}$$

Combining (3.7), (3.9) shows that $d(yz)0, \;\; \forall \;\; y, z \in A$.
In particular $d(xrx) = 0, \;\; \forall \;\; x, y \in A, r \in N$ and since d acts as homomorphism on A, we have

$$d(xr)d(x) = 0 = xd(r)d(x) + d(x)rd(x), \;\; \forall \;\; x, y \in A, \; r \in N.$$

In view of (3.9), this gives us $d(x)Nd(x) = 0, \;\; \forall \;\; x, y \in A$.
Case (ii) : Suppose d acts as an antihomomorphism on A. Note that $a_0 = 0, \;\; \forall \;\; a \in A$, by Lemma 2. Now according to Lemma 3(ii),

$$d(y)xd(y) = xyd(y), \;\; \forall \;\; x, y \in A. \tag{3.10}$$

$$d(y)xd(y) = d(y)yx, \;\; \forall \;\; x, y \in A. \tag{3.11}$$

Replacing x by $xd(y)$ in (3.10) and using Lemma 1, we have

$$d(y)xyd(y) + d(y)xd(y)y = xd(y)yd(y), \;\; \forall \;\; x, y \in A. \tag{3.12}$$

Subatituting xy for x in (3.10), we have,

$$d(y)xyd(y) = xy^2d(y), \;\; \forall \;\; x, y \in A. \tag{3.13}$$

Right multiplying (2.10) by y, we get

$$d(y)xd(y)y = xyd(y)y, \;\; \forall \;\; x, y \in A. \tag{3.14}$$

Replacing x by y in (2.10), we have $d(y)yd(y) = y^2d(y)$ and left multiplying this relation by x, we get

$$xd(y)yd(y) = xy^2d(y), \;\; \forall \;\; x, y \in A. \tag{3.15}$$

Using (3.13), (3.14), (3.15) and (3.12),one can easily obtain $xyd(y)y = 0 \ \ \forall \ \ x, y \in A$. Hence $yryd(y)y = 0$ and $yd(y)yryd(y)y = yd(y)0 = 0, \forall \ \ y \in A, r \in N$ and by semiprimeness

$$yd(y)y = 0, \ \ \forall \ \ y \in A. \tag{3.16}$$

According to (3.14), we get $d(y)xd(y)y = 0, \ \ \forall \ \ x, y \in A$. Using this relation in (3.11), we get

$$d(y)yxy0, \ \ \forall \ \ x, y \in A. \tag{3.17}$$

Replacing x by $xd(y)$ in (3.17), we have $d(y)yxd(y)y = 0 = d(y)yrxd(y)yx, \ \ \forall \ \ x, y \in A$ and $r \in N$. Hence

$$d(y)yx = 0, \ \ \forall \ \ x, y \in A. \tag{3.18}$$

Using (3.18) and (3.11), one can obtain $d(y)xd(y) = 0 = d(y)xrd(y)x, \ \ \forall \ \ x, y \in A, r \in N$. Hence ,

$$d(y)x = 0, \ \ \forall \ \ x, y \in A. \tag{3.19}$$

Therefore

$$x(z)d(yx)x = 0$$

$$xd(z)(yd(n) + d(y)n)x = 0$$

$$xd(z)yd(n)x + xd(z)d(y)nx = 0, \ \ \forall \ \ x, y, z \in A, \ n \in N.$$

In view of (3.19), this gives $xd(z)d(y)nx = 0 = xd(z)d(y)nxd(z)d(y)$.
Hence $xd(z)d(y) = 0, \ \ \forall \ \ x, y, z \in A$. Since d acts an an anti homomorphism on A, we have $xd(yz) = 0, \ \ \forall \ \ x, y, z \in A$. So that $xyd(z) + xd(y)z = 0, \ \ \forall \ \ x, y, z \in A$. By (3.19) we now get $xyd(z) = 0 = xd(z)ryd(z)$. By taking x instead of y we get $xd(z) = \ \ \forall \ \ x, y, z \in A$. Recall (3.19) we have now $d(xy) = 0$ and $d(xxr) = 0, \ \ \forall \ \ x, \in A$ and $r \in N$. Thus $d(xr)d(x) = 0$. Hence the proof.
We have some consequences of the theorem.
Corollary : Let N is semi prime right near ring and d a derivation of N. If d acts as an homomorphism on N or as an anti homomorphism on N then $d = 0$.

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